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Improved wide-field emmetropic human eye model based on ocular wavefront measurements and geometry-independent gradient index lens

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<https://drive.google.com/drive/folders/1KwWwVLihpycCFUc1hXZmb6J9tI9HGPx0?usp=sharing>

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There is a need to better understand the peripheral optics of the human eye and its correction. Current eye models have some limitations to accurately predict the wavefront errors for the emmetropic eye over a wide-field. The aim here was to develop an anatomically correct optical model of the human eye that closely reproduces the wavefront of an average Caucasian-only emmetropic eye across a wide visual field. Using an optical design program, a schematic eye was constructed based on ocular wavefront measurements of the right eyes of thirty healthy young emmetropic individuals over a wide visual field (from 40° nasal to 40° temporal and up to 20° inferior field). Anatomical parameters, asymmetries and dispersion properties of the eye's different optical components were taken into account. A geometry-independent gradient index (GRIN) model was employed to better represent the crystalline lens. The RMS wavefront error, wavefront shapes, dominant Zernike coefficients, nasal-temporal asymmetries and dispersion properties of the developed schematic eye closely matched the corresponding measured values across the visual field. The developed model can help in the design of wide-field ophthalmic instruments and is useful in the study and simulations of the peripheral optics of the human eye.

OCIS codes: (330.4060) Vision modeling; (330.7326) Visual optics, modeling; (170.4460) Ophthalmic optics and devices; (080.2740) Geometric optical design;

1. INTRODUCTION

There has been a steady effort to improve the understanding of, and model the optical performance of the human eye [1–19]. Schematic eyes are helpful for teaching visual optics, understanding vision and designing ophthalmic lenses and optical instruments for eye examination. Moreover, such models are also useful for refractive surgical procedures [20] and intra-ocular lens implants designs and calculations [21]. The expected rise in myopia prevalence [22] and age-related macular degeneration [23] has increased the need to understand peripheral vision and its correction over a wide field of view, for how to understand and control myopia progression and to help people with central visual field loss [24–32].

Over the years, many schematic eyes have been developed which reproduce the optical performance of a human eye with variable success [18]. Some famous examples are: Gullstrand's model based on spherical surfaces [2], Lotmar's model with aspherics [3], Blaker's model with accommodation [4], Liou and Brennan's model with gradient index lens [9], Goncharov and Dainty models with gradient-index lens [5, 6], Navarro's models for wide-angle eye [13, 14], including age and accommodation dependence [15], Kooijman's aspherical model [11], Kong's model for the Chinese eye based on anatomical and wavefront measurements [12], Atchison's models for myopic eye (both symmetrical and asymmetrical versions) [8] and Polans' wide-angle model based on a large set of ocular

wavefront measurements [18]. A comprehensive review of the aberration prediction of different eye models and their accuracy with respect to the measured aberration of an average eye has been presented by Polans [18, 33]. As shown in their Figure 2 [18] and Figure 2S [33], most of the well-known eye models over-predict the root-mean-square wavefront error (RMSWFE) at wide angles both in horizontal and vertical eccentricity. In addition, the predicted Point Spread Function (PSF) and the wavefront shapes do not match the measured quantities at wide-viewing angles.

Out of these eye models, the model by Polans [18, 33] gives the best match of the predicted versus measured aberrations (both RMS values, the shape and dominant Zernike terms) over a wide visual field. This model is based on known anatomical parameters of the human eye, a GRIN model for the crystalline lens and measured wavefronts of 101 persons over a $\pm 40^\circ$ horizontal eccentricity and 10 persons over $\pm 25^\circ$ vertical eccentricity for a 7.8 Diopters range of mean spherical refractive error (M) (from -4.6 to +3.2 Diopters). In contrast to some other models which are symmetrical around the optical axis, this model includes a decentered and tilted lens to better reproduce the anatomical and optical asymmetries present in the human eye. The chromatic dispersion and spatial index variation of the GRIN lens are modelled by a specialized surface equation in Zemax [18, 34]. This model typically represent a highly myopic eye with large defocus aberration. A shortcoming in the Polans model is the choice of the GRIN medium ray tracing resolution parameter $\Delta T = 3.562 \text{ mm}$, which is rather high and unrealistic. In a GRIN region, the ray path is constantly changing due to local index gradient. However in ray tracing software Zemax, the ray path is calculated in small steps assuming a constant index between each step, and the parameter ΔT controls the physical distance traced by a ray after which the refraction is applied. The ray tracing accuracy and aberration calculation of the model depends on this particular parameter ΔT . For example, if one chooses a more realistic value of $\Delta T = 0.5 \text{ mm}$ during ray tracing (meaning that the refraction will be applied recursively after each step of 0.5 mm in the GRIN region), the on-axis RMSWFE is nearly doubled in the Polans model. Another serious weakness in the Polans model is the unphysiological spatial profile of the refractive index distribution inside the GRIN lens. In figure 1, we have plotted the GRIN profile based on the modeled equation given in his article in Table 1 [18] and the modeled profile does not represent a realistic crystalline lens, as the crystalline lens has a concentric shell-like index profile [15, 16, 35]. In addition, the GRIN lens equation used in his model does not adapt to geometrical changes of lens shape, and thus it is not valid for optimization, simply because the coefficients are not geometrically invariant. Any changes in the curvatures, conic constants or lens thickness does not influence the spatial index profile and it can yield erroneous refractive index values that may even fall outside the physiological range. Moreover, the positive value of the conic constant of the posterior lens surface (Conic constant = 0.821, see Table 1 [18]) is unrealistic and there is evidence for that this should be negative [5, 8].

A Chinese-only eye model was proposed by Mei-mei Kong [12] based on the on-axis wavefront measurements and known anatomical parameters of the human eye. This model uses specialized written routines in C-language to model the dispersion properties of the GRIN lens in the Zemax program. The line-of-sight (LOS) wavefront prediction of the model is in good agreement with the measurements, however this model does not include the off-axis wavefront information during model construction.

A very good refraction-dependent eye model was developed by Atchison [8] which includes GRIN lens (though not geometry independent), aspheric corneal, lens and retinal surfaces. It has both symmetrical and asymmetrical versions (with tilt and denetration of the lens and the retina). It is a wide-angle model with good match for the lower order aberrations, but no higher order wavefront information was used to fine-tune the model over a wide field.

Another age-dependent wide-angle GRIN lens based model is presented by Goncharov [5]. A nice improvement in this model is that the GRIN lens equations are constructed in such a way that the iso-indicial lines of constant refractive index follow the outer optical surfaces of the lens. The focus here was to match the longitudinal spherical aberration of the model to the measurements. The model is symmetrical with no distinction between the nasal and the temporal field. The same authors developed another narrow-angle GRIN lens eye model [6] using reverse ray tracing, with measured wavefronts over -5deg to $+5\text{deg}$ in both meridians. This eye model is symmetrical around the optical axis, with good match between the measured and the reconstructed wavefront over a small angle. However, in this work, the GRIN model is not geometry independent.

A recent GRIN lens based age and accommodation dependent eye model was developed by Navarro [15]. The GRIN lens spatial profile is geometry independent and follows the outer surface contours with a kink at the anterior and the posterior hemisphere interface. The cornea and the lens are tilted and decentered to faithfully represent the asymmetry in a real eye. In addition, the irregular corneal shape was modeled by including additional Zernike deformation terms to the basic biconic equation. Both corneal surfaces are biconic (with different radii of curvature and conic constants on the two orthogonal meridians). As stated in his paper, this is an on-axis model which has good match of lower and higher order aberrations on-axis, but off-axis wavefront information was not included in the model construction. However, due to the inclusion of eye asymmetries, realistic iso-indicial GRIN profile and biconic surfaces, this model can be thought to be useful over a wide-angle. We have included the wide-field off-axis performance of this eye model in the results below and compared it to our measurements.

As we show in the later part of the article, the abovementioned schematic eyes have limitations in faithfully reconstructing the wavefront errors for the emmetropic eye at the field extremes. In the present work, we propose an alternate generic wide-field emmetropic Caucasian-only schematic eye that faithfully reconstructs wavefronts across the field of view that matches well with the measured wavefronts. The anatomical parameters of the eye, the nasal-temporal asymmetry in the eye performance and chromatic aberrations were accounted for during the model development. The model construction was based on ocular wavefront measurements of thirty healthy young emmetropic individuals over a wide visual field (from 40° nasal to 40° temporal and up to 20° inferior visual field, in steps of 10°). Here, the recently proposed geometry-independent contour-following GRIN lens models by Bahrami [35–37] was implemented in the model for the first time. This was done to make the model realistic and better suited for further optimization (for example to develop personalized eye models). The article is organized as follows: the wavefront measurement procedure and the eye model construction steps are described in the Methods section. The performance of the developed model is verified against the wavefront measurements and other eye models in the Results section, followed by further discussion and a conclusion including some ideas on further work.

Fig. 1. Spatial refractive index profile $n(z, \rho)$ of the Polans GRIN lens model [18]

2. METHODS

A. Ocular wavefront measurements

A high-definition Shack-Hartmann open-view aberrometer (COAS-HD VR, Complete Ophthalmic Analysis System High Definition Vision Research, AMO Wavefront Sciences, Albuquerque, NM) was used to measure monochromatic aberrations along on-axis as well as off-axis. The light source used was a superluminescent diode which emits 840 nm infrared light. The wavefront sensor is based on an 83×62 array of lenslets each being $108 \mu\text{m}$ in diameter. The pupil magnification factor was 0.68, sampling the exiting wavefront in steps of $158 \mu\text{m}$ at the pupil plane. There are approximately 500 sample points for a pupil diameter of 4 mm. The aberrometer provides a map of the the wavefront error, and this map is fitted to OSA standard Zernike polynomials [38] (please see Appendix A for further details about the measurements). There are different ways to choose the aperture to fit the measured wavefront to Zernike coefficients (large-circle (LC), small-circle (SC), stretched-elliptical (SE)), which are explained in detail in [26, 39]. We have opted for the small-circle aperture method (inscribed circle of 4 mm diameter) within the bigger elliptical pupil for fitting the on-axis and off-axis wavefront to Zernike coefficients. The advantage of this method over the LC and the SE methods is that the coefficients can be treated in the same manner as for the foveal measurements and a direct comparison of the coefficients can be made between different viewing angles. Further details of the instrument can be found in [29]. One such example of the measured wavefront is given in Figure 2. A mean wavefront was constructed at each eccentricity by averaging the corresponding Zernike coefficients of all the participants at that field angle. The mean Zernike coefficient values at different eccentricities are given in Table 1 for reference. A Zemax lens file, as we show in Code file 1 (Ref. [40]), is also provided showing the mean-measured wavefront as well as the Zernike coefficients at different eccentricities.

Fig. 2. An example of the measured wavefront error. The inscribed circle has a diameter of 4 mm. The wavefront data inside this circle is used for calculations in this article. The outer elliptical pupil shape shows the full extent of the light wave exiting from the eye.

B. Generic wide-field eye model development in Zemax

The eye model is based on ocular wavefront measurements (right eye only) of thirty healthy young emmetropic Caucasians (12 Males, 18 Females), average age: 24 years, standard deviation: ± 3 years, age range: from 20 to 29 years, subjective refraction: between ± 0.50 DS and ≤ -0.50 DC, uncorrected visual acuity of 1.0 (logMAR 0.0) or better. The wavefront measurements were done across the horizontal and the vertical field of view from 40° nasal to 40° temporal visual field and up to 20° inferior field in steps of 10° while the participant viewed an object at 3 meter distance. Average on-axis Mean-Spherical-Error (M) was -0.08D . The measured wavefront was fitted to a set of orthonormal Zernike polynomials up to 6^{th} order [25, 27, 29, 38, 41, 42] over a circular exit pupil diameter of 4 mm.

Table 1. Mean Zernike Coefficients vs eccentricity, OSA=Optical Society of America notation following [38], Zenmax=Zemax notation following Noll[41], T= Temporal visual field, N= Nasal visual field, I= Inferior visual field. RMS wavefront error (calculated from all Zernike terms) and Peak-to-valley wavefront error is given in the last two rows, the data are in wavelength units at $\lambda = 555 \text{ nm}$, Zernike coefficients normalization radius=2 mm

OSA	Zemax	On-axis	10T	20T	30T	40T	10N	20N	30N	40N	10I	20I
OSA(2,0)	Z04	0.1470	0.4220	0.2180	0.3850	0.2780	0.4850	1.0490	1.4220	1.5430	0.4340	1.007
OSA(2,-2)	Z05	0.0646	0.2118	0.2777	0.3568	0.4150	-0.0539	-0.1457	-0.2729	-0.4240	-0.1568	-0.3724
OSA(2,2)	Z06	-0.0781	-0.0727	0.2420	0.8040	1.4418	0.2553	1.0541	2.0605	3.1037	-0.3736	-1.0643
OSA(3,-1)	Z07	0.0163	0.0098	0.0088	0.0466	0.0763	-0.0068	0.0116	0.0100	0.0344	0.1191	0.2060
OSA(3,1)	Z08	0.0938	0.1822	0.3103	0.4238	0.5870	-0.0683	-0.2014	-0.3153	-0.5091	0.0671	0.0637
OSA(3,-3)	Z09	-0.0654	-0.0671	-0.0292	0.0004	0.0380	-0.0665	-0.0240	0.0326	0.1197	-0.0799	-0.0432
OSA(3,3)	Z10	0.0185	0.0242	0.0246	0.0214	0.0302	0.0070	-0.0148	-0.0290	-0.0825	0.0206	0.0012
OSA(4,0)	Z11	0.0163	0.0142	0.0138	-0.0048	-0.0394	0.0278	0.0254	-0.0010	-0.0919	0.0184	0.0370
OSA(4,2)	Z12	0.0033	-0.0200	-0.0308	-0.0440	-0.0781	0.0012	-0.0104	-0.0436	-0.1425	0.0138	0.0040
OSA(4,-2)	Z13	0.0131	0.0056	0.0186	0.0108	0.0092	-0.0010	-0.0050	-0.0102	-0.0082	0.0026	-0.0012
OSA(4,4)	Z14	0.0155	0.0042	-0.0084	-0.0126	-0.0170	0.0114	0.0090	-0.0128	-0.0581	0.0064	0.0098
OSA(4,-4)	Z15	0.0047	0.0104	0.0158	0.0172	0.0022	-0.0030	0.0090	0.0212	0.0448	0.0098	0.0048
OSA(5,1)	Z16	-0.0165	-0.0144	-0.0168	-0.0236	-0.0300	0.0084	0.0204	0.0202	-0.0048	-0.0042	0.0024
OSA(5,-1)	Z17	-0.0128	-0.0084	-0.0030	-0.0084	-0.0112	-0.0076	-0.0170	-0.0116	-0.0068	-0.0218	-0.0322
OSA(5,3)	Z18	-0.0026	-0.0024	0.0008	0.0050	0.0074	-0.0052	0.0018	0.0038	-0.0208	-0.0082	-0.0070
OSA(5,-3)	Z19	0.0038	0.0028	-0.0010	-0.0060	-0.0042	0.0034	0.0004	0.0018	0.0028	0.0084	0.0124
OSA(5,5)	Z20	-0.0056	-0.0098	-0.0046	-0.0040	-0.0018	-0.0022	0.0030	0.0110	0.0032	-0.0010	0.0024
OSA(5,-5)	Z21	-0.0008	-0.0004	0.0052	-0.0062	-0.0216	0.0016	-0.0014	0.0010	-0.0012	-0.0042	-0.0078
OSA(6,0)	Z22	0.0046	0.0042	-0.0044	-0.0036	-0.0002	-0.0046	-0.0072	-0.0066	0.0088	0.0022	-0.0052
OSA(6,-2)	Z23	-0.0059	-0.0002	-0.0042	0.0008	-0.0010	-0.0032	-0.0046	-0.0036	0.0032	0.0040	-0.0022
OSA(6,2)	Z24	-0.0005	0.0070	0.0056	0.0024	0.0056	0.0002	0.0036	0.0010	0.0168	-0.0014	-0.0032
OSA(6,-4)	Z25	0.0008	-0.0020	-0.0020	-0.0018	0.0020	0.0024	-0.0020	-0.0042	0.0086	-0.0016	-0.0004
OSA(6,4)	Z26	-0.0020	-0.0010	0.0040	-0.0002	0.0046	-0.0002	0.0010	0.0068	0.0162	-0.0002	0.0010
OSA(6,-6)	Z27	0.0019	-0.0014	-0.0048	-0.0072	-0.0018	-0.0000	-0.0056	-0.0058	0.0038	0.0010	0.0068
OSA(6,6)	Z28	-0.0022	-0.0046	0.0002	-0.0074	-0.0126	-0.0034	-0.0108	-0.0064	0.0038	-0.0054	-0.0008
RMS WFE		0.2159	0.5173	0.5317	1.0507	1.6373	0.5595	1.5058	2.5341	3.5301	0.6149	1.5255
P-V WFE		1.1142	2.3649	2.9712	5.2849	8.6696	2.5098	6.7584	10.8537	16.5845	3.0735	6.9050

Anatomical parameter limits of the eye (thickness, curvature, conic constant, tilt and decenter values of different parts) were taken from the literature [33] and incorporated during the model development. An optics modeling program, Zemax [34] was used to fit the measured wavefront data to a schematic eye using built-in damped-least-square minimization algorithm. Both tilt and decentration of the crystalline lens and the cornea with respect to the optical axis was varied during the optimization. This resulted in a much improved match of the modelled to the measured aberrations.

A schematic diagram of the 'Right Eye' as modelled in Zemax is shown in Figure 3. The retina is taken as the source object and the origin of the coordinate system. Horizontal meridian is placed along the x-axis and vertical meridian along the y-axis in Zemax (+x-axis nasal retina, +y-axis superior retina), while +z-axis is the light propagation axis. This way, the coordinate system of the physical eye in Zemax becomes similar to that of Zernike polynomial definition [38]. Light rays are traced outward passing through the lens, iris and the cornea towards an open field of view. A multi-configuration setup is used in Zemax to define each field angle separately. Appropriate object heights are defined on the retina to get from 40° nasal to 40° temporal visual field angle and to 20° inferior visual field angle (in increments of 10°) in the outside world [43]. The iris is modelled by a flat dummy surface co-located with the anterior lens surface and it does not control the size of the light beam exiting the model eye. A circular opening of fixed diameter 4 mm is placed after the cornea perpendicular to the chief ray by appropriately tilting it at each field angle separately, and it is defined as the 'Aperture Stop' surface in Zemax as shown in Figure 3. The size and shape of the light beam exiting the model eye in Zemax is controlled only by this circular opening. By defining appropriate x- and y-decenters with respect to the optical axis on this surface, the circular opening is placed such that the chief ray at each field angle passes through the center of the iris as shown in Figure 3 [43]. In addition, the 'Float by Stop Size' option is turned on in Zemax. This special procedure is adopted to make sure that the exit pupil and the light beam exiting the model eye does not become elliptical at extreme field angles, since our measured wavefront is always taken inside a constant circular diameter of 4 mm at each field angle without any pupil squeezing. The system is modelled in the 'Afocal Mode' in Zemax. This feature takes a flat surface located at the exit pupil of the optical system as the 'Reference Wavefront' for subsequent wavefront error calculation. By adopting these steps, the wavefront and the Zernike coefficients calculated in Zemax become directly comparable to the corresponding quantities in the Shack-Hartmann measurement at all field angles. For accurate ray tracing, the 'Real Ray Aiming' option is also enabled in the software, so that the 'Aperture Stop' surface circular opening is fully filled with a grid of rays for all field points avoiding any pupil distortion effects. A grid of 128 × 128 rays is used which gives sufficient sampling points inside the exit pupil to faithfully calculate the wavefront and the Zernike coefficients avoiding undersampling artifacts. A flow chart describing the model development procedure is shown in Figure 4. The intricacies of Zemax modeling can be further understood by examining the supplementary Zemax file uploaded to the figshare website [44].

The model development process starts by writing a custom 'Weighted Cost Function' (called 'Merit Function' in Zemax) and minimizing it using the built-in constrained damped-least-square algorithm. The cost function includes the mean Zernike coefficients up to 6th order at each eccentricity and targeting these to their desired average values from the measurement as given in the Table 1. It also includes possible upper and lower limits of the anatomical parameters of the eye as taken from the literature [12, 27, 33, 45] such as: curvatures and conic constants of the cornea, lens and the retina as well as thickness limits of the cornea, aqueous, lens and vitreous. In the 'Cost function', more weight is given to those Zernike coefficients which have a higher value. Similarly, the radii of curvature are given more weight as compared to the conic constants of surfaces. For the decentration and tilt angles of the cornea and the lens with respect to the retina, limits of ±0.3 mm and ±3° are chosen respectively [18]. The decentration and tilts are implemented with a pair of 'Coordinate-break, Coordinate-return' surface pair immediately before and after the surfaces of interest. The 'Coordinate return' surface reverses the decentration and tilt introduced by the first 'Coordinate-break' surface and it brings the optical axis back to its original orientation.

The Navarro model [13] was taken as the starting point and the following additional degrees of freedom were added to it: a biconic surface on the front cornea, conic surface on the rear cornea, geometry independent GRIN profile of the crystalline lens, x,y,z tilts and x, y decenters on the whole lens and on the whole cornea. The refractive index dispersion equations of different surfaces (cornea, aqueous, vitreous) were adapted from Polans work [18, 43] and will not be repeated here (see Table 2 of Polans' paper or Table 4 of Jaeken's paper for these coefficients). The GRIN lens parameter ΔT in Zemax was put as 0.05 mm to improve the axial resolution of the ray tracing through the GRIN region. During optimization, the following quantities were declared as 'Variable': surface radii of curvature and conic constants, intermediate spacings, x,y,z tilts and x,y decenters with appropriate limits. The optimization routine was run in Zemax until it converged to a minimum. After the first optimization run, the z-rotation angle of the lens and the cornea were found to be 13.9 deg and -0.54 deg, respectively. Such small values do not have strong influence on the eye's aberrations. To simplify the model, these values were reset to zero, and further optimization was run to obtain the finalized eye model. Please note that such optimization is a semi-automatic process where after a few iterations, a close inspection is often needed on the developed model. For example, if some parameter of the eye (such as thickness of a surface, conic constant or curvature value) became unrealistic during optimization, the optimization is stopped and the offending parameter has to be manually re-entered to a realistic value. Similarly, if a particular Zernike coefficient is far away from the measured value, it is given more weight during the optimization. In addition to the Zernike coefficients, the RMSWFE values at all field angles are also targeted to be equal to the measured values.

B.1. Geometry-independent GRIN lens model

In our work, we have used the geometry-independent GRIN lens model by Bahrami including the dispersion effects [35–37]. This model faithfully represents the iso-indicial profile of the real lens including age-related effects [35–37]. Please note that the Bahrami GRIN model presented in [35–37] is an extension of the geometry-independent GRIN model by Navarro in [15] with the addition of an extra term bz^3 (please see equation 6 of [37]) to avoid discontinuity of iso-indicial contours at the equatorial interface joining the anterior and the posterior hemispheres. For this GRIN model, the spatial refractive index profile $n(x, y, z)$ equation is posed in such a way that constant index surfaces inside the lens always follow the lens outer contours. Thus, if there is a change in the lens radii of curvatures, conic constants or thickness, this model adjusts itself accordingly to always give a concentric shell-like spatial refractive

index profile. Moreover, the refractive index always varies between two well defined values, index at the center n_c and index at the surface n_s even if the lens shape changes. For these reasons, this model is particularly suited for optimization in Zemax. Since such a GRIN model is not available in Zemax, a custom C-program routine was written based on the equations provided in their papers [35–37], and compiled to a Dynamic Link Library .DLL file that can be called from Zemax. These files can be downloaded from the www.figshare.com website (Ref. [44]). Some details of the model are given here for completeness, additional details of the modeling are provided in Appendix B. The GRIN refractive index profile is given by the equation:

$$n(\zeta) = n_c + (n_s - n_c)(\zeta^2)^p \quad (1)$$

where the parameter p in the exponent can be employed to account for age-related dependence of the GRIN lens (this is a fixed parameter in our work), ζ is the normalized distance from the center of the lens, and n_c and n_s are the refractive indices at the center and at the surface of the GRIN lens, respectively.

The chromatic dispersion of the refractive index is represented by [15, 36]:

$$n(\lambda) = A + B/\lambda^2 + C/\lambda^4 + D/\lambda^6 \quad (2)$$

where λ is the wavelength (in nanometers) and A, B, C, D are constants (these are different for the center and the surface of the GRIN lens). For the average age of 24 years for our subjects and based on the age-dependent model in Table 3 of Navarro's paper [15], the GRIN parameters used here are: $p = 2.888$, for the center: $A = 1.414, B = 6521.218, C = -6.111 \times 10^8, D = 5.908 \times 10^{13}$, and for the surface: $A = 1.356, B = 6428.455, C = -6.024 \times 10^8, D = 5.824 \times 10^{13}$, lens thickness = 3.5 mm, anterior GRIN region thickness = 2.109 mm. In figure 5, we have plotted the spatial refractive index profile showing the concentric shell like structure following the lens outer contours based on the lens parameters given in this paper.

Fig. 3. (a) Schematic eye (Right eye) and coordinate system (b) Zemax setup to match the pupil shape, wavefront and Zernike coefficients between the measurements and the Zemax model at different field angles

Fig. 4. Flow chart for the development of the generic eye model

3. RESULTS AND DISCUSSION

A. The Caucasian-only schematic wide-field eye

The finalized schematic eye specifications are given in Table 2. A Zemax lens file of the developed wide-field eye model, as we show in Code file 2 (Ref. [44]), is also provided for the community. The Zemax file and the associated Glass file contain all the information

Fig. 5. Spatial refractive index profile $n(z, \rho)$ of the geometry-independent GRIN lens for our model

Table 2. Wide-field emmetropic eye model specification, CC=Conic Constant, index value given at $\lambda = 555$ nm, geometry independent GRIN model parameters $p=2.888$, $\Delta T = 0.05$ mm, anterior region thickness $t_a = 2.1$ mm, posterior region thickness $t_p = 1.4$ mm

Surface Description	Index	Y-Radius, CC (mm)	X-radius, CC. (mm)	Thickness (mm)	X, Y Decenter (mm)	X, Y, Z Tilt (deg)
Conic, Retina	1.336	14.00, 0.247	Same	16.957		
Coord. Break				0	-0.27, 0.092	0.91, -1.643, 0
Conic, Lens Back	GRIN	6.884, -3.286	Same	3.50		
Conic, Lens Front	1.336	-14.304, -3.546	Same	0		
Flat, Iris	1.336			0		
Coord. Return				3.05	0.371, -0.036	-0.91, 1.643, 0
Coord. Break				0	0.194, -0.213	1.241, 2.983, 0
Conic, Cornea Back	1.376	-6.323, -0.038	Same	0.55		
Biconic, Cornea Front		-7.757, -0.272	-7.744, -0.361	0		
Coord. Return					-0.222, 0.225	-1.241, -2.983, 0

about the eye model, dispersion parameters of different eye regions and the 'Cost Function' used. In the developed schematic eye, the surface radii of curvature, conic constants, spacings, tilt and decenter values are within the statistical range [33]. Our schematic eye has an axial length of 24.06 mm and paraxial optical power of 60.74 Diopters. In Table 3, these values are compared with other well-known schematic eyes and are found to be similar. Our model has axial chromatic focal shift ($M_{red} - M_{blue}$) of 1.00 Diopter, which is also comparable to other schematic eye models [18]. Please note that to implement the tilt/decenter of the lens and the cornea, we have used the automatic feature of Zemax, where one defines the desired tilt/decenter on a 'Coord. Break' surface, and Zemax automatically calculates the reverse tilt/decenter on the 'Coord. Return' surface thus bringing the coordinate system back to its original alignment.

B. RMS wavefront error

A measure commonly used to judge the accuracy of a schematic eye model is to compare its RMS wavefront error prediction against measured values. In Figure 6, we have plotted the RMSWFE of different models against the mean-measured values (including both lower and higher order Zernike coefficients up to 6th order as given in Table 1), both for the horizontal and the vertical eccentricities. These calculations were performed while keeping a fixed exit pupil diameter of 4 mm at each field angle for all models as explained before. It is evident from the measurements that the eye has asymmetric performance across the visual field. Most of the schematic eye models (Atchison symmetrical [8], Navarro 1999 [10], Kooijman [11], Liou-Brennan [9], Goncharov's models [5, 6], Jaeken 2011 [43]) are unable to reproduce this asymmetrical RMSWFE performance. This is because these models are implemented with rotationally symmetrical surfaces around the optical axis. The model recently developed by Polans [18, 33] is able to reproduce the asymmetrical behavior of the eye. However, this model is built on persons having on-axis mean spherical refractive error (M) ranging from -4.6 to +3.2 Diopters [18]. Thus as evident in Figure 6, Polans' model RMSWFE prediction does not match the measured values of emmetropic persons in our study. In the Polans model, there is a large defocus aberration present indicating a myopic eye. The model developed by Navarro [15] incorporates the corneal and lens asymmetries (tilt, decenters, biconic surfaces, Zernike deformation terms on the cornea) as well as iso-indicial GRIN model of the lens. But still, it has much higher RMSWFE on the temporal field side as compared to the measurements. The Atchison's asymmetric model [8] also has a much higher RMSWFE on the temporal field side. Our developed model has much better match with the mean-measured RMSWFE values in both the horizontal and the vertical eccentricity, and it correctly reproduces the temporal-nasal asymmetry in the eye as well. Some additional details of the previous models are provided in

Table 3. Comparison of different eye models, AL= Axial Length, LT= Lens Thickness, CR= Corneal Radius (Avg.), ACD= Anterior Chamber Depth, VCD= Vitreous Chamber Depth

Model	AL (mm)	Optical Power (Diopters)	LT (mm)	CR (mm)	ACD (mm)	VCD (mm)	AL/CR	VCD/CR
Kooijman 1983 [11]	24.20	60.18	4.00	7.80	3.60	16.60	3.102	2.128
Liou & Brennan 1997 [9]	23.95	60.29	4.02	7.77	3.66	16.27	3.082	2.086
Navarro 1999 [13]	24.00	60.63	4.00	7.72	3.60	16.40	3.109	2.124
Atchison 2006 [8]	23.58	61.40	3.60	7.77	3.70	16.28	3.035	2.095
Goncharov 2008 [6]	23.90	60.02	3.69	7.76	3.61	16.60	3.079	2.139
Kong 2009 [12]	23.83	60.79	3.81	7.94	3.74	16.28	3.001	2.050
Jaeken 2011 [43]	23.95	60.34	4.02	7.77	3.66	16.27	3.082	2.094
Navarro 2014 [15]	23.78	61.29	3.496	7.669	3.63	16.10	3.100	2.099
Polans 2015 [18]	23.77	64.12	3.53	7.72	4.20	16.04	3.077	2.077
Our Model	24.057	60.74	3.50	7.7505	3.60	16.957	3.104	2.188

Appendix C.

For the Polans model following Table 1 in his paper [18], the nasal and superior retina are placed on the -x axis and -y axis respectively in Zemax (private communication), while in our case, they are placed on the +x axis and +y axis, respectively. For the Atchison models, we put SR=0.0 D in his model to make the calculations in this paper. For the Navarro 2014 model, we put Age=24 years and accommodation D=0.0 in his model equations. Using these values in his model, we got VCD=15.789 mm which resulted in on-axis RMSWFE of 1.449λ . This showed that the retina was slightly out of focus for the given model equations. For this paper, we changed the VCD to 16.1 mm to bring the retina in focus, which resulted in on-axis RMSWFE of 0.6375λ (at $\lambda = 555 \text{ nm}$).

Fig. 6. RMS wavefront error versus eccentricity of various eye models as compared to the mean-measured wavefront. The error bars correspond to the standard deviation in the measured data set over the 30 tested participants. $\lambda = 555 \text{ nm}$, constant exit pupil diameter=4.0 mm at each field angle

C. Zernike coefficients

In addition to the RMSWFE values, it is also important to evaluate how well the predicted wavefront shape matches the measured wavefront. One way to do this is to decompose the wavefront error into Zernike orthonormal polynomials, and compare the dominant Zernike terms. In figure 7, we have plotted the dominant Zernike coefficients against horizontal and vertical eccentricity, both for the measured and predicted data set. There is a good match between the model and the measured values for oblique and vertical astigmatism, defocus, horizontal and vertical coma up to the field extremes. For the on-axis, the defocus term OSA(2,0) is slightly higher than the measured value, which also increases the on-axis total RMSWFE. However, this mismatch can be adjusted in the model by slightly refocusing the retina (for example by sliding the lens by 0.1 mm towards the retina). This will however result in slightly worst off-axis match. The oblique trefoil aberration OSA(3,3) and primary spherical aberration OSA(4,0) terms have a good match up

till $\pm 20^\circ$ field angle, but these terms have much higher values than the measured ones at the field extremes. In the Polans model, there is a close match between the model and the measurements for the primary spherical aberration term, but as discussed before, his GRIN profile is totally different from the iso-indicial GRIN profile that we have used. Perhaps the iso-indicial GRIN profile of our lens needs some correction terms to obtain a good match for the OSA(4,0) term.

The defocus term OSA(2,0) and vertical astigmatism term OSA(2,2) show significant asymmetry between the nasal and the temporal field, while the remaining terms are more or less similar on both sides. These terms also have the largest magnitude among all the Zernike terms. Among the higher-order terms, the horizontal coma OSA(3,1) is the dominant aberration and has the largest magnitude in the horizontal field. This term is well matched by the model up to $\pm 30^\circ$ field, but at the field extremes, the model gives higher values than the mean-measured values. Similarly, vertical coma OSA(3,-1) has the largest magnitude along the inferior visual field. Earlier studies have shown that the magnitude of coma increases with age [46] and is a major contributor to the degradation of the peripheral image quality. When designing optical devices to improve peripheral retinal image quality, the main higher order aberration that needs to be corrected will be coma. Currently, it is suggested that to improve peripheral visual function in AMD subjects with central field loss, intraocular lenses should be designed in such a way that they can correct the peripheral defocus, astigmatism and coma [47].

D. Mean Sphere (M) and Astigmatism (Cyl.)

Mean Sphere (M) and Astigmatism (Cyl) terms were calculated from the low-order Zernike terms [25] and are plotted in Figure 8. There is a good match between the measured values and the values from the model up to the field extremes. In addition, the nasal-temporal asymmetrical performance of the measured eyes is reproduced by the eye model as well.

E. Chromatic aberrations

The chromatic performance of the schematic eye is shown in Figure 9. The mean sphere (M) plot vs horizontal eccentricity for red ($\lambda = 671$ nm) and blue color ($\lambda = 475$ nm) match very well with Figure 2 in [43]. The chromatic focal shift CFS ($M_{red} - M_{blue}$) values as a function of eccentricity matches well with Figure 3 in [43] and Figure 6 in [18]. The on-axis CFS is approximately 1.0 Diopter both for our model and the measured values reported. The CFS increases off-axis and it is slightly higher on the nasal field side as compared to the temporal field side, and this trend is also similar for our model and the reported values in the literature. This shows that the dispersion properties of the eye's refractive surfaces are correctly represented in the schematic eye.

F. Wavefront shape, Point Spread Function and Aerial image simulation

For visual comparison, we have tabulated images of the wavefront shape, PSF and Letter 'E' image convolutions in figure 10, for different fields of view, both for the measured and the developed eye model. There is a good overall qualitative and quantitative match between the two sets. It can be noticed in the second column that the exit pupil in the Zemax model is a circle for all field angles, showing that there is no pupil squeezing.

G. Point Spread Function for previously published models

In figure 11, we have simulated the PSF at different eccentricities for some of the previously published eye models. These results can be compared with the PSFs calculated from the measurements to ascertain the accuracy of the published models. The emmetropic models developed by Jaeken [43] and Goncharov [5, 6] are symmetrical around the optical axis, and hence are unable to differentiate between the nasal and the temporal fields. In addition, their PSF is much broader at field extremes due to higher wavefront error. The asymmetric emmetropic model by Atchison [8] has a good match with the measured PSF on the nasal field side, but not on the temporal field side where it has higher RMSWFE than the measurement. For the Polans model as mentioned before, it represents a myopic eye with large defocus aberration. To make a direct comparison with our measurements which are for emmetropic subjects, we put a Zernike phase plate with defocus term OSA(2,0) on the exit pupil, and varied its magnitude until the RMSWFE on-axis was the same as the mean-measured RMSWFE. This step essentially removes the excessive defocus aberration from the Polans model. After that, we calculated the PSF for various field angles. By comparing the corresponding PSF images in figure 11, it can be seen that the PSFs for the Polans model are comparable to the measurements, and the nasal-temporal asymmetry is also reproduced. This simulation shows that it is possible to use the Polans model for emmetropic eye by balancing out the on-axis defocus term. However as mentioned before, the Polans model has a number of shortcomings (too large ΔT parameter, positive conic constant on the posterior lens surface, unphysiological GRIN profile) making it structurally incorrect. For the Navarro's emmetropic model [15], it can be observed that the PSFs are very different from the measured results. Even though this model includes realistic details about the eye (tilts, decenters, biconic surfaces, iso-indicial GRIN model), still it is not so successful in reproducing the off-axis PSFs.

The generic model was kept as simple as possible without sacrificing accuracy. For example, on the front corneal surface, in addition to the biconic surface option that we chose in our model, one can add extra deformation terms in the form of Zernike polynomials to get a better wavefront fit between the measurements and the eye model [7, 15]. This would add further complexity to the model, and was not included. The back corneal surface could also be made biconic [12, 15, 18] with the possibility of slightly different curvatures and conic constants in the x- and the y- meridians. However for this surface, the differences in the curvatures and conic constants in the x- and the y- meridians as reported in the literature [12, 15, 18] are rather small. Since this surface has much less optical power than the front corneal surface, this surface was kept purely rotationally symmetrical. Thus, the cylindrical aspect of the wavefront was taken into account by choosing a biconic surface on the front corneal surface only.

In the Polans model [18], only the lens is tilted and decentered with respect to the retina. We started with this option during the model development. However, this choice gave a poor match between the measured and the reproduced wavefronts. Moreover, it is reported in the literature that the cornea, lens and retina are not centered with respect to each other [8, 15]. Thus, both the lens and the cornea were allowed to be tilted and decentered independently.

Fig. 7. Plots of dominant Zernike terms versus eccentricity. Blue curves = horizontal field, red curves = inferior field. The error bars correspond to the standard deviation in the measured data set over the 30 tested participants. $\lambda = 555$ nm, constant exit pupil diameter=4.0 mm at each field angle

The optimization algorithm in Zemax minimizes the weighted error between the target values and the desired values. After deciding upon the “Cost Function” and running the Zemax optimization, our model converges to a unique solution with realistic surface shapes, conic constants, thickness, tilts and decenters. However, one can obtain slightly different eye models by changing weights on different Zernike coefficients as desired. For example, one can get a better match for the primary spherical term by putting more weight on it in the “Cost Function”, but this might give a poorer match with other Zernike coefficients. It is up to the user to further modify the model to suit a particular need, for example to develop personalized eye model of a person with given refractive errors.

We have modeled the eye in such way that the exit pupil is circular with a diameter of 4 mm at all field angles and the light propagation is from the retina towards the open world. This was done to correctly match the measured wavefronts with the ones calculated by Zemax modelling. In order to use the eye model for retinal image simulations, light propagation direction should be chosen from the open world towards the retina. Additionally, one should put the ‘Aperture Stop’ on the Iris and give it a fixed diameter as desired. This way, the incoming light beam diameter and shape (both on-axis and off-axis) will correctly represent the physical eye.

4. CONCLUSIONS

In summary, the measured wavefronts across horizontal and vertical eccentricities of thirty healthy young emmetropic persons have been used to construct a wide-field generic eye model. A geometry-independent GRIN model was employed to better represent the crystalline lens. The dispersion properties and anatomical parameters of the different parts of the eye have been taken into account to achieve a realistic model. In this model, the lens and cornea are slightly tilted and decentered with respect to the retina to better reproduce the measured wavefront across the field of view. The model can faithfully reproduce the RMS wavefront error values as well as the wavefront shape across the field of view for an average emmetropic eye. It can also reproduce the temporal-nasal asymmetries found in the eye performance across the field of view with a chromatic aberration performance that matches values reported in the literature. The main limitation of the model is that the primary spherical aberration term OSA(4,0) and oblique trefoil term OSA(3,3) are too high beyond $\pm 20^\circ$ field. Perhaps the iso-indicial GRIN profile of the lens needs some correction terms to better reproduce the OSA(4,0) term at the field extremes.

In this work, we have used the wavefront information only from the central 4.0 mm diameter circular part of the exit pupil. In future, we plan to use wavefront information from a bigger elliptical pupil to build a model which will be better suited for situations where the eye pupil is larger than 4.0 mm and possibly elliptical. In addition, by measuring wavefronts for different age groups and with different refractive errors, one could extend the present model further.

Fig. 8. (a) Plots of Mean Sphere (M) and (b) Astigmatism (Cyl) versus eccentricity. Blue curves = horizontal field, red curves = inferior field. The error bars correspond to the standard deviation in the measured data set over the 30 tested participants. $\lambda = 555$ nm, constant exit pupil diameter=4.0 mm at each field angle

Fig. 9. The Chromatic performance of the schematic eye, constant exit pupil diameter=4.0 mm at each field angle (a) Mean Sphere vs eccentricity M_{red} and M_{blue} (b) Chromatic focal shift $(M_{red} - M_{blue})$ vs eccentricity

5. DISCLOSURES

The authors declare that there are no conflicts of interest related to this article. The study was approved by the local ethics committee in Linköping and the participants gave their informed consent after the nature and the intent of the study had been explained. The study protocol was designed in accordance with the tenets Declaration of Helsinki.

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APPENDIX A: REMOVAL OF ACCOMMODATIVE DEMAND IN THE DEFOCUS TERM

The measurements in this study were performed with a natural pupil in a dimly illuminated room with the left eye occluded while the subjects viewed a red LED light at a distance of 3 meters. The target was non-accommodative, but given the young age of the subjects, control measurements were carried out and revealed that subjects tended to accommodate approximately 0.33 Diopters. This bias of 0.33 Diopters was corrected for in the defocus term $OSA(2,0)$. For a pupil radius $R_p = 2$ mm, this amounts to $-0.192 \mu\text{m}$. This value was added to the $OSA(2,0)$ coefficient reported by the wavefront sensor to correct the bias in all field angles. The eye model and the Zernike coefficients reported in this article are with this correction.

APPENDIX B: DETAILS OF THE GEOMETRY INDEPENDENT GRIN LENS MODEL

- It is mentioned in Bahrami's 2014 paper [37] that for the real root of the cubic equation, $j = 0$ should be chosen in the equation 13, both for the anterior and the posterior hemisphere of the GRIN region. To make our Zemax C- routine more general and suitable for different shapes of the GRIN lens, we have provided the option to choose any root $j = 0, 1, 2$. This can be set in the

Fig. 10. Wavefront error (in units of λ), PSF and Letter 'E' image convolution, Mean-measured vs model eye. Constant exit pupil diameter=4 mm at each field angle, scale on the Wavefront plot box = $4 \text{ mm} \times 4 \text{ mm}$, scale on the PSF box = $12.46 \text{ mrad} \times 12.46 \text{ mrad}$ unless stated otherwise, scale on the Letter 'E' box = $46.2857 \text{ mrad} \times 46.2857 \text{ mrad}$, $\lambda = 555 \text{ nm}$, It can be seen that the shape of the exit pupil remains circular without any pupil squeezing for the off-axis fields

Fig. 11. Point Spread Function simulation for various eye models, constant exit pupil diameter=4 mm at each field angle, scale of the PSF box = $12.46 \text{ mrad} \times 12.46 \text{ mrad}$ except on 40T and 40N images where it is $24.92 \text{ mrad} \times 24.92 \text{ mrad}$, $\lambda = 555 \text{ nm}$

'Extra Data Editor' corresponding to the GRIN surface in Zemax. One should choose a root (usually $j = 0$ but not always) and then plot the GRIN index profile in Zemax to see if it is iso-indicial, and if not, one should choose the next root (individually for each hemisphere).

- For ray tracing in a GRIN region, one has to provide Zemax with the derivatives $dn/dx, dn/dy, dn/dz$ at the spatial location x, y, z inside the GRIN region. Since it was difficult to derive the expressions for the analytical derivatives, we found the derivatives numerically. For example: $dn(x, y, z)/dx \approx (n(x + dx, y, z) - n(x, y, z))/dx$, and similarly for the other derivatives. The spatial differential dx (and hence dy and dz) can be chosen by the user in the 'Extra Data Editor' in Zemax corresponding to the GRIN surface. We did a verification test for a simpler polynomial-equation based GRIN surface (whose analytical derivative expressions are straight forward to be found). For a value of $dx = 0.001 \text{ mm}$, the analytical and the numerical derivatives gave same results in terms of ray aberrations and optical path difference aberrations. Hence this value was chosen as the default during simulations in this article.
- In Bahrami's J. Biomed. Opt. paper [35], there appears to be a typing error in equation 9. The printed equation gave wrong profile for the iso-indicial GRIN region. Since in Bahrami's paper, the iso-indicial profiles are correctly plotted, we concluded that it was a typing mistake. We derived this equation again using Symbolic Toolbox in MATLAB, which is given below and this is the one we have implemented in our model.

$$z_c = 0.5(k_p - k_a)^{-1} \left(2t_a + 2t_p - 4r_a - 4r_p + 2t_a k_a + 2t_p k_p + (32r_a r_p - 16r_a t_a - 16r_a t_p - 16r_p t_a - 16r_p t_p + 8t_a t_p + 4t_a^2 k_a + 4t_a^2 k_p + 4t_p^2 k_a + 4t_p^2 k_p + 16r_a^2 + 16r_p^2 + 4t_a^2 + 4t_p^2 - 16r_a t_a k_p - 16r_p t_a k_a - 16r_a t_p k_p - 16r_p t_p k_a + 8t_a t_p k_a + 8t_a t_p k_p + 4t_a^2 k_a k_p + 4t_p^2 k_a k_p + 8t_a t_p k_a k_p)^{0.5} \right) \quad (3)$$

APPENDIX C: FURTHER DETAILS OF OTHER EYE MODELS

- Polans Model: Based on 101 subjects along the horizontal meridian, 10 subjects along the vertical meridian, 95% Caucasian subjects, normal viewing conditions without ophthalmic correction, without cycloplegia, average age 27.5 ± 7.2 years, spherical equivalent refraction (M) ranging from -4.2D to +3.6D.
- Atchison's Model: Based on 121 emmetropic and myopic participants aged 25 ± 5 years (age range 18–36 years). Subjective refraction was performed without cycloplegia and the mean spherical refraction (SR) was +0.75 D to -12.38 D. 63% of the total group was female. The measurements and hence modeling apply for the unaccommodated state.
- Goncharov JOSAA Model: Based on five subjects with average age: 20 years, ages ranged from 31 to 47 yr, spherical equivalent corrections ranged from 1.50 DS to -2.00 DS and on-axis astigmatism was 0.75 D or less for all subjects. The eyes were dilated with 1% cyclopentolate. Dilated pupil sizes were approximately 6.5 to 8.0 mm.
- Goncharov Optics Express Model: Based on 30-year old eye and some data from his previous model.
- Jaeken Model: The right eyes of five emmetropes (refractive error mean and std: -0.38D and 0.35D, age: 30 ± 2.5 years) and six mild myopes (refractive error: $-2.91D \pm 1.57D$, age: 31 ± 3 years) were measured. Measurements were done with the natural pupil of the subject.
- Navarro 2014 Model: The range of ages considered is from 20 to 70 years. This is a model of the emmetropic eye at each age.

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