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Optimum Signal Design in UWB Communications

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Abstract

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In this thesis, the optimum signal design in UWB (Ultra Wide Band) communications based on a Generalized Uncertainty Principle is presented.

The purpose of this thesis work is to see how the finite time duration constraint can be optimized with respect to the corresponding energy constraint within a prescribed frequency band of interest. The thesis work further focuses on an arbitrary and UWB communication channel. Technically, the optimization criterion is to maximize the energy within a prescribed frequency band at the channel output while maintaining a fixed transmitted power at the channel input.

Theoretical derivations and the analyses of numerical examples demonstrate the possibilities of applying the eigenvalue problem to the design of an optimum window.

Keywords: Optimum signal design, Generalized Uncertainty Principle, UWB (Ultra Wide Band) communication, Eigenvalue problem, Prolate spheroidal wave functions, Energy constraint.

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1 Introduction

This chapter presents an introduction to the thesis topic. Furthermore, a discussion of the problem, the purpose of the thesis work and questions along with the thesis structure are presented.

1.1 Background

Since the inception of the time-frequency analysis theory and technology, its application potential has been recognized and its rapid development has provided a powerful tool for solving many technical problems. The time-frequency analysis has found applications in speech recognition, image processing, radar signal processing and seismic signal processing. An optimal time-frequency distribution (TFD) needs to be performed such that the distribution is expressed as a series of signal components and suppressing cross-terms [1].

In the traditional signal analysis, the signal is generally considered to be stationary. The signal can be described either in the time domain or the frequency domain after being Fourier Transformed (FT). Despite its many advantages, the FT is only suitable for stationary signals, which, unfortunately, is not the case in many practical engineering applications, where most signals are non-stationary. For this kind of signal, we introduce the joint description method of time and frequency domain.

The statistical properties of non-stationary signals, such as speech signals, radar and sonar echo signals, vary with time. The time-energy density and the frequency-energy density do not fully describe the physical conditions. In particular, although it is possible to determine which frequencies appear in the signal from the spectrum, determining when these frequencies exist, remains a challenge. Therefore, we need to describe how the spectrum contents change with time and what the time-varying spectrum is. The time-frequency joint-description of the signal is, thus, introduced.

The basic idea of Short Time Fourier Transform (STFT) is to intercept a signal with the window function, assuming that the signal is stable in the window, and then to analyze the signal by using Fourier transform, while determining the features of the signal for that particular time and frequency. Since the concept of STFT is direct and simple, it has been widely applied in many areas.

1.2 Traditional methods of signal analysis

- Short Time Fourier Transform

The Short Time Fourier Transform (STFT) is a powerful method for analyzing non-stationary signals, because it provides important information about the change of the frequency content. Mathematically, the STFT is represented by

$$\begin{aligned} STFT(\tau, f) &= \int_{-\infty}^{+\infty} x(t)w(\tau - t)e^{-j2\pi ft}dt \\ &= \int_{-\infty}^{+\infty} X(\alpha + f)W(\alpha)e^{j2\pi\alpha\tau}d\alpha, \end{aligned} \tag{1}$$

see [2].

Therefore, the STFT is performed by shifting a window of constant size along the time axis of a signal, such that the signal is divided into segments in which it is considered stationary, and, the FT can be applied.

During the past two decades, the STFT has gained more significance due to the growth in computer science. The major advantage of STFT is that it can provide the time-frequency location of a particular signal of interest. Examples of some of its applications are as follows:

- Gabor Transform

The Gabor Transform can be defined as a specific method when a Gaussian window is used in the STFT. It is, in fact, also widely used due to its low leakage in the time-frequency domain [2]. The Gabor transform is given by

$$G(t, \omega) = \int_{-\infty}^{+\infty} e^{\frac{-(\tau-t)^2}{2}} e^{-j\omega\tau} x(\tau)d\tau. \tag{2}$$

- WignerVille distribution

The Time-Frequency Distributions (TFD's) describe the energy content of a signal as a function of both time and frequency [3]. The Wigner Distribution (WD) is represented by

$$W_z(t, f) = \int_{-\infty}^{+\infty} z(t + \frac{\tau}{2})z^*(t - \frac{\tau}{2})e^{-j2\pi f\tau}d\tau, \tag{3}$$

and is of great interest due to a number of attractive properties. However, it also has spurious cross-components and high noise sensitivity, both of which obscure the true signal features. Therefore, the WD is often convolved with a two- dimensional smoothing function that suppresses the cross-components at the expense of signal energy concentration.

The Wigner-Ville Distribution (WVD) of a signal $s(t)$ is given by its analytic associate $z(t)$, as

$$W_z(t, f) = F\left\{z(t + \frac{\tau}{2})z^*(t - \frac{\tau}{2})\right\}. \quad (4)$$

As shown above, the WVD, in contrast to the WD, emphasizes the use of the analytic signal and acknowledges Ville's contribution to deriving the context of signal-processing in 1948 [4].

- Hermite functions

Hermite functions are utilized as modulating functions which were introduced by Loeb and Cahen. All the modulating functions and their derivatives necessary in the higher order systems, are uniformly constructed from the higher order Hermite functions. This has made it easier for the method to be applied, and it is applicable without any special measures regardless of whether a system has initial values, or not [5].

1.3 Ultra wide-band wireless communication

Ultra wide-band (UWB) signal waveforms are pulses with extremely short duration transmitted without modulation by a continuous carrier frequency, and are suitable for baseband encoding schemes for secured information transmission. Ultra wide-band wireless communication is a method that uses pulses with very short time intervals (less than 1 ns) for communication without a carrier wave. It is a carrierless communication technology that uses nanosecond to picosecond nonsinusoidal narrow pulses to transmit data.

The outstanding qualities of UWB include strong anti-interference performance, high transmission rate, large system capacity and very low transmission power. The transmission powers of UWB systems are very low, communication equipment can use less than 1mW transmission power to achieve communication. Low transmission power greatly extends the operation time of a power system. Moreover, when the transmission power is small, the electromagnetic radiation has little effect on the human body, which widens the area of application [6].

1.4 Purpose and Research Questions

A classical result, given in [8] describes how a signal pulse with a finite time duration constraint can be optimized with respect to its energy concentration within a prescribed frequency band of interest. This problem can be formulated as an eigenvalue problem for an integral operator with the classical sinc-kernel, and the solution is given by the classical prolate spheroidal wave functions [8, 9]. The discrete time version of this formulation is sometimes referred to as an optimum window design, (see, for example, the classical Kaiser window [10, 11, 12, 13]).

Here, we are considering a generalized version of the classical formulation, which incorporates the frequency response of an arbitrary, given channel. The optimization criterion is to maximize the energy within a prescribed frequency band at the channel output while maintaining a fixed transmitted power at the channel input, based on a finite time duration signal. The optimization formulation leads to an eigenvalue problem for an integral equation and is solved numerically by using quadrature methods and an eigendecomposition of the corresponding matrix.

The research questions concentrate on the following items:

- How can a signal pulse with a finite time duration be optimized with respect to its energy concentration within a prescribed frequency band of interest?

- How can the eigenvalue for an integral operator be found with the classical sinc-kernel when the solution is given by the classical prolate spheroidal wave functions?

- How can the fixed transmitted energy be maintained, such that the received energy is maximized after passing an arbitrary and given channel, which is the UWB communication channel?

1.5 Structure

This thesis is divided into several chapters:

Chapter 1 covers the introduction to the thesis. This Chapter, provides a background on the related topics, such as optimum signal design, UWB communications, and the Uncertainty Principle. Furthermore, a summary of the discussions around the thesis problem, purpose, research questions, and the structure is presented.

Chapter 2 provides the theoretical parts which are the foundations of the thesis.

Chapter 3 gives some empirical examples regarding the maximum eigenvalue, and energy constraints between the time and frequency domains.

Chapter 4 presents a discussion and an analysis of the results.

Figure 1 illustrates the structure of the thesis.

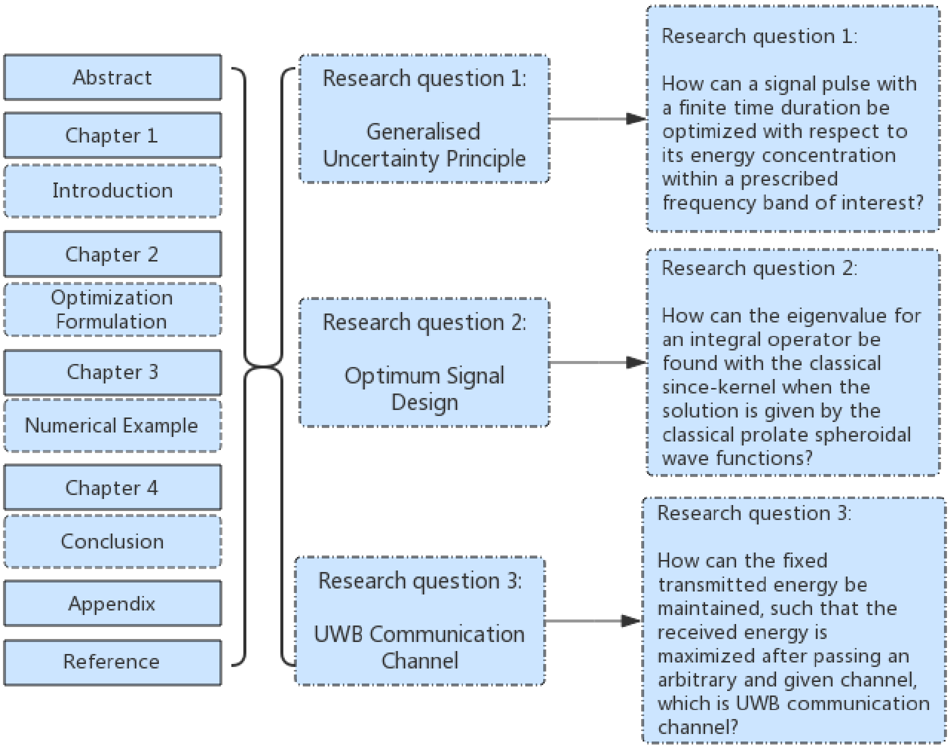


Figure 1: Structure of the thesis

2 Optimization Formulation

Consider the transmission of a signal pulse $x(t)$ with the time-domain constraint

$$x(t) = 0, \quad |t| > \frac{T}{2} \tag{5}$$

and the total transmitted energy

$$E_\text{t} = \int_{-T/2}^{T/2} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega, \tag{6}$$

where $X(\omega)$ is the Fourier transform given by

$$X(\omega) = \int_{-T/2}^{T/2} x(t)e^{-\text{i}\omega t}dt. \tag{7}$$

The received signal $y(t)$ is the convolution of the input signal $x(t)$ with the impulse response $h(t)$ of the communication channel, so that

$$Y(\omega) = H(\omega)X(\omega) \tag{8}$$

is the Fourier transform of the received signal, and where $H(\omega)$ is the frequency response of the channel. Figure 2 shows the linear system including input signal $x(t)$, impulse response $h(t)$ and received signal $y(t)$.

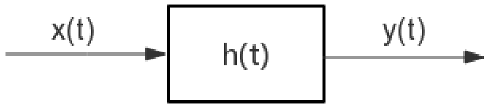


Figure 2: Linear system

The energy E_r received within a prescribed frequency band of interest $[-\pi B, \pi B]$ at the output of the communication channel is given by

$$
\begin{aligned}
E_{\mathrm{r}} &= \frac{1}{2\pi}\int_{-\pi B}^{\pi B} Y(\omega)Y^{*}(\omega)\mathrm{d}\omega \\
&= \frac{1}{2\pi}\int_{-\pi B}^{\pi B}\left[H(\omega)\int_{-T/2}^{T/2} x(t)e^{-\mathrm{i}\omega t}\mathrm{d}t \times H^{*}(\omega)\int_{-T/2}^{T/2} x(u)e^{\mathrm{i}\omega u}\mathrm{d}u\right]\mathrm{d}\omega \\
&= \int_{-T/2}^{T/2} x(t)\int_{-T/2}^{T/2}\frac{1}{2\pi}\int_{-\pi B}^{\pi B}|H(\omega)|^{2}e^{-\mathrm{i}\omega(t-u)}\mathrm{d}\omega x(u)\mathrm{d}u\mathrm{d}t \\
&= \int_{-T/2}^{T/2} x(t)\int_{-T/2}^{T/2} K(t,u)x(u)\mathrm{d}u\mathrm{d}t \\
&= \langle x(t), Kx(t)\rangle,
\end{aligned} \tag{9}
$$

where $\langle\cdot,\cdot\rangle$ denotes the scalar product $\langle x(t), y(t)\rangle = \int_{-T/2}^{T/2} x(t)y(t)\mathrm{d}t$ and K is the integral operator defined by

$$
Kx(t) = \int_{-T/2}^{T/2} K(t,u)x(u)\mathrm{d}u, \tag{10}
$$

where the integral kernel is

$$
K(t,u) = \frac{1}{2\pi}\int_{-\pi B}^{\pi B}|H(\omega)|^{2}e^{-\mathrm{i}\omega(t-u)}\mathrm{d}\omega. \tag{11}
$$

Note that if $|H(\omega)| = 1$ within the frequency band, we have that

$$
K(t,u) = B\frac{\sin \pi B(t-u)}{\pi B(t-u)}, \tag{12}
$$

which is the classical sinc-kernel [8]. It assumed that $E_{\mathrm{r}} = \langle x(t), Kx(t)\rangle$ is a positive definite quadratic form.

Consider an optimization problem where we wish to maximize the in-band energy of the received signal for $|\omega| \leq \pi B$ subjected to the constraint that the transmitted pulse is limited in time within $|t| \leq T/2$, and that the total transmitted pulse energy is constrained. The corresponding (non-convex) optimization problem can be formulated as

$$
\begin{aligned}
&\textit{maximize} \quad \langle x(t), Kx(t)\rangle \\
&\textit{subject to} \quad \int_{-T/2}^{T/2} x^{2}(t)\mathrm{d}t = E_{\mathrm{t}},
\end{aligned} \tag{13}
$$

where E_{t} is the prescribed transmitted pulse energy. The solution to (13) can be obtained from the eigenvalue problem

$$
\int_{-T/2}^{T/2} K(t,u)\phi(u)\mathrm{d}u = \lambda\phi(t), \tag{14}
$$

as the properly scaled eigenfunction $x(t) = \phi_{\max}(t)$ corresponding to the largest eigenvalue $\lambda_{\max}$.

3 Numerical Example

As a numerical example, we consider the design of an optimum design in UWB communications based on the Generalized Uncertainty Principle. The composite Simpson's rule is novelly used to solve the eigenvalue problem (14), which indicates the core method of the optimum design.

Fig.3 shows the classical prolate spheroidal function and eigenfunction with the time-frequency parameters including a frequency band width of B=10 and a time period of T=1. In this test program, the total transmitted energy E_t of the classical prolate spheroidal function signal with a finite time duration constraint in the time domain is normalized to be 1 and the largest eigenvalue λ_{max} is then calculated to be 1. Consequently, the signal pulse with a finite time duration constraint can be optimized corresponding to λ_{max}, as well as, the classical prolate spheroidal function is estimated perfectly by eigenfunction.

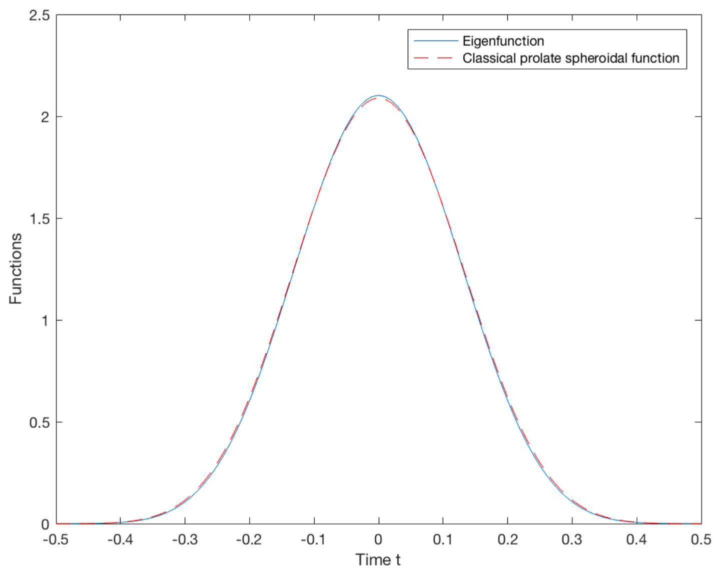


Figure 3: Test program: Classical prolate spheroidal function and Eigenfunction with B=10, T=1

Figure 4 displays the optimal window function when T=1, and BT=10. The corresponding maximal in-band power, which varies with respect to the value of BT, is illustrated in Figure 5: As BT increases, The maximal in-band power indicated by λ, which is from the eigenvalue problem (14), approximates to 1. This is especially the case when the value of BT is 3, which can be seen as a separation point, meaning that the in-band power increases rapidly when the value of BT is less than 3 and then plateaus as it approaches 1. The changeable law links in with the form of window function as BT increases, which can be seen in Figrue 6.

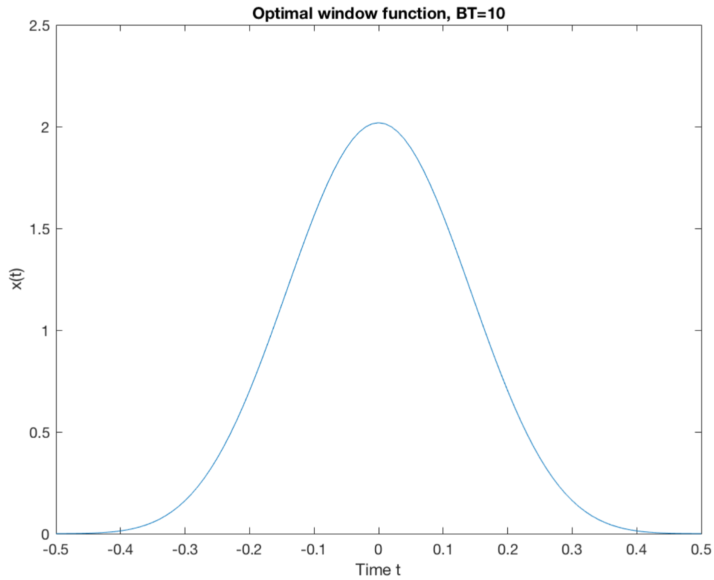


Figure 4: Optimal window function, T=1, BT=10

The Kernel function $K(\tau)$ is an impulse response corresponding to the raised cosine spectrum in terms of the function $H(\omega)$ appearing in (11). The raised cosine frequency characteristic is given as,

$$|H(f)|^2 = X_{rc}(f) = \begin{cases} T & \left(0 \leqslant |f| \leqslant \frac{1-\beta}{2T}\right) \\ \frac{T}{2}\left\{1+\cos\left[\frac{\pi T}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right]\right\} & \left(\frac{1-\beta}{2T} \leqslant |f| \leqslant \frac{1+\beta}{2T}\right) \\ 0 & \left(|f| > \frac{1+\beta}{2T}\right), \end{cases} \tag{15}$$

where β is called the *rolloff factor* and takes values in the range $0 \leq \beta \leq 1$. $H(f)$ expresses the frequency response of the transmitter, see [7].

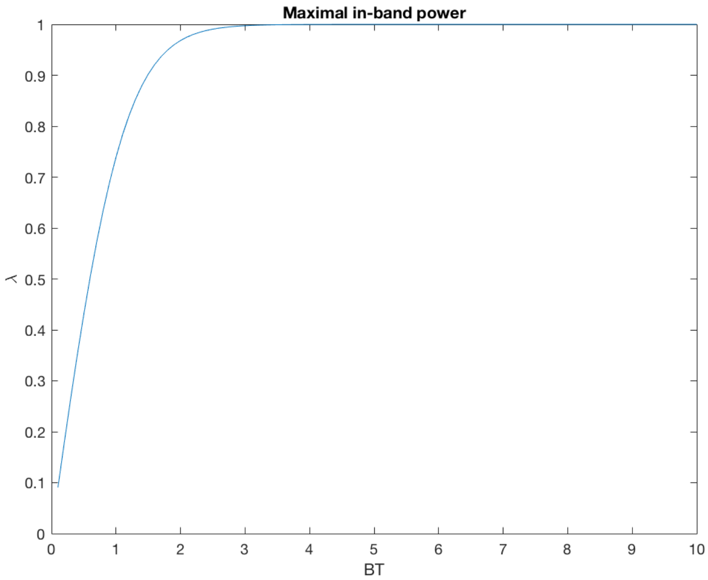


Figure 5: Maximal in-band power with respect to BT

The pulse $K(\tau)$, having the raised cosie spectrum, is

$$K(\tau) = \frac{\sin(\pi\tau/T)}{\pi\tau/T}\frac{\cos(\pi\beta\tau/T)}{1-4\beta^2\tau^2/T^2} = \text{sinc}(\pi\tau/T)\frac{\cos(\pi\beta\tau/T)}{1-4\beta^2\tau^2/T^2}, \tag{16}$$

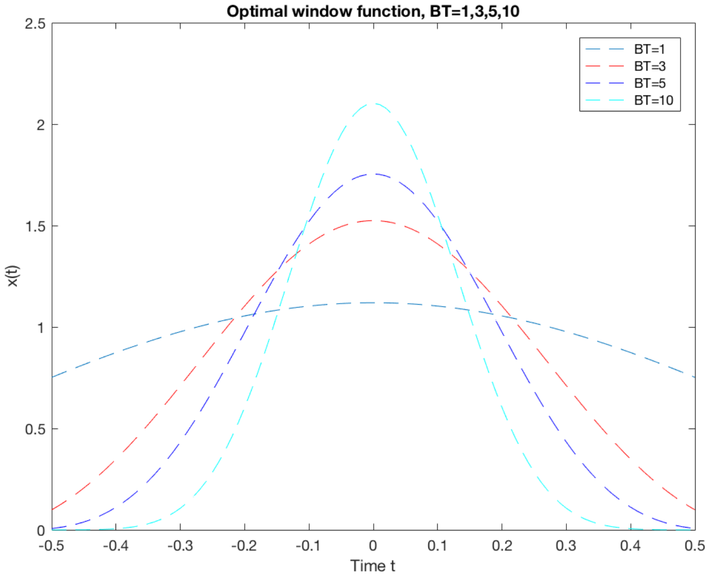
where $\tau = t - u$. Note that for β =0, the pulse reduces to $K(\tau) = \text{sinc}(\pi\tau/T)$, see [7].

Figure 6(a) presents a comparison of the sincfunction forms with BT=1,3,5,10. As the time period is locked to T=1, the sincfunction presents a convergence distribution as BT increases. The gradual decrease of the function spillover reflects the gradual concentration of the energy within the band.

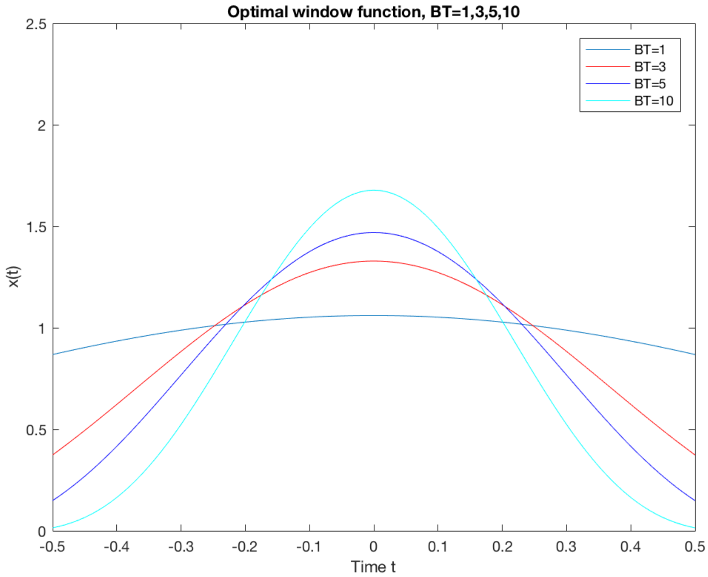
Figure 6(b) shows the comparison of eigenfunction forms of the raised cosine spectrum with BT=1,3,5,10, when β is chosen to be 0.5. The change of forms is similar to the sincfunction but is flatter.

Figure 7 illustrates the raised cosine spectral characteristics and the corresponding pulses for β =0, 0.5 and 0.8 when BT=10. As β increases, the forms tend to have gentler slopes, which is opposite to trend displayed for the parameter BT.

Figure 8 displays a comparison of the corresponding maximal in-band power expressed by λ, for different β =0, 0.5 and 0.8, with respect to BT. As can be seen, the *rolloff factor* β increasingly exerts a tremendous influence on the forms of the maximal in-band power.



[a]



[b]

Figure 6: Comparison of sincfunction and eigenfunction forms with BT=1,3,5,10

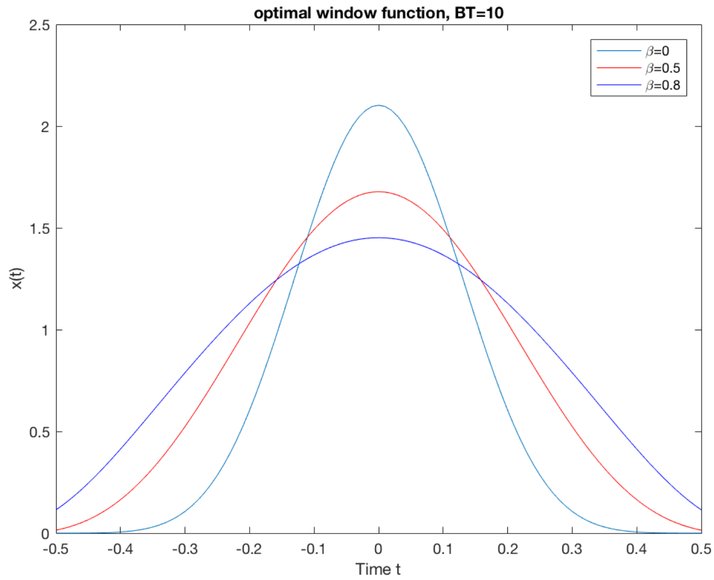


Figure 7: Comparison of eigenfunction forms for different β =0,0.5,0.8

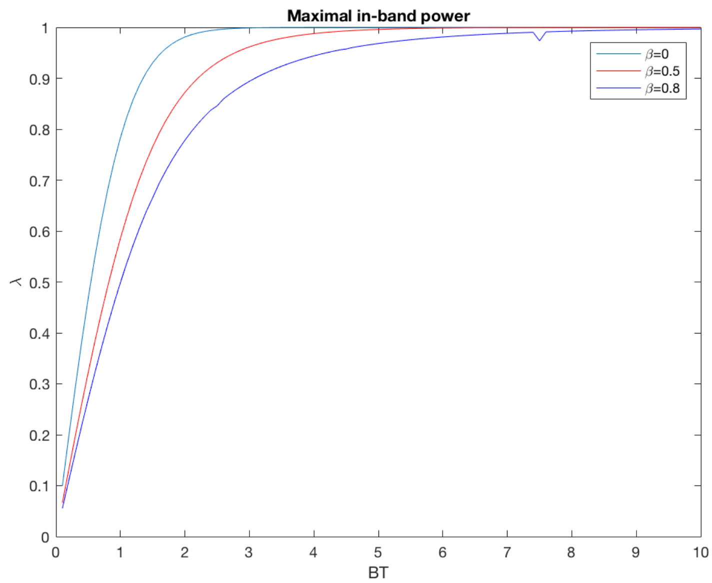


Figure 8: Comparison of the maximal in-band power for different β =0,0.5,0.8

In this design example, we make the following observations: the sincfunctions with finite time duration constraints can be optimized by maximizing the energy within a prescribed frequency band at the channel output. The smooth characteristics of the raised cosine spectrum of a particular signal pulse, means that it can be used widely in practical applications. When the value of *rolloff factor* β is larger, it is clearly seen that the speed of the increase of the in-band power decreases.

4 Conclusions

In this thesis, the problems related to optimum signal design based on the Generalized Uncertainty Principle have been discussed and solved.

The theoretical derivations solved the eigenvalue problem for an integral operator with the classical sinc-kernel. The solution was given by the classical prolate spheroidal wave functions. More precisely, the optimization formulation led to an eigenvalue problem for an integral equation, which was solved numerically by using the quadrature methods and an eigendecomposition of the corresponding matrix.

Finally, the Generalized Uncertainty Principle provided the theoretical basis for this thesis work. A more in-depth research would be focused on the reduction of computational complexity and the enhancement of the accuracy of the eigenvalues.

Appendix

A. Discretization by Nyströms method

The operator eigenvalue problem (14) is approximated by using a suitable quadrature rule similar to Nyströms method for integral equations of the first or second kind [14]. Here, the quadrature rule for an arbitrary integrand $f(t)$ is of the form

$$\int_{-T/2}^{T/2} f(t)dt = \sum_{i=1}^{n} w_i f(t_i) + E_n(f), \tag{17}$$

where $t_i \in [-T/2, T/2]$ are the prescribed quadrature points, w_i the quadrature weights and $E_n(f)$ the error term, *cf.*, [9, 14]. The discrete approximation of the eigenvalue problem (14) is then given by

$$\sum_{j=1}^{n} w_j K(t_i, u_j)\phi_n(u_j) = \lambda\phi_n(t_i), \tag{18}$$

where $i = 1, \ldots, n$, and $\phi_n(t)$ denotes the solution to the approximated integral equation [14]. The system (18) can be written in matrix form as

$$K\phi = \lambda\phi, \tag{19}$$

where $K$is an $n \times n$ matrix with elements

$$[K]_{ij} = w_j K(t_i, u_j), \tag{20}$$

and ϕ is an $n \times 1$ eigenvector vector of signal samples

$$[\phi]_i = \phi_n(t_i), \tag{21}$$

and λ denotes the eigenvalue.

B. Quadrature rule based on rectangular approximation

A simple discretization is obtained by employing uniformly sampled quadrature points

$$t_i = -T/2 + (i-1)h, \tag{22}$$

for $i = 1, \ldots, n$, where $h = T/n$, and a piecewise constant (rectangular) approximation of the integrand yielding

$$w_i = h, \tag{23}$$

for all $i = 1, \ldots, n$. This simple quadrature rule will work, but its convergence properties are expected to be inferior to the composite Simpson's rule and the composite Gauss quadrature described below.

C. Composite Simpson's rule

The elementary Simpson's rule is given by

$$\int_a^b f(t)dt = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right] + E(f), \tag{24}$$

where $h = \frac{b-a}{2}$ and $E(f)$ is the error term. The elementary Simpon's rule is exact for polynomials of order less then or equal to 3, *cf.*, [9, 15]. The composite Simposon's rule is obtained by applying the elementary rule to a sequence of subintervals yielding

$$\int_a^b f(t)t = \frac{h}{3}\left[f(t_1) + 4f(t_2) + 2f(t_3) + \cdots \right.$$
$$\left. + 2f(t_{n-2}) + 4f(t_{n-1}) + f(t_n)\right] + E_n(f), \tag{25}$$

where $h = \frac{b-a}{n-1}$, n is odd and the quadrature points are given by

$$t_i = a + (i-1)h, \tag{26}$$

for $i = 1, \ldots, n$. The error term is given by

$$E_n(f) = -\frac{b-a}{180}h^4 f^{(4)}(\eta), \tag{27}$$

where $a < \eta < b$, see [9].

Using the composite Simpson's rule, the discretization in (19) and (20) is given by the quadrature weights

$$[w_1 \cdots w_n] = \frac{h}{3}\left[1\ 4\ 2\ 4\ \cdots\ 2\ 4\ 1\right], \tag{28}$$

where $h = \frac{T}{n-1}$, n is odd and the quadrature points are given by $t_i = -T/2 + (i-1)h$ for $i = 1, \ldots, n$.

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