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Analysing students’ graphicacy from a national test
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The present study explores a task containing graphical artefacts from the Swedish national test (Nationella provet – NP) for a sub-sample of grade 9 students’ solutions. The sub-sample comprising of 115 students’ solutions to the task is closely analysed, using an analytical construct founded on “identification” as well as the “critical-analytical” approach to problem solving. Based on this construct it is observed that a sizable number of students’ solutions follow a visual strategy with strong reliance on everyday forms of expression. Given the nature and purpose of NP we posit that students use the methods and tools that reflect general school practice. The analysis used in this study is perceived as problematising the assessment of competency from high stake tests, and the educational setting in general.

Keywords: Assessment, examinations, graphical artefacts, misleading diagram, competencies.

Introduction

National tests generally play a vital role in the evaluation of educational systems, informing educational reforms and policy implementations as well as serving as a local comparative instrument (Eurydice, 2009; cf Skolverket, 2016). This implies that results from national tests can be used to explore the effects of curriculum documents on teaching and learning. Some researchers, (e.g. Boesen 2006) have suggested that centrally administered tests can influence instructional practices, for instance by guiding the time set aside to teaching specific topic strands in the mathematics classroom. Given the orientation towards competencies in the mathematics classroom, it is also envisaged that the Swedish national test (Nationella provet – NP) might be a suitable indicator of teaching and learning of mathematics competencies. Thus, national tests can be perceived as having the potential to provide insight on students’ fluency with mathematical concepts, as well as mathematics competencies (NCTM 2000; Niss & Højgaard, 2011; Skolverket, 2011; see also Sáenz, 2008).

The goals of the study

The aim of the present study is to gain insight into the strategies and approaches that some students at the end of the compulsory school in Sweden employ, as they interact with a mathematics task containing graphical artefacts. This study is based on written solutions from NP. The purpose is to explore the strategies and tools used to solve this task, as well as to determine the potential provided by a model focusing on mathematical tools and forms of expression as a means of exploring students’ mathematics competency.

Background and theoretical construct

Several methods have been used to analyse students’ responses to test items (e.g. Åberg-Bengtsson, 2005; Goodchild & Grevholm, 2007). In some of these studies the focus has been on general performance with microanalysis done at topics strand levels. While there are models proposed for examining the response to tasks (e.g. Gal, 2002, 1998; Ben-Zvi & Arcavi, 2001; Watson & Callingham, 2003), these seem to suggest general skills needed to solve the task with a bias towards statistical literacy. Friel, Curcio and Bright (2001) developed a three-tier model entirely devoted to
graphical artefacts: without loss of generality this can be collapsed into two tiers (cf. Gal, 1998; Bertin, 1983; Olande, 2013). While there are models for analysing students’ test results, there is a dearth of studies exploring the use of subject specific tools and forms of expression in students’ solutions.

In the present study an analytical construct (see Olande, 2013), focusing on the application of mathematics tools and forms of expression while solving test items, is employed. In this construct response to test items is perceived as being oriented towards identification and critical-analytical approaches. Items eliciting the identification approach are largely “self-evident” and as such, problem solvers might not need to unpack their mathematical skills entirely in order to solve the problem. On the other hand, items seen as demanding a critical-analytical approach require focused engagement from the problem solver. For example, this could be in the form of a critical analysis of underlying factors in the task, evaluation and selection of appropriate tools for interacting with the task, as well as reporting the solution with relevant subject specific forms of expression.

The construct guiding the present study borrows from a socio-semiotic paradigm where the emphasis is on artefacts as a means of coming to know. Radford (2008) posits that the investigation of students’ interaction and use of semiotic means of objectification is a methodological way of accounting for learning (see also Radford, 2003; Vygotsky, 1978). It is recognised that a sign or symbol does not exist in isolation but is always bound with intentions, motives and the objects of action (Roth, 2008; Olande, 2014). Thus, in a critical-analytical approach, “being critical” encompasses more than visually interrogating a graphical artefact, but also includes the means and the tools used in the sense making process. The assumption about what can be perceived as a non-hierarchical path to cognition is significant for the analytical framework: i.e. it is access to tools and forms of expression that is perceived as largely determining a problem-solving trajectory.

The research questions

Thus, the concern of the study is to outline and analyse tools and forms of expressions used by students as they interact with a task from NP. This task was picked from section C of the national test, a section that requires students to justify in one way or another how they arrive at their solutions. The task (figure 1) was selected for further analysis given that it provided a combination of visual-identification as well as critical-analytical components. This way of assessing the task is different from the marking process employed by graders while awarding credit to students’ solutions. The marking scheme used by graders did not indicate assessment of diverse solutions provided by students but largely gave written statements as guidelines. Thus item a) scored full credit when the solution contained the expression TRUE with corresponding justification such as making a comparison based on the sizes of the shaded areas. For item b) partial credit was awarded where the solution contained the expression FALSE with corresponding justification indicating an understanding that the pie charts express different quantities e.g. “Australia has more medals than Spain”. Full credit was awarded where the solution, in addition to the general statement FALSE, explicitly made comparisons based on mathematical forms of expression e.g. computation with fractions. A reliability check conducted by an independent entity (Skolinspektionen, 2010) from a representative sample of Swedish students for section C of the test, indicated that for items scoring grade G - (for the purpose of this study partial credit) in 51% of the cases the graders gave higher grades than the assessors. In 27% of the cases there was correspondence in the credit award. For items scoring VG - “full credit” in 37% of the cases
the graders’ credit award was higher than that of the assessors, with 41% indicating correspondence in the credit award.

From a competency perspective (Skolverket, 2011), this task might be considered to focus on developing competencies in the following: using and analysing mathematical concepts, use of appropriate mathematical methods to solve problems, and the use the of mathematical forms of expression to discuss, reason and give account of questions, calculations and conclusions. However, given that the task explicitly called for a justification of the student’s given solution, it was generally perceived as eliciting a critical-analytical approach. The success rate for item 9a was 78% for both girls and boys, while the success rate for item 9b was 47% and 55% for girls and boys respectively.

**Figure 1: Task No. 9 from NP Sweden**

The research questions are outlined as:

1. What range of tools and forms of expression are made manifest as students interact with the task containing graphical artefacts?

2. How does tool selection and use impact on the characteristics of the task solution provided?

**Task analysis**

Students’ solutions to task No. 9 were closely analysed with respect to: a) forms of expression (see Table 1 for typical examples of student responses) mathematical, everyday, graphical or one-word and b) tool use/sign of tool accessibility – mathematical operators and symbols. With regard to tool use seven approaches to task solving were identified, namely: i) visual comparison of graphical artefact, ii) critical - questioning the production of the graph, iii) fraction, iv) proportion, v) percent, vi) division, vii) multiplication and viii) other solutions.
Mathematical

Britain ≈ 5/12 ≈ 0.42 40% gold (18.8 medals)

Australia ≈ 35/120 ≈ 0.30 30% gold (13.8 medals)

Answer: true

Everyday

True: Britain obtained most gold in its diagram and Australia more silver.

Graphical

Yes! Britain’s bit is larger

One-Word

False

Yes Spain’s bit is larger

Table 1: Forms of expression identified from test task

Based on aspects of identification and critical-analytical approaches (Olande, 2013) and aspects of Sfard’s (2008; cf. Schleppegrell, 2007) categories of mathematical discourse, the author developed a coding scheme and analysed the students’ solutions. The focus was on the forms of expression and mathematical tools – concepts used in working out a solution, rather than the correctness of students’ solutions. The correctness or otherwise of the item solution was pegged on the test score awarded by test graders within the framework of the test situation. Significantly, within this coding scheme the perception of the forms of expression was as follows: 1) everyday – the use of causal expressions wherein aspects of the obviousness of solution are embedded. In Table 1, while the type of language used in the solution is everyday, it is apparent that there are quantities being compared: “Britain obtained most gold” and “Australia more silver” 2) mathematical – the use of mathematical concepts and methods in the solution 3) graphical – the use of illustrations in an attempt to amplify the visual aspects of the task. In the illustration given in Table 1 above while the solution for item b) did not receive credit award from the graders, the solution appears to be appealing to the visual faculties in a comparison exercise. Zooming in on gold or silver, the student seem to be posing the question can’t you see they are different?

Results and analysis

The success rate for task No. 9 seems to indicate that students did not have as much difficulty with aspects of item 9a as compared to item 9b. This can be explained in part by the nature and the array of tools needed for effective interaction with the different items. Thus, the different forms of expression of the items were analysed.

Table 2: Forms of expression identified from item 9a

<table>
<thead>
<tr>
<th>Form of expression</th>
<th>Everyday</th>
<th>Mathematics</th>
<th>One-word</th>
<th>Graphical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct responses</td>
<td>0.72</td>
<td>0.15</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>Incorrect responses</td>
<td>0.11</td>
<td>0.00</td>
<td>0.60</td>
<td>0.00</td>
</tr>
<tr>
<td>No response</td>
<td>0.00</td>
<td>0.05</td>
<td>0.08</td>
<td>0.00</td>
</tr>
</tbody>
</table>
For this item, a majority (72%) of the students’ solutions used everyday forms of expression, of these 89% provided successful solutions. While only 15% of the students used mathematical forms of expression, the success rate was relatively high at 95%. Students providing a one-word response to the task gave the majority of unsuccessful solutions. For this item it was observed that more than 50% of the students used visual comparison and/or the comparison of totals.

The general pattern for forms of expression for item 9b was no different for item 9a (see Table 2).

<table>
<thead>
<tr>
<th>Form of expression</th>
<th>Everyday</th>
<th>Mathematics</th>
<th>One-word</th>
<th>Graphical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full credit</td>
<td>0.50</td>
<td>0.32</td>
<td>0.13</td>
<td>0.04</td>
</tr>
<tr>
<td>Partial credit</td>
<td>0.43</td>
<td>0.04</td>
<td>0.13</td>
<td>0.00</td>
</tr>
<tr>
<td>Incorrect responses</td>
<td>0.27</td>
<td>0.02</td>
<td>0.79</td>
<td>0.75</td>
</tr>
<tr>
<td>No response</td>
<td>0.00</td>
<td>0.05</td>
<td>0.08</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 3: Forms of expression identified from item 9b

It is noteworthy that students providing one-word answers and those using graphical strategies recorded rather higher rates of incorrect responses. Evidently, students’ results indicate that the use of forms of expressions reflects one way or another on the success rate of their solutions. Consequently, it is of particular interest to explore the subject specific tools and forms of expression used in solving the task, and how these might have impacted on the quality of the response provided. This analysis is conducted for item 9b (see figure 2).

Figure 2: Frequency of tool use and credit award

From figure 2 it is shown that most of the solutions awarded full credit included the application of mathematical symbols and calculation. For these solutions the predominant tools of manipulation/calculation were fraction, division and percent. The solutions providing incorrect responses seem to be largely based on visual strategies.
From the results it is apparent that students using mathematical tools and forms of expression had a relatively higher success rate. The predominant tools used in solving the task were fraction, division and percentage. The frequency with which fraction and division are used is not entirely unexpected since these tools are closely related; the same applies to some extent to percent. It could be suggested that the task in some way elicits the use of fraction: It is not uncommon for mathematics teachers to use pie charts in the teaching of, or in the introduction of, the topic strand of fraction. From this approach there is a natural connection to percentage; a circle divided into two gives two halves – this is often perceived as 50-50 (%). The results also indicate that some of the students using these tools (fraction, percentage) experienced some difficulty in application. It is worth noting that the students using multiplication obtained full credit. These students seem to employ a tool that might not be considered “self-evident” for the task. This necessitated using the tool in a creative manner, thus indicating a higher level of confidence and “procifiency” in using the tool.

For the students scoring full credit for the items, it is observed that most of them are proficient in the use of item specific tools namely, fraction, division and multiplication. From the students’ solutions it seems that only a small number indicate some aspect of interrogating the purpose of the graphics. Indeed there seems to be a general scarcity of solutions communicated using subject specific forms of expression, that is, mathematical language.

**Discussion**

The present study sought to apply a construct focusing tools and forms expression in solving mathematics tasks containing graphical artefacts. The purpose was to outline the usage of tools and forms of expression, and the quality of the solution thereof. A deeper analysis of task No. 9 provided more insight into strategies and tools used in interacting with graphical artefacts. It is observed that it is in part the grasp of the tool in use that determines the quality of the solution given. Based on the level of confidence in tool use, it is possible for the test taker to interrogate the task from different perspectives. Based on what can be considered as overlapping tool use (see figure 2), the results are perceived as suggesting that “reading” a graphical artefact can be a complex undertaking that might involve *reading the graph – reading within the graph – reading beyond the graph* (cf Friel et. al., 2001). In the case of this task, there is indeed a different array of tools available to the students as they solve the task. However, it is communication using subject specific forms of expression that appears to be wanting – this might be an indicator that as much as the tools are available and “visible” to some of the students, the competency to apply and organize the same to produce a sound solution is a major challenge. This was observed in the case of students indicating knowledge of appropriate tools needed to solve the task, but apparently lacking the necessary skills to effectively apply the same in a problem-solving situation. Thus the observation made in the present study underscores the importance of having a solid foundation in the use of mathematical tools and forms of expression (concepts) in different settings. The importance of the use of subject specific forms of expression is also observed. For item b) there was higher correspondence in credit award between the graders and assessors as compared to item a) which did not elicit the use of the subject specific forms of expression as such. The analytical framework employed in the present study also helped to identify the strengths and weakness in students’ written solutions, thus providing valuable indicators for developing classroom practice.
Given the interest in, and the focus on mathematics competencies in the Swedish mathematics classroom, the present study can also be perceived as drawing attention to the practice of assessment: if the concept of mathematical competencies implies imparting aspects of such skills as mathematicians use in the processes of mathematisation (cf Niss & Højgaard, 2011; Sfard, 2008), then assessment practice might need to be refocused to examine such use of tools and forms of expression that enable the learner to understand, and to participate in, activities within the mathematics community.

References


Olande, O. (2013). *Students’ narratives from graphical artefacts: Exploring the use of mathematics tools and forms of expression in students, graphicacy*. Sundsvall: Mid-Sweden University.


