Analysis of a Prefabricated Concrete Skew Angle Slab Bridge

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Abstract

Prefabricated concrete elements are widely used in the construction industry today. With advantages such as time savings, increased safety at the construction site and minimized material usage, prefab becomes a major challenger to the traditional on-site casting construction method. However, constructing a bridge in concrete still presents challenges when using prefab as a construction method. Hence, more research in the area is needed.

This master thesis has been studying the behavior of a prefabricated skew angle slab and the connection between the slab and wall elements of a bridge. The study was conducted using a finite element software, where three 3D-models of skew angle slabs were created. The three models had different skew angles (0, 15 and 30 degrees) and crossed the same path. The models could represent both the slab and the slab-wall connection.

The finite element analysis showed that slabs with angles up to 15 degrees could be designed as a straight bridge. However, when the skew angle increases to 30 degrees, the behavior of the slab and connection changes significantly. Furthermore, the results show that a stress concentration occurs in the obtuse corner and that the stress increases when the skew angle increases. Moreover, there is a slight uplift in the acute corner when the skew angle increases to 30 degrees.

Keywords: Finite Element Model, Skew Angle Slabs, Slab Bridge, Prefabrication.
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1 Introduction

The world is facing many challenges and problems today considering the state of the environment. Global warming, pollution caused by overuse of the planet’s resources and emissions of carbon dioxide are steadily increasing [1]. Hence, the European Union (EU) has established environmental goals to limit global warming. In numbers, the goal is to reduce the carbon dioxide emissions by 20 % from 1990 to 2020. Furthermore, there are countries that have created even stricter goals than the EU as ambition. For example, Sweden has the national ambition to reduce emissions by 40 % from 1990 to 2020 [2]. To achieve these goals, all industries need to cooperate and take responsibility.

A third of the world’s carbon dioxide emissions originate from the construction industry [1]. Therefore, there is a global interest to increase sustainability in the industry. A considerable part of the industry’s emissions comes from the production of construction materials. One of the most debated materials is concrete. Because of its versatile properties such as moldability, durability, and strength, concrete is a common material to use when building houses, roads, bridges etc. [3]. The manufacturing of concrete begins with extracting the cement from burnt lime. Then, the cement is mixed with stone aggregate and water. The cement production demands much energy and emits a considerable amount of carbon dioxide. Furthermore, bigger and more complex structures require high strength concrete that consists of a greater part of cement.

In the infrastructure industry, there is a constant need for new and improved designs and solutions. Sweden alone is estimated to have more than 16 000 road bridges and that number is constantly increasing [4]. When constructing bridges, the main material used is concrete. A common method of construction is on-site casting, which lately has been challenged by prefabrication (pre-fab) methods, which are safer and more time efficient. However, on-site casting has its advantages compared to prefabrication, because of its flexibility. For example, the prefab method is limited to the factory molds’ dimensions, angles, and sizes. The dimensions of the molds are determined by both space and capacity of the transport vehicles delivering the prefab elements to the construction site. On-site casting does not have these limitations. [5]

1.1 Background and problem description

During recent years, prefabrication has increased in the construction industry because of its time efficiency compared to on-site casting. Furthermore, the production of the concrete elements happens in a factory with a controlled climate in premade molding casts, which results in a final product with higher qualities regarding strength and usage of time and material. The
elements are optimized depending on amount of material used. Moreover, the production method is based on an assembly line principle, which is also safer for the workers. With the use of pre-fab, the construction time can be reduced and major environmental consequences and job related accidents could be avoided [5]. According to Håkan Sundqvist [6], the most common accidents on a construction site during on location casting occur because of form work failure. In the factory, form work failure poses little or no risk for human life or injuries.

Bridges have not undergone the same transition from casting on-site to using prefabrication elements. One of the reasons for this is that prefabrication has a bad reputation among designers in the bridge field [5]. However, when designing a straight bridge, prefabrication is a feasible construction method. But, when the intersecting roads cross each other with an angle, the pre-fab method has limitations resulting from its lack of flexibility. A prefabricated bridge generally consists of different elements. The reason for this is to make transportation of the elements possible from the factory to the construction site. There are three base components for a straight slab bridge: slab and wall elements and distance beams as shown in Figure 1. The number of elements can be changed to fit the specific construction site, to get different span widths and widths of the bridge.

![Figure 1: A prefabricated straight slab bridge, with different elements.](image)

The industry is looking for an effective method for constructing a prefabricated skew angle bridge. Figure 2 illustrates two alternatives on how to use prefabrication as a solution for skew angle bridges. One solution is to use straight slab elements and let the road cross diagonally. But, this will leave areas of the bridge redundant and lead to material waste. Also, the design may not be aesthetically appealing. The more efficient solution is to use skewed slabs with the same angle as the crossing road. The biggest
difference between the two solutions is the difference in stress and deflection distribution.

Figure 2: Two alternatives where a road is crossing by a bridge with an angle that is not perpendicular, so called skew angle: a prefabricated bridge with straight slab elements (left) and a prefabricated bridge with skewed slab elements (right).

In Figure 3, some of the terms used in this study are visually explained. The figure shows a skew angled slab.

Figure 3: Technical terms used in this master thesis.
1.2 Aim and purpose

The aim of this thesis is to:

- Establish FE-models that are able to represent the behavior of the slab and the connection between the slab and wall element, for various skew angles, regarding stresses and displacement.

- Investigate the stress and deflection distribution changes in the slab due to the skew angle.

- Investigate the response in the connection for different skew angles.

The purpose is to investigate the possibilities and limitations when prefabricating a skew angled concrete bridge. Furthermore, the purpose is to provide a solution for the bridge without unused areas and thus minimizing material usage that results in reduced environmental consequences. Additionally, the ambition of this project is to contribute and widen the knowledge of the influence of the skew angle when designing a prefabricated concrete bridge.

1.3 Hypothesis and limitations

1.3.1 Hypothesis

The skew angle will affect the response of the bridge in numerous ways such as deflection, stress concentration, bending moment, and shear stresses. Furthermore, the skew will lead to a larger torsional moment that may affect the slab element performance.

A finite element model of a bridge element will monitor the slab’s limitations and predict which parts of the slab element are exposed to the highest stresses.

1.3.2 Limitations

The limitations for this paper are:

- the study carried out will only investigate bridges with the skew angle of 0, 15 and 30 degrees,

- the opening underneath the bridge will have the same width, 5 meters, for all the skew angles,

- the width of the slab will be 2.4 meters for all slabs,
- the finite element analysis will be focused on the slab element and the connection between the slab and the wall element of the bridge,

- the slabs will be simply supported,

- the bridge used as reference object is designed by the Swedish company Abetong AB,

- the finite element analysis will not include non-linear material behavior,

- the materials used in the analysis will be concrete and steel,

- interaction in the joints between the slab elements will not be considered.

1.4 Reliability, validity and objectivity

Using a finite element software to create a model for the construction will lead to a more valid result when analyzing the stress of the bridge. This is because a numerical FEM analysis is widely used in research and industry today. However, the model is not calibrated by experimental data that would give further reliability for the model. Hence, the work conducted in this thesis is strictly based on theoretical data.

The reference object is designed by the company Abetong AB. However, the company has not influenced the results. The results and analysis were obtained by the authors.

1.5 Authors’ contribution

This thesis is a cooperative work, in which both authors have equally contributed to all parts of the study conducted.
2 Literature Review

In this chapter, previous studies of reinforced skew angle concrete bridges will be presented and reviewed. Research about the behavior of skew angle slabs are of great interest. Studies about the effect of skew angle slabs using a finite element modeling are also included in this section. Linear and non-linear FE-models will be reviewed.

2.1 The behavior of a skew angle bridge

In this section, articles and reports regarding the behavior of concrete slab with various skew angles are reviewed.

The load path, regarding all types of loads, in skew slabs tends to take the shortest path through the strip of area (E) to the obtuse corners. Increasing the skew angle leads to a decrease of the length between the obtuse corners. The maximum deflection occurs near the obtuse corners. Figure 4 shows the direction of moment flow in a skew angle bridge [7].

![Figure 4: Direction of moment flow in a skew slab (taken from Dhar et al. [7]).](image)

The effect of the skew angle on a bridge slab can be summarized by:

- increase in transverse moment,
- decrease in longitudinal moment,
- significant torsional moments in the deck slab,
- high reaction and torsion near the obtuse corner,
- possible uplift in the acute corner,

and is shown in Figure 5.
The Swedish Transport Administration [8] shows that a skew slab with an angle lower than 10 degrees can be considered as a straight slab with the theoretical span with the same parallel span. However, multiple studies have stated that skew angle slabs up to 15-20 degrees can be considered as a straight slab [7]. According to Menassa et al. [9], a bridge with a skew angle less than 20 degrees can be considered as straight.

2.2 Finite element modeling of concrete skew slabs

In this section, articles and reports examining skew bridges with finite element software are reviewed. The main focus is to review and take part of the possible results and previous problem when studying a skew slab with a FE-software.

A study conducted by Vikash Khatri et al. [10] compared two methods when analyzing skew bridges, the grillage methods, and the Finite Element Method. The grillage methods are used for structural analysis and design software. In the report, Vikash Khatri et al. stated that the Finite Element Method was recommended to use when analyzing bridges. The conclusion of the study was that the numerical finite element analysis resulted in a solution closer to the exact theoretical solution.

2.2.1 Deflection

A study conducted by Sindhu et al. [11] presents a parametric study of a skew angle bridge using finite element analysis. The result shows the effect of skew concrete bridges with various angles regarding critical structural

Figure 5: Summarize of behavior for a skew slab (taken from Dhar et al. [7]).
parameters. For instance, when the skew angle increases, the maximum deflection will also increase. The reason for this is that the force flow is taking a shortcut between the two obtuse corners of the support lines. Moreover, the maximum deflection occurs near the obtuse corners of the slab.

Arindam Dhar et al. [7] studied the effect of the skew angle on slab deflection and stress for a simply supported T-section beam bridge. In their study, the software Abaqus was used and it was stated that Abaqus is a useful tool when calculating the deflection in skew slabs. The result showed that the deflection of the obtuse corner grows linearly until a skew angle of 45 degrees. After 45 degrees, the curve gradually evens out. On the other side of the support, the acute corner is decreasing for angles between 0 and 65 degrees. The deflection in the acute corner is reduced by 60%. Figure 6 shows the results of the change in average deflection for various skew angles. Furthermore, the study presents an increasing deflection in the obtuse corner and a decreasing deflection in the acute corner for a slab with a higher skew angle.

![Figure 6: Difference in average deflection for various skew angles, for the acute and obtuse corner (taken from Dhar et al. [7]).](image)

The stress in the acute corner gradually approaches zero stress when the skew angle reaches 65 degrees. Furthermore, the biggest change of stress in the acute corner happens between 30 and 45 degrees of skew.

This is illustrated in Figure 7 as well as the change in shear stress (S13) in the acute corner.
A possible result of increased skew angle is an uplift in the acute corner. The elevation is most likely to happen when the skew angle gets higher than 50 degrees.

2.2.2 Failure analysis of skew angle bridges

An article review conducted by Bagge et al. [12] investigates different failure on various types of bridges. All the studies that were reviewed examined field tests. The research investigated a total of 40 different types of failure tests on 30 bridges around the world. The main conclusion was that 28% of the test objects failed in an unpredictable way. This indicates that it is challenging to predict what type of failure will occur in a bridge. Hence, the predictions that are made during the dimensioning need to be accurate in order to avoid unexpected failure modes. Accordingly, it is important that models are valid and that the non-linear behavior of the construction is considered when predicting failure modes.

A study made by Abozaid et al. [13] shows that increasing the compressive strength of the reinforced concrete leads to a major effect on the cracking load, ultimate load and ductility factor. But there is no major effect on deflection, steel stresses and principal concrete strain. As stated by Fatemi et al. [14], a skew slab with the same compressive strength and the same geometry as a straight slab cannot sustain the same ultimate load.

To make the model accurate, it can be calibrated with experimental results. This is done by Fatemi et al. [14] when investigating skewed angled bridges.
using ultra-high performance fiber reinforced concrete. Furthermore, it is of high importance to consider the non-linearity of the concrete when calculating the load bearing capacity during and after cracking of the material. Furthermore, a skew angle slab shows greater displacements for the same load and boundary conditions compared to those of a straight slab [13].

A study performed by Abozaid et al. [13] explored the non-linear behavior of a skew slab bridge under traffic loads. The data were collected from a finite element model with non-linear material properties. The study compared failure loads for a bridge with different skew angles. It was found that with a bigger skew angle the failure load got smaller. Moreover, the conclusions of Abozaid et al. study are comparable with earlier mentioned research [13]- [15] conclusions.

2.2.3 Moment

A result was obtained by Raj and Phani [15], where finite element models of skewed highway bridges were studied. It was found that the torsional moment increased proportionally with the skew angle. The effect of this will be a greater risk of elevation (lifting) of the slab corners. There is no need to consider a torsional moment when constructing a straight bridge because of its small magnitude.

Regarding the longitudinal bending moment, previous research shows that a growing skew angle results in a reduced bending moment [15], [11]. This is caused by the force flow between the support and the slab. The presented articles show similar results regarding deformation, skew angle limits, etc.
3 Theory

In this chapter, calculation models and material properties that are used in this thesis will be presented.

3.1 Concrete

Concrete is a hard and robust material with a long life span that is widely used in the construction industry. The material is a composition of mainly water, stone aggregate and cement. The properties of hardened concrete are reminiscent to those of stone. Therefore, concrete is frequently used when constructing bridges [16].

3.1.1 Strength classes

The strength properties of concrete are different when subjected to tension or compression. The compressive strength is higher than the tensile strength. These properties must be considered when designing concrete structures that are exposed to tensile stresses; otherwise, there is a major risk for tensile failure. To minimize the risk of tensile failure in the construction, steel bars are molded into the concrete where the tensile stresses occur. Steel has a higher tensile strength than concrete and will compensate for the concrete’s lack of tensile properties. Hence, the interesting property is the compressive strength of the concrete and the tensile strength of the steel bars [17].

In the industry today, there are several strength classes that label the strength of the concrete. The strength classes are graded after the strength capacity of the concrete, obtained in cube and cylinder compression tests. The classes are then defined by the results of this test; for example, C35/40, where 35 refers to the cylindrical compressive strength and 40 to the cubical strength in MPa. In Eurocode 2 [18], the characteristic cylindrical compressive strength is called $f_{ck}$ and the cubical $f_{ck,cub}$. The Eurocode also expresses the mean values for compressive strength $f_{cm}$ and tensile strength $f_{tcm}$. It is stated that the strain at maximum stress is $\varepsilon_{cl}$ and the maximum strain at failure is $\varepsilon_{cul}$. These values are given in Table 1 for common concrete classes.
Table 1: Properties of concrete for different strength classes.

<table>
<thead>
<tr>
<th></th>
<th>C20/25</th>
<th>C25/30</th>
<th>C30/37</th>
<th>C35/45</th>
<th>C40/50</th>
<th>C45/55</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{ck})</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td>(f_{ck,cub})</td>
<td>25</td>
<td>30</td>
<td>37</td>
<td>45</td>
<td>50</td>
<td>55</td>
</tr>
<tr>
<td>(f_{cm})</td>
<td>28</td>
<td>33</td>
<td>38</td>
<td>43</td>
<td>48</td>
<td>53</td>
</tr>
<tr>
<td>(f_{cm})</td>
<td>2.2</td>
<td>2.6</td>
<td>2.9</td>
<td>3.2</td>
<td>3.5</td>
<td>3.8</td>
</tr>
<tr>
<td>(\varepsilon_{cu1})</td>
<td></td>
<td></td>
<td>3.5‰</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values for Young’s Modulus, \(E_{cm}\), are estimated to be:

\[
E_{cm} = 22 \left(\frac{f_{cm}}{10}\right)^{0.3}
\]  

3.1.2 Water Cement Ratio

The water cement ratio, \((wcr)\), is a key factor that affects the compressive strength. \(wcr\) denotes the ratio between amount of water and cement. The strength and durability of the concrete increases with a lower \(wcr\). However, concrete with a low \(wcr\) can be more challenging to cast because of its higher viscosity. The \(wcr\) and the strength class also affect which type of failure that will occur in the concrete structure. A higher strength means a higher risk of a brittle failure [17].

3.1.3 Behavior of concrete in compression

As mentioned earlier, concrete’s ability to withstand compressive stresses is superior to its ability to resist tensile stresses. Hence, concrete is used where a higher compressive strength is needed. Thus, the compressive behavior is a key factor when designing a concrete structure. The compressive behavior of a material can be illustrated in a stress-strain curve. According to Eurocode 2 [18], concrete can be described to have a linear behavior for stresses lower than \(0.4f_{cm}\), as shown in Figure 8.
The non-linear stress-strain curve is usually used when an advanced analysis is conducted. In Eurocode 2 [18], the relationship between the compressive stress $\sigma_c$ and strain $\varepsilon_c$ can be calculated by

\[
\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta},
\]

(2)

\[
\eta = \frac{\varepsilon_c}{\varepsilon_{c1}},
\]

(3)

\[
k = 1.05E_{cm} \left(\frac{\varepsilon_{c1}}{f_{cm}}\right).
\]

(4)

The calculations results in estimations of the stress-strain curve for strength classes under C50/60. The curves are shown in Figure 9.
3.1.4 Behavior of concrete in tension

When concrete is exposed to tension, there is a risk for cracking. There are different ways to predict under which tensile stress the section will crack. One way is to perform a splitting test of a cylinder made of the specific strength class. However, this method has been shown to be challenging to perform correctly. Thus, in Eurocode 2 [18], the tensile strength $f_{ctm}$ can be calculated based on the compressive strength, see Table 1. For strength classes under C50/60, the mean tensile strength can be calculated as

$$f_{ctm} = 0.30 f_{ck}^{2/3}$$  \hspace{1cm} (5)

There are, however, times when the mean value of the tensile strength is insufficient for calculations. In these cases, the tensile strength can be assumed to be normally distributed. In Eurocode 2 [18], the 5% and 95% fraction can be calculated by

$$f_{ctk0.05} = 0.7 f_{ctm}$$  \hspace{1cm} (6)
$$f_{ctk0.95} = 1.3 f_{ctm}$$  \hspace{1cm} (7)

The distribution is illustrated in Figure 10, where 95% of the concrete has a tensile strength higher than what is calculated in equation (6).

![Figure 10: Normal distribution according to Eurocode 2 [18].](image)

The stress-strain curve for concrete in tension can be described as elastic before the section cracks. The limit where the section starts to crack is calculated in equations (6)-(7). When the concrete has cracked, the stress-strain curve becomes nonlinear, and the section loses much of its strength. The tensile behavior of concrete is illustrated in Figure 11.
3.2 Steel

In a reinforced concrete structure, steel bars have the purpose of handling tensile stresses. The tensile behavior of steel differs from concrete. Steel is linear-elastic until it reaches its yield stress, \( f_{yk} \). Then the deformation increases until the ultimate tensile stress capacity \( f_t \) is reached. The stress-strain curve for steel in tension is shown in Figure 12 [18].

3.3 Failure modes of reinforced concrete

3.3.1 Bending failure

Reinforced concrete can fail in different ways when it is exposed to a bending moment [17]. When a simply supported beam is loaded with a uniformed distributed load, as shown in Figure 13 a), the beam will bend. This will cause compressive stresses in the upper part of the beam and
tensional stresses in the lower, as shown in Figure 13 b). It is found that the beam can fail in four different modes.

- Compressive failure in the concrete – ductile behavior of the beam: the steel has reached its yield stress and the concrete fails before the steel has reached its ultimate stress, see Figure 14.

- Compressive failure in the concrete – brittle behavior of the beam: the concrete reaches its capacity before the steel has reached its yield stress. The concrete collapses in the compressive zone, see Figure 14.

- Tensile failure in the steel – ductile behavior of the beam: The steel reaches its ultimate limit before the concrete has reached its capacity. This leads to the failing of the steel, see Figure 15.

- Tensile failure in the steel – brittle behavior of the beam. The steel fails immediately when the first cracks in the concrete appear, see Figure 15.
3.3.2 Shear failure

Besides bending failure, concrete can also fail in shear. There are typically two different shear failure modes for reinforced concrete. The first way the beam can fail is when the shear force exceeds the load bearing capacity and a diagonal crack occurs near the support, shown in Figure 16 a). However, when the reinforcement is sufficient to avoid the diagonal failure, the diagonal forces can crush the concrete near the supports. This is illustrated in Figure 16 b).

---

3.4 Loads

3.4.1 Traffic load

Road bridges are exposed to traffic load. Traffic loads refer to the effects that traffic has in horizontal and vertical direction of the carriageway. There are different kinds of traffic load that need to be taken into account. Vertical loads refer to distributed loads and point loads caused by the weight of vehicles. Horizontal loads are breaking and acceleration loads. When predicting the behavior of the slab and connection, distributed load can be used [19].
3.4.2 Self-weight

Self-weight refers to the weight of the structure itself. Self-weight is a permanent load and includes the supporting construction as well as railings and other permanent installations. The weight of the structure is directly connected to the density and volume of the building material. Temporary structures that are used during the building progress such as scaffolding and other material aids are not to be added in the self-weight of the finished construction. Table 2 shows the heaviness and density of common building materials.

Table 2: Heaviness and density of the materials used in this report.

<table>
<thead>
<tr>
<th>Material</th>
<th>Heaviness [kN/m³]</th>
<th>Density [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>24</td>
<td>2400</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>25</td>
<td>2500</td>
</tr>
<tr>
<td>Steel</td>
<td>77</td>
<td>7850</td>
</tr>
</tbody>
</table>

3.4.3 Lateral earth pressure

The lateral earth pressure affects geotechnical structures such as tunnels and basements. A bridge is designed to withstand the lateral earth pressure where the soil is acting on the wall element connected to the bridge. Figure 17 illustrates how the lateral earth pressure occurs on the wall element of a bridge [20].

![Figure 17: Earth pressure effect on the wall element of the bridge.](image)

When designing bridges, there is a great chance of encountering geotechnical problems. First, the initial stress state in the soil needs to be
established. To define the initial stress, the coefficient of the earth pressure, \( K \), needs to be calculated. There are three types of the coefficient \( K \): at-rest, active and passive [20], [21].

At rest, \( K_0 \), refers to when the soil is untouched and no work has been done, for example excavation. Active lateral earth pressure, \( K_a \), occurs when the soil is pushing into the supporting construction and then is allowed to relax. If the construction is pushing against the surrounding soil mass, it is called passive earth pressure. According to the Swedish Transport Administration, when designing a bridge the earth pressure should be at rest. The reason for this is that the backfill, the subbase material, needs to be fully compact and the structural element should be designed for this pressure [20], [19].

The at-rest earth pressure coefficient, \( K_0 \), is given by

\[
K_0 = 1 - \sin(\phi'_{dim}),
\]

where \( \phi' \) is the value of the friction angle as a function of the storage density of the backfill material. A safety factor, \( F_\phi \), is used, as a rule, to calculate the design value for the friction angle, \( \phi'_{dim} \), and is calculated with following equation:

\[
\phi'_{dim} = \arctan \left( \frac{\tan(\phi')}{F_\phi} \right).
\]

To calculate the at-rest earth pressure, \( p_0 \), the depths below the surface, \( h \), the dead weight, \( \sigma' \), of the soil and the distributed load, \( q \), need to be defined. The lateral earth pressure at rest is given by

\[
p_0 = K_0 \left( h \sigma' + q \right).
\]

3.4.4 Thermal expansion

When a material gets warmer, it tends to expand. This can cause problems when constructing a structure that will be exposed to temperature differences. For a prefabricated bridge element, the temperature difference between the factory and constructing location can lead to major internal stresses. The thermal strain, \( \epsilon_{thermal} \), can be calculated by multiplying the coefficient of thermal expansion for the material, \( \alpha \), with the difference in temperature, \( \Delta T \). To calculate the thermal stress, Hooke’s law can be applied, where thermal strain and Young’s modulus \( E \) determine the stress \( \sigma \) [18].

\[
\epsilon_{thermal} = \alpha \Delta T
\]

\[
\sigma_{thermal} = \epsilon_{thermal} E
\]
3.5 The Finite Element Method

The Finite Element Method, (FEM), is a numerical calculation method that is able to approximately solve numerous engineering problems. The method can be adapted to solve problems within different fields such as structural, heat flow, fluid flow, etc. FEM utilizes partial differential equations and boundary conditions to derive a Finite Element Method formulation of the specific problem. The analyzed part is divided into smaller elements and the overall deformation is represented by the displacement of the nodes. The different elements create a mesh covering the part. The FEM formulation is applied for each element and the equations are assembled to formulate the entire part [22].

When using FEM for solving structural problems, the equations follow a pattern where the stiffness of the element is assembled in a matrix, the so-called stiffness matrix. The stiffness matrix $K$ is multiplied with the nodal displacement vector $a$, which results in the load vector containing boundary loads $f_b$ and body loads $f_l$. This can be formulated as:

$$Ka = f_b + f_l$$  \hspace{1cm} (13)

By repeating this for all elements and assembling them, the whole system can be modeled. This process can be demanding to do manually for larger problems. Hence, there are many advantages gained by computerization of these operations and letting a FEM-software solve the problem [22].

3.5.1 Brigade/Plus

Brigade/Plus is an Abaqus-based finite element software that is customized for finite element analysis, (FEA), of structures, mainly bridges. The software can implement the behavior and effect of reinforcement embedded in the structure. Different part dimensions and material properties can be taken into account and assembled in 2D or 3D, which enables complex and detailed analysis of reinforced concrete structures [23].

3.5.2 Material properties

When modeling a structure with the Finite Element Method, the material properties are of great importance. In Brigade/Plus, materials can be assigned different properties depending on what type of analysis will be carried out. For example, both linear and non-linear behavior can be implemented. Materials with different behavior in tension and compression can also be analyzed. This is useful when modeling concrete structures because of its strength properties [23].
3.5.3 Embedding

When modeling reinforced concrete in Brigade/Plus, the properties of the concrete and the reinforcement need to interact to predict the structure behavior. In Brigade/Plus, this can be achieved by constraining the reinforcement with the solid concrete with an embedding constraint. The embedding constraint enables a host element to deform in the same way as the embedded region by translating the displacements of the host nodes onto the nodes of the embedded region. Furthermore, the host element needs to be a solid 3D element and the reinforcement a 3D beam element. The embedding constraint makes it possible to predict the deformation and stresses for the structure, considering the interaction between the concrete and reinforcement [23].
4 Methods and implementations

In this chapter, the methods and implementations used in the project will be presented. The project is based on models created in the software Brigade/Plus, as well as on Eurocode 2 [18] and rules and requirements of the Swedish Transport Administration [19]. Furthermore, references object of a straight prefabricated bridge provided by the company Abetong AB has been used. The reference object will provide strength properties of concrete and the amount of reinforcement bars, based on Eurocode requirements.

4.1 Reference object

A prefabricated concrete bridge designed by Abetong AB was used as the reference object. The reference object is a straight slab bridge located in Sweden. For this project, the interesting parts were the slab and the connection between the slab and wall elements. The model of the reference object is illustrated in Figure 18.

The connections between the slab and wall elements were assembled with threaded bolts with a diameter of 20 mm. The wall elements have a heel on top which allows the wall to push against the slab. The lower edge of the slab has the same shape for a tight fit. The connection between the wall and the slab elements is shown in Figure 19.
The same materials used in the reference object were used in this study. The quality and other data for the different materials are shown in Table 3.

Table 3: Material quality and properties.

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Strength class</th>
<th>wcr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>C35/45</td>
<td>≤ 0.45</td>
</tr>
<tr>
<td>Quality</td>
<td>B500B</td>
<td>40 mm</td>
</tr>
</tbody>
</table>

4.2 Calculations of external loads

4.2.1 Traffic load

In this project, the purpose is not to design the slab but to investigate the stress and deflection distribution between a straight slab and a skewed slab. The traffic load is an approximation of a typical load on a small bridge. In this case, the distributed load acting on the surface of the slab is estimated to be 20 kN/m².
4.2.2  **Self-weight**

The concrete has the density 2400 kg/m$^3$ and steel has 7850 kg/m$^3$. However, Brigade/Plus does not allow the combination of the steel and concrete as a composite material when stating the slab’s density. This was solved by using the density 2500 kg/m$^3$ [18], which is an estimation of the density of reinforced concrete (steel and concrete combined).

4.2.3  **Lateral earth pressure**

The friction angle $\phi'$, safety factor $F_\phi$, design friction angle $\phi'_{dim}$, heaviness of soil $\sigma$, traffic load, $q$, and the depth below surface is found in Table 4.

*Table 4: Data used for calculation of the lateral earth pressure.*

<table>
<thead>
<tr>
<th>Input</th>
<th>Friction angle, $\phi'$ [°]</th>
<th>Safety factor, $F_\phi$</th>
<th>Design friction angle, $\phi'_{dim}$ [°]</th>
<th>Dead weight, $\sigma$ [kPa]</th>
<th>Traffic load, $q$ [kPa]</th>
<th>Height, $h$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>1.3</td>
<td>37.6</td>
<td>22</td>
<td>20</td>
<td>3.5</td>
<td></td>
</tr>
</tbody>
</table>

The calculation results in a lateral earth pressure as shown in Figure 20. The calculations follow equations (8)-(10).

![Figure 20: Illustration of the lateral earth pressure affecting the wall element.](image)

To transform the distributed load from the earth pressure, equivalent reaction forces can be calculated as a simply supported beam. In Figure 21, the simply supported beam is shown. The force that will act on the slab element is the reaction force at point B. The force can then be distributed along the width and height of the slab, which will create pressure acting
along the slab edge. The reaction force in point A goes into the distance beams.

![Figure 21: Simply supported beam with earth pressure as distributed load.]

The resulting reaction forces and distributed load along the slab edges are presented in Table 5.

<table>
<thead>
<tr>
<th>Loads</th>
<th>Reaction force (A) [kN/m]</th>
<th>Reaction force (B) [kN/m]</th>
<th>Distributed load [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>138.7</td>
<td>85.77</td>
<td>1.715</td>
</tr>
</tbody>
</table>

### 4.2.4 Thermal expansion

The temperature difference, $\Delta T$, Young’s modulus, $E$, area, $A$, and the coefficient of thermal expansion for concrete, $\alpha$, with their values are presented in Table 6.

<table>
<thead>
<tr>
<th>Indata</th>
<th>Temperature difference, $\Delta T$ [°]</th>
<th>Young’s modulus, $E$ [GPa]</th>
<th>Area, $A$ [m$^2$]</th>
<th>Coefficient of thermal exp, $\alpha$ [1/°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>27</td>
<td>34</td>
<td>0.096</td>
<td>12E-06</td>
</tr>
</tbody>
</table>

The input data are used to calculate the resulting thermal expansion force. The calculations follow Equations (11)-(12). The resulting thermal expansion pressure acting on the slab is presented in Table 7.
Table 7: Resulting load resulting from thermal expansion on edge.

<table>
<thead>
<tr>
<th>Thermal expansion pressure [MPa]</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11.016</td>
</tr>
</tbody>
</table>

4.2.5 Summary of external loads

All loads that are considered in this study are summed up in Table 8.

Table 8: Summary of the different loads that affect the slab

<table>
<thead>
<tr>
<th>Load</th>
<th>Load type</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traffic load</td>
<td>Pressure</td>
<td>20</td>
<td>kN/m²</td>
</tr>
<tr>
<td>Lateral Earth pressure</td>
<td>Pressure</td>
<td>1.715</td>
<td>MPa</td>
</tr>
<tr>
<td>Lateral Thermal expansion</td>
<td>Pressure</td>
<td>11.016</td>
<td>MPa</td>
</tr>
<tr>
<td>Self-weight</td>
<td>Gravity</td>
<td>9.81</td>
<td>m/s²</td>
</tr>
</tbody>
</table>

4.3 Finite element models in Brigade/Plus

Three 3-dimensional models of a reinforced concrete slab were created in Bridge/Plus with different skew angles, with the same material properties as the reference object. Boundary conditions and loads were established to represent the behavior of the slab and the connection between the wall and the slab elements for various types of skew angles. The different models were based on a road crossing with a span of 5 m and the width of the slabs of 2.4 m. The different geometries of the slabs are shown in Figure 22 and Table 9.

Figure 22. The skew angle has effect on the geometry of the slab. Increasing the skew angle requires the free edge to be length increases as well.
Table 9: Geometry for the different models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Skew angle [°]</th>
<th>Thickness [m]</th>
<th>Width [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 deg</td>
<td>0</td>
<td>0.4</td>
<td>2.4</td>
</tr>
<tr>
<td>15 deg</td>
<td>15</td>
<td>0.4</td>
<td>2.4</td>
</tr>
<tr>
<td>30 deg</td>
<td>30</td>
<td>0.4</td>
<td>2.4</td>
</tr>
</tbody>
</table>

4.3.1 Model parts

Six parts were created for the reinforced concrete slab. The concrete slab was made as a solid element and the reinforcement was made as a wire/beam element. For the straight slab model, 0° skew angle, the amount and geometry of the reinforcement needed was based on the reference object. However, for the slabs with 15° and 30° skew angles, the geometry changed but the amount of reinforcement was kept the same. The different parts and their geometry are shown in Figure 23. The positions of the steel bars are shown in Figure 25.

The number of rebars stayed the same for all the models but the length was changed to fit the specific skew angle slab. The geometry, length and number of rebars for the straight slab are presented in Table 10.
Table 10. Reinforcing list for a straight slab

<table>
<thead>
<tr>
<th>Rebar</th>
<th>Diameter [mm]</th>
<th>Quantity</th>
<th>Length [m]</th>
<th>Height [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stirrup Big</td>
<td>12</td>
<td>19</td>
<td>2.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Stirrup Small</td>
<td>12</td>
<td>124</td>
<td>0.2</td>
<td>0.32</td>
</tr>
<tr>
<td>C-bar</td>
<td>12</td>
<td>4</td>
<td>2.32</td>
<td>-</td>
</tr>
<tr>
<td>Tension bar</td>
<td>20</td>
<td>23</td>
<td>4.92</td>
<td>0.20</td>
</tr>
<tr>
<td>Compression bar</td>
<td>12</td>
<td>17</td>
<td>4.92</td>
<td>0.20</td>
</tr>
</tbody>
</table>

4.3.2 Partition

For the solid part of the concrete slab the elements were divided in partitions. The reason for creating partitions was to simplify the working process in the model. A partition allows the user to work more efficient, for example, the models can be meshed more evenly and focused on areas of interest. Partitions were made in the center of all the holes, see Figure 24. These partitions were used when setting boundary conditions for the slab. When analyzing the finished model, paths were made by means of the partitions.

4.3.3 Material properties

The materials that were used in the model were concrete and steel. The concrete class was C35/45 and the steel had the quality B500B.
Brigade/Plus, the material properties need to be entered. When doing a linear analysis the Young’s modulus and the respective Poisson’s ratios are sufficient for the materials. The calculations for concrete follow Equation (1).

Young modules of elasticity, $E_{cm}$, density $\rho$ and Poisson’s ratio, $v$, are all included in the elastic material model. The values for the material properties are given in Table 11. Both materials were modeled as linear-elastic.

Table 11: Material properties used in Brigade/Plus.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho$ [kg/m$^3$]</th>
<th>Young’s modulus, $E_{cm}$ [GPa]</th>
<th>Poisson’s ratio, $v$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>2500</td>
<td>34</td>
<td>0.2</td>
</tr>
<tr>
<td>Steel</td>
<td>7850</td>
<td>200</td>
<td>0.3</td>
</tr>
</tbody>
</table>

4.3.4 Assembly of the parts

In the assembly module of Brigade/Plus the reinforcement bars and the concrete slab were assembled. Starting with the reinforcement bars, all the different bars were assembled using the drawings for the reference object. Last, the reinforcement bars were inserted inside the concrete solid part. The reinforcement was created as wires and the concrete as a solid for a more accurate result compared to a shell element that also could be used. The assembled rebar web is shown in Figure 25.
The reinforcement needs to be embedded together with the concrete in the software, to make the concrete, and reinforcement into a composite material. An embedded constraint was used to specify the reinforcement interaction with the surrounding concrete. This solution captures the composite material’s behavior in a realistic way. The input data for the constraint are stated in Table 12.

Table 12: Embedded constraint values used in Brigade/Plus.

<table>
<thead>
<tr>
<th>Embedded Region</th>
<th>Weight factor roundoff tolerance</th>
<th>Absolute exterior tolerance</th>
<th>Fractional exterior tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00E-06</td>
<td>0.0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

4.3.5 \textit{Step}

The order in which the different loads were applied was determined in the step module in Brigade/Plus. Each load was placed in a separate step. The order assigned, and the different steps that were created are presented in Table 13.
Table 13: Order and loads in the different step used in Brigade/Plus.

<table>
<thead>
<tr>
<th>Order</th>
<th>Step</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Step 1</td>
<td>Traffic load</td>
</tr>
<tr>
<td>2</td>
<td>Step 2</td>
<td>Self-weight</td>
</tr>
<tr>
<td>3</td>
<td>Step 3</td>
<td>Earth pressure</td>
</tr>
<tr>
<td>4</td>
<td>Step 4</td>
<td>Thermal expansion</td>
</tr>
</tbody>
</table>

4.3.6 Loads

Loads on the slab were; traffic load, earth pressure, thermal expansion and self-weight. The traffic load was a distributed load on the top surface of the slab and was acting in the negative direction of the Y-axis. Earth pressure and thermal expansion were lateral pressure load acting on the supported edge, in positive Z-direction. The thermal expansion was placed on the upper part of the edge. The earth pressure was set to act on the lower part of the heel. The self-weight was introduced as a body force acting on the slab and acting in the negative Y-axis direction with the gravity value of 9.81 m/s². The loads acting on the slab are shown in Figure 26.

![Figure 26: Load direction and point of impact for the different loads.](image)

The load types, values and units for the loads that was acting on the slab are presented in Table 8.

4.3.7 Boundary conditions

The boundary conditions applied on the slab were created to reflect the real behavior of the slab in a realistic way. The vertical boundary condition was set along the support at the slab edges, creating a simple supported slab. This is shown in Figure 27 a) where the line is fixed in Y-direction.
The horizontal boundary conditions were placed inside the holes in the solid concrete element, see Figure 27 b). On the loaded side, the holes were fixed in transversal direction that is X-direction. On the other side where the loads did not act, the holes were fixed in both X- and Z-direction. In this way, the unloaded side can compute the behavior of the slab.

4.3.8 Mesh

When selecting the element type, it is of great importance to choose one that suits the model. The concrete part of the model was three-dimensional and of irregular shape, which makes the linear tetrahedral element the only alternative. Smaller elements give a result closer to reality. The mesh was liner along the supports (mesh size 0.05 m) compared to the middle of the slab (mesh size 0.1 m). The mesh web for the different models is shown in Figure 28. For the reinforcement bars, the global element size is set to 0.1 m.
Figure 28: Example of how the mesh appears like along the edge of the slab.
5 Results and analysis

In this chapter, the results from the finite element analysis will be presented. The results and analysis will be divided into two parts; slab and connection. The results are based on the finite element models of the slab and connection for 0, 15, and 30 degrees of skew angle.

5.1 Slab

As presented in chapter 4.3, three different slabs were created with different skew angles. The reaction forces, longitudinal stresses along edge A-B and C-D and vertical deflection for the different skew angles will be compared and analyzed in this chapter. The results will be presented along and between partitions made on the slab, shown in Figure 29. The normal forces, earth pressure and the thermal force, are applied along A-C. The paths A-C and B-D are in the bottom of the slab. A-B, A-D, C-D and C-D are all placed on the top surface of the slab. The paths and the results are presented for the top side of the slab.

![Figure 29: Paths made based on the partitions of the slab.](image)

5.1.1 Vertical reaction forces

The reaction forces for each node, in the y-direction, along the supports are presented in Figure 30. The supports are located between A-C and B-D, see Figure 29.
To check if the total reaction force is correct, the traffic load 20 kN/m² can be multiplied by the slab area and compared with the output file where the sum of all nodal reaction forces are summarized. This comparison is shown in Table 14.

Table 14: Comparison between hand calculated and computer-generated total reaction force.

<table>
<thead>
<tr>
<th>Skew angle</th>
<th>Hand calculation [kN]</th>
<th>Output file [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 deg</td>
<td>360</td>
<td>360</td>
</tr>
<tr>
<td>15 deg</td>
<td>372</td>
<td>371.9</td>
</tr>
<tr>
<td>30 deg</td>
<td>416</td>
<td>415.8</td>
</tr>
</tbody>
</table>

The results show that for a slab with 0 degrees of skew, the reaction force is evenly distributed along the supports. However, when the angle increases to 15 degrees the force gradually increases to the obtuse corners, B and C, from zero at the acute corners, A and D. The slab with 30 degrees displays the same gradual increase with a slightly lower slope then for 15 degrees. Furthermore, the holes in the slabs affects the reaction force distribution. According to the results obtained from the models, the reaction force increases between the connection joints.

Based on the results it can be concluded that when the skew angle increases, the maximum reaction force goes to the obtuse corner of the slab. A similar result was obtained by A. Dhar et al. [7] and B. Sindu et al. [11] who discovered a comparable distribution of the reaction forces along the
supports. A comparison between tendency curves of the results obtained in this thesis and A. Dhar et al. is illustrated in Figure 31.

5.1.2 Longitudinal stresses, parallel to the free edges

The results from the stress analysis along the path between C-D show that there is a little difference between 0 and 15 degrees of skew. In both cases compressive stresses will occur along the path. However, when the skew angle increases, the longitudinal stress distribution changes substantially. According to the results from the FE-analysis, the stress changes from compression in the obtuse corner to tension in the acute corner. Moreover, the maximum stress is significantly higher for the slab with 30 degrees of skew compared with 0 and 15 degrees. The results are illustrated in Figure 32.
The path between A-B shows similar results as between C-D. Tensile stresses occur in the obtuse corner and compressive stresses in the acute corner. The slabs with 0 and 15 degrees of skew behave similarly. For 30 degrees, the highest stresses occur in the corners of the slab. The results are shown in Figure 33.

Figure 32: Distribution of longitudinal stress along path C-D, stress in Z-direction (parallel to the edge C-D), in the top surface of the slab.

Figure 33: Distribution of longitudinal stress along path A-B in the top surface of the slab (stress in Z-direction, parallel to the edge A-B)
Along the diagonal paths between the acute corners of the slabs, A-D, the stresses for 0 and 15 degrees correlate. However, the 30 degree skew angle affects the stress distribution greatly. The result shows that there is a tension stress concentration at the corners when the skew angle is 30 degrees. For the slabs with 0 and 15 degrees of skew angle compressive stresses will occur. This is illustrated in Figure 34.

![Figure 34: Distribution of longitudinal stress along path A-D in the top surface of the slab (stress in Z-direction, parallel to edges C-D and A-B).](image)

Between the obtuse corners, points C-B, the stress distribution is similar for 0 and 15 degrees. There are little differences in the stresses between path A-D and C-B for skew angles 0 and 15 degrees. For the slab with 30 degrees skew, the stress is purely compression and the stress curve has a similar shape as for path A-D. This is shown in Figure 35.
Figure 35: Distribution of longitudinal stress along path C-B, in the top surface of the slab (stress in Z-direction, parallel to edges C-D and A-B).

5.1.3 Summary

The distribution of the longitudinal stresses along the analyzed paths shows that there are little differences between slabs with a skew angle of 0 degrees and 15 degrees. As stated in the TDOK [8], bridges with a skew angle smaller the 10 degrees can be considered as a straight bridge, regarding static behavior. However, other studies have shown that slabs with a skew angle up to 20 degrees can be considered as straight [9] [7]. The results in this report indicate that slabs with skew angles between 0 degrees and 15 degrees behave in a similar way. However, when the skew increases to 30 degrees the stress distribution changes a lot. A reason for this is that the moment created from the earth pressure and the thermal expansion increases for bigger skew angles. This increased moment is a result of the increased distance between the forces and the supported side because of the higher skew angle.

5.1.4 Vertical Deflection

The vertical deflection between points C-D is shown in Figure 36. The shapes of the curves are similar for all skew angles. However, the maximum deflection along the path is greater for 30 degrees and least for the straight slab.
Along the other edge, A-B, the deflection correlates for the slabs with a skew angle 0° and 15°. However, there is a tendency of uplift of the acute corner when the skew angle is 30 degrees. The maximum vertical deflection does not differ significantly between different skew angles. The results are shown in Figure 37.

Earlier studies have also found that there is an extended risk of uplift in the corners when the skew angle of the slab increases [11] [7]. According to Dahr et.al [7] the uplift is caused by the torsional moment that occurs when the skew angle increases.
The vertical deflection along path A-D shows similar results as path A-B. However, there are more differences in maximum deflection for the different skew angles. This is shown in Figure 38.

The deflection between C-B correlates for all skew angles. There are however a slight difference in maximum deflection along the path. The deflection is shown in Figure 39.
To visualize the results of the distribution of vertical deflection, color plots for the different skew angles are presented in Figure 40. The plots illustrate the change in behavior when the skew angle increases.
As shown in Figure 40, the behavior of vertical deflection changes significantly for different skew angles. For a straight bridge, the distribution of the vertical displacement is symmetrical, where the maximum deflection occurs in the middle of the span. For a slab with 15 degrees of skew, the deflection distribution gets less symmetrical and the slab starts to bend out of the plane. This behavior gets more evident when the skew increases to 30 degrees. This behavior is caused by the moment from the earth pressure and the thermal expansion. These loads want to compress the slab which results in a bending moment in the plane as shown in Figure 41.
5.2 Slab – wall connection

The results from the FE-analysis of the connections are presented in this chapter. In total, three paths are analyzed, the middle, acute, and obtuse all on the unloaded side which is shown in Figure 42 a). The line along which the connection stress is analyzed is shown in Figure 42 b).

Figure 42: Illustration of the skewed slab with the analyzed connection joints. a) Location of the analyzed joints. b) Path where the joints are analyzed.
5.2.1 Normal stresses along the connection path

The stress in the connections in a straight slab is presented in Figure 43. The results show a clear correlation for the stress distribution between the different joints. The highest stress is in the lower part of the connection and the lowest stress at the top of the joint. The highest compressive stress is around 10 MPa for all the joints.

![Figure 43: Stresses in the joints for a straight slab along the analyzed path.](image)

The stresses in the connection for a slab with 15 degrees of skew are shown in Figure 44. The stresses are distributed evenly throughout the different joints. However, the joint at the obtuse corner takes a little more stress than the acute joint. The maximum stress is considerably higher in the slab with 15 degrees compared with the slab with 0 degrees of skew.
For the connection on the slab with 30 degrees of skew, the stress distribution in the joints changes drastically. In the obtuse corner, compressive stresses up to 300 MPa will occur in the connection. The joint in the middle will take the least stress compared to the other joints in the 30 degree slab. In the acute joint, tensile stresses will occur up to 120 MPa. The stress distribution for the joints in the slab with 30 degrees of skew is shown in Figure 45.
Figure 45: Stresses in the joints for 30 degrees of skew angle along the analyzed path.

The connection will handle the moment created from the horizontal forces by creating a resulting moment in the different joints, shown in Figure 46.

Figure 46: Force distribution in the connections resulting from the horizontal loading.
6 Discussion

6.1 Method discussion

The methods used in this study are well known and acknowledged in the engineering field. The methods have shown to agree with reality when comparing Finite element modeling with real life field tests. However, in this study, the results obtained are purely based on theoretical data. This affects the study’s credibility.

The boundary conditions applied on the slab were simplified based on theoretical assumptions made by the authors, which may result in a stricter constraint between the slab and the connection than reality. This could affect the end result of the analysis carried out in this thesis.

The temperature load acting on the slab assumes that the wall elements are fixed and not able to deform when the slab expands. Since the theoretical elongation for the slabs are approximately 1.6 mm, the impact from the temperature load would have minimal effect on the slabs in reality. This may result in a higher stress in the slab and the connection compared to reality.

6.2 Result discussion

The research made in this thesis is purely based on theoretical knowledge and has not been compared with any field tests. However, the results obtained correlate with previous research made on the subject.

6.2.1 Slab

As mentioned, the results from this study agree with results from other studies regarding deflection, stress distribution, and reaction force in the slab. This indicates that the model is valid and can be used to analyze the connection as well.

6.2.2 Slab - wall connection

The results from the connection analysis are hard to validate because no previous studies have been found. However, the models used in the slab analysis are regarded as valid. The connection analysis of the same models can therefore also be regarded as valid. Moreover, more research needs to be done regarding the connection between the slab and the wall to verify the results obtained in this thesis. For example, a non-linear analysis can be done for a more detailed analysis of the slabs. Furthermore, a higher number of skew angles could be analyzed to display when the connection starts to
handle in-plane bending moment. The same could be said about the slab analysis.

The stress calculated in the connection for a 30 degree slab is approximately 300 MPa which is much higher than the concrete's strength. This indicates that the strict boundary condition as well as the temperature load cause a high pressure on the connection.
7 Conclusions

7.1 Slab

7.1.1 Longitudinal stress distribution

The study conducted shows that the slab with a skew angle of 15 degrees behaves in a similar way as the straight bridge. Furthermore, when the skew angle increases to 30 degrees, the stress distribution radically changes compared to 0 and 15 degrees of skew.

7.1.2 Vertical deflection

The analysis of the vertical deflection shows that there is a risk of uplift in the acute corners for a slab with 30 degrees of skew. Moreover, the deflection is distributed similarly for all skew angle slabs studied.

7.2 Slab-wall connection

The results from the slab-wall connection analysis display that a higher skew angle results in higher loads on all connections. This is caused by the increased moment in the plane when the angle gets higher. Furthermore, when the skew angle is 30 degrees, the acute and the obtuse joints take forces in different directions to handle the increased in-plane moment. For a straight slab and a slab with 15 degrees of skew, the loads are evenly distributed among the joints.
8 References


