Master Thesis

Portfolio optimization

A DCC-GARCH forecast with implied volatility

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Abstract

This thesis performs portfolio optimization using three allocation methods, Certainty Equivalence Tangency (CET), Global Minimum Variance (GMV) and Minimum Conditional Value-at-Risk (MinCVaR). We estimate expected returns and covariance matrices based on 7 stock market indices with a DCC-GARCH model including an ARMA (1.1) process and an external regressor of an implied volatility index (VIX). We then simulate returns using a rolling window of 500 daily observations and construct portfolios based on the allocation methods. The results suggest that the model can sufficiently estimate expected returns and covariance matrices and we can outperform benchmarks in form of equally weighted and historical portfolios in terms of higher returns and lower risk. Over the whole out-of-sample period the CET portfolio yields the highest mean returns and GMV and MinCVaR can significantly lower the variance. The inclusion of VIX has marginal effects on the forecasting accuracy and it seems to impair the estimation of risk.

Keywords

DCC-GARCH, Portfolio Optimization, Certainty Equivalence Tangency (CET), Global Minimum Variance (GMV) and Minimum Conditional Value-at-Risk (MinCVaR), Implied volatility index (VIX).
Preface

This thesis was conducted during the spring semester of 2019 at Linnaeus University (LNU) in Växjö.

We would like to express our utmost gratitude to Maziar Sahamkhadam and Håkan Locking for their valuable guidance and support during the process of writing this thesis. We would also like to thank Magnus Willesson and Ola Nilsson, and our fellow students who have contributed to the process of writing our thesis.

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<td>Adaptive Market Hypothesis</td>
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<td>AR</td>
<td>Autoregressive</td>
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<td>ARCH</td>
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<td>ARMA</td>
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<td>BF</td>
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<td>Box-Ljung test</td>
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<td>Constant Conditional Correlation</td>
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<td>Certainty Equivalence Tangency</td>
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<td>CVaR</td>
<td>Conditional Value-at-Risk</td>
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<td>Dynamic Conditional Correlation</td>
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<td>Equally Weighted Portfolio</td>
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<td>Generalized Autoregressive Conditional Heteroscedasticity</td>
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<td>Global Minimum Variance</td>
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<td>Historical portfolio</td>
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<td>MGARCH</td>
<td>Multivariate Generalized Autoregressive Conditional Heteroscedasticity</td>
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<td>Modern Portfolio Theory</td>
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<td>MVO</td>
<td>Mean-Variance Optimization</td>
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<td>SR</td>
<td>Sharpe Ratio</td>
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<td>STARR</td>
<td>Stable Tail Adjusted Return Ratio</td>
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<td>SW</td>
<td>Shapiro-Wilk test</td>
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<td>Value-at-Risk</td>
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<td>Volatility Index</td>
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<td>VIX²</td>
<td>The squared value of VIX</td>
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Chapter 1

1 Introduction

This chapter initially presents the background of the thesis which starts in a discussion about risk and how it can be measured and managed. Afterwards, the problematization discusses problems regarding how the characteristics of input parameters affect model output. Further, the purpose, research question, and contribution are formulated. The chapter ends with presenting limitations and the disposition of the thesis.

1.1 Background

Managing risk is crucial for portfolio managers and investors. History has repeatedly proven that enormous financial losses can occur when risk is not adequately controlled. A fair example, not too far back in time, is the bankruptcy of one of America's largest investment banks, Lehman Brothers, who became one of the victims of the subprime crisis. Events like these have pushed for advancements of more trusted methods to monitor financial risk as they have grown more important (Bormetti et al., 2007).

The concept of risk has long been discussed and researched. Knight (1921) said the term risk has been used loosely and concluded that risk is the measurable uncertainty and the part which cannot be measured is called uncertainty. This implies that risk is not just associated with a negative but also with a positive outcome.

In finance, to estimate risk, variance of returns has often been used as a measurement. Variance includes both the upside and downside risk of returns. It could be argued that upside risk should not viewed as something negative but rather desirable as actual returns exceed the expected. This shortcoming has in recent times caused variance to be questioned as a proxy for risk since high returns can imply high variance. Instead, other risk measurements which only considers the downside risk, the probability of loss, such as Value-at-Risk (VaR) have been suggested (Bodnar & Zabolotskyy, 2017). Although, determining the best measurement of risk is ambiguous, some claim that downside risk measurements are preferable (Bodnar & Zabolotskyy, 2017). While Fantazzini (2004), claims that many professional practitioners prefer to use traditional measures such as sample variance.
Apart from using the realized volatility extracted from actual asset returns, implied volatility derived from option pricing can be used. While the estimated volatility is a measure of realized volatility, the implied volatility serves as the market’s expectations of future volatility (Poon & Granger, 2003) and is believed to be more forward-looking compared to volatility based on historical values (Koopman et al., 2005). It is said to be a good proxy for future volatility and is applicable for volatility forecasting as it is relieved from some of the estimation errors associated to volatility measures (Koopman et al., 2005).

Managing risk through diversification has been utilized within portfolio management for a long time. Diversification is based on the notion that portfolio risk is derived from the covariance between assets (Fabozzi et al., 2007). Even if risk can be greatly reduced by means of diversification, it cannot be eliminated completely. Markowitz (1952) claimed that this is partly due to the inter-correlation between assets and interplay and complexity of financial markets. The subprime crisis in 2007 serves as a fair example of how internationally intertwined the financial markets have become. Where events and volatility in foreign markets can affect domestic markets (Chou, Lin & Wu, 1999; Brooks & Henry, 2000; Li, 2007).

Managing portfolio risk can also be done through portfolio optimization. The seminal framework, Mean-Variance Optimization (MVO), by Markowitz (1952) has proven to be a prominent optimization method which is still widely used by financial practitioners today (Tu & Zhou, 2011). Markowitz (1952) argued that portfolios which maximize the expected return for any given level of risk are characterized as efficient, as they optimize the combination of the expected return and covariance matrix. Finding these efficient portfolios is appealing to any investor or portfolio manager, where the optimal level of risk is dependent on the investors own risk-aversion. Sharpe (1966) introduced a new performance measure, the Sharpe Ratio (SR), which facilitated the computation and localization of the efficient portfolios. Markowitz’s work proved to be prominent for Modern Portfolio Theory (MPT) and remains a keystone in portfolio selection. The optimal portfolio which both minimizes the overall risk and maximizes the expected return are desirable. Markowitz (1952) however, dismiss this optimal portfolio and claims that the two assumptions together are unrealistic.
Within MVO the overall risk is determined by the covariances between all assets in the portfolio. However, the development of mathematical- and economic theory have led to the questioning of the input parameters estimations. These are essential to the optimization problem and portfolio selection. The sample mean, and sample covariance have been commonly used in practice, however, since financial time-series are not constant over time they can provide poor estimates. Also, the number of estimated covariances for the sample covariance increase rapidly when more assets are included, which can make it impractical to use (Fabozzi et al., 2007).

Asset pricing theories such as the Capital Asset Pricing Model (CAPM) by Sharpe (1964) have been developed to more intuitively explain and estimate the associated risks and expected return. CAPM assumes that the portfolio risk is dependent on systematic risk, defined by the market portfolio variance and the unsystematic risk. CAPM is a single-factor model which incorporates the market risk premium as a factor in the modeling. In the calculation of the covariance matrices, the single-factor models offer a simpler way compared to MVO. Instead of computing covariances between all assets, only the covariance with the market factors needs to be calculated for each asset and from those the whole portfolio covariance is estimated, which decrease the numbers of calculations drastically (Fabozzi et al., 2007).

Multi-factor models such as Fama & French (1993)’s three-factor have been developed as extensions to single-factor models. Where additional factors are included in the model, which potentially can improve the estimation and explain the systematic risks of assets. The number of needed estimates of covariance matrices for multi-factor models increases compared to single-factor models as the covariance with additional factors needs to be computed. However, it is still significantly fewer than the traditional sample covariance matrix. Despite some computational improvements compared to sample mean and covariance, these models can be demanding and time-consuming to use when simulating future values. Some factor models also assume that the variance of factors is constant over time which they are not in practice, which implies that the model could be mis-specified. Factors that had good explanatory power in the past, might not yield good estimates today or in the future. Additionally, there is no established consensus of which factors should be included (Fabozzi et al., 2007).
Apart from asset means and variance being different over time, financial returns sometimes show signs of serial-correlation, which implies that they are dependent on past returns. To account for these effects, different autoregressive (AR) models have been used. Several models could also be fitted with an autoregressive moving average (ARMA) processes to account for autocorrelation (Fabozzi et al., 2007).

The interest from and investors and improvements in computer speed have pushed the literature forward for new estimation techniques. The earlier arguments to use factor models as they reduce the number of estimations are not as convincing anymore even if the new techniques still put a structure on the estimation of the covariances. Ever since Engle (1982), published the original autoregressive conditional heteroskedasticity (ARCH) model, a methodology framework used for forecasting, the literature has been extended with new estimation techniques and improved models in attempts to produce more accurate forecasts, such as the generalized ARCH (GARCH) by Bollerslev (1986). The statistical aspects of some GARCH models are beneficial as they can account for the time-varying means and variances of asset returns, as well as for autocorrelation. One of these is the dynamic conditional correlation (DCC)-GARCH, introduced by Engle (2002) which allows for the correlation between assets to vary over time. Extensions of this type of models have been done with additional risk factors such as an implied volatility index (VIX) but whether they have improved the forecasting accuracy is uncertain. Some research claims that the inclusion of additional risk measures improves the GARCH forecasting (Cochran et al., 2012; Kanas, 2012; 2013). However, the results are not conclusive, where in some cases the effects have only been marginal (Kambouroudis & McMillan, 2016). Some even claim that it does not yield improved accuracy (Day & Lewis, 1992; Koopman et al., 2005).

1.2 Problematization

Understanding portfolio performances continues to be a well-discussed topic in the context of empirical finance. It covers test of potential market beating strategies, test of the efficient market hypothesis, test of the capital market model and how to forecast the variances and covariances used in forming optimal portfolios. In this thesis we focus on the last part of this well discussed topic and we do not test commonly known investment
strategies based on firm characteristics or valuation nor do we attempt to test theories such as efficient market hypothesis (EMH) or CAPM.

The higher stock market volatility of past years serves as a fair example for the increased volatility, which has indisputable implications for asset pricing and portfolio- and risk management (Kambouroudis & McMillan, 2016). The estimation of volatility plays a central role in risk management and portfolio selection and therefore there will always exist a need to provide accurate volatility forecasts. This has been concluded within previous literature but without an established consensus regarding the preferred approach in financial markets (Kambouroudis & McMillan, 2016; Kim, Jung & Qin, 2016). The most frequently used approach to estimate expected returns and covariances is to compute the sample mean and sample covariance matrix. It is essential to recall that these inputs solely rely on the fact that past data also serves as “a good estimate for the future” (Fabozzi et al., 2007). Expected returns are often shown not to be covariance stationary, meaning that they vary over time. Therefore, historical performance will, in general, serve as a poor proxy for estimating future performance (Fabozzi et al., 2007). Financial returns sometime display signs of autocorrelation and heteroskedasticity, which suggests that the variance and covariance of assets are time-varying. Hence, the sample covariance matrix can get outdated quickly and Ledoit & Wolf (2003b) showed that it provides insufficient estimates.

Other approaches to estimate the expected return and covariance matrix is therefore needed. Previous studies have been towards the use of GARCH forecasting (Akgiray, 1989; Engle, 2001), which has also proven to yield accurate estimates of downside risk (So & Yu, 2006; Kuester et al., 2006; Weiß & Supper, 2013). GARCH models take the non-covariance stationarity and heteroskedasticity into consideration which is important for capturing the changing market conditions. They can also be extended with an ARMA process to account for autocorrelation.

These quantitative models are important for forecasting because they provide estimates for portfolio optimization. However, they bring an element of risk. The models are dependent on historical data of the input, expected return and covariances, and they forecast values based on the specifications. Since these inputs can vary and behave differently over time, the models can become mis-specified or outdated (Fabozzi et al., 2007). A robust model is therefore important for portfolio management and the
applicability of optimization methods. Incorrect assumptions, model misspecifications, or out-of-sample performance which does not hold under scrutiny could be costly for investors. Without robust testing, practitioners and investors cannot use models with great confidence. It is therefore important to critically review models on a regular basis as well as to test their applicability and usefulness in estimating the input parameters of different asset classes (Fabozzi et al., 2007). In addition, the lack of consensus regarding improved forecasting accuracy by the inclusion of external risk factors makes further research necessary.

1.2.1 Purpose

The main purpose of this thesis is to evaluate a DCC-GARCH model’s ability to adequately forecast returns and covariances in the attempt to find optimal portfolios, by simulating step-ahead returns and subjecting the allocation to optimization criteria and constraints. Further, we evaluate the effects of including an external regressor, in form of a volatility index, in the forecast.

1.2.2 Research question

Can the DCC-GARCH model forecast expected returns and covariance matrices efficiently to form optimized portfolios which can outperform benchmarks in terms of expected returns and our measures of risk, and does the inclusion of VIX improve the performance of the portfolios?

1.2.3 Contribution

Our paper contributes to existing research regarding the DCC-GARCH model with updated results of out-of-sample performance of optimized portfolios on a new set of recent data. The optimization criteria are Certainty Equivalence Tangency (CET), Global Minimum Variance (GMV) and Minimum Conditional Value-at-Risk (MinCVaR) and have, to the extent of our knowledge and research, not been estimated, evaluated, and
compared together with our suggested model. In addition, we contribute with result regarding whether the inclusion of a volatility index improves forecasts and how it performs during tranquil and volatile time periods.

Previous research such as Sahamkhadam et al. (2018) have been calling for additional attempts to improve the volatility forecasting by including additional factors. Researchers are constantly estimating, extending and evaluating already existing models on newer sets of data, by using different asset classes, or by adding new elements to contribute with new empirical results in their designated area of research. The empirical contribution and results will be of interest to other researchers and financial agents.

1.3 Limitations

Due to the vast body of literature regarding and affiliated to portfolio optimization, we are forced to limit ourselves to what we believe to be the most relevant. To write extensive reviews of models which are not employed in this thesis are deemed to be irrelevant. However, some factor models which are not employed here are covered briefly due to their relevance in portfolio optimization. Additionally, the number of existing GARCH models are also extensive, some of them are brought up as comparisons and to cover relevant topics in which the DCC-GARCH model is not employed to a great extent. Regarding previous research on VIX, we have limited the thesis to articles which employ it in the variance equation, as it can be added to the mean equation as well.

We also want to mention that MVO is described due to Markowitz pioneering work and its relevance in obtaining two of our investment strategies, as well as the portfolios of the efficient frontier. Further, we do not attempt to test theories such as CAPM or the efficient market hypothesis but as they could be used to motivate and understand the models we use and as the topics are related, we discuss them. The focus rather lies on estimating and forecast the covariance matrices and returns of assets and creating portfolios based on risk minimization or risk adjusted return maximization optimization criteria.

Our greatest limitation to work against have been the time. This is reflected in some of the choices we have made, especially for the forecasting and simulations sections. One example would be to test the robustness of the estimation window length, however, this
would require us to re-run the simulations which are time-consuming. Other limitations will be pointed out and motivated along the way.

1.4 Disposition

Following this first chapter, the thesis is outlined as follows:

- Chapter 2. Theoretical Framework
  Provides an introduction to relevant economic theories for portfolio optimization where the efficient market hypothesis, random walk behavior, behavioral finance, the adaptive market hypothesis, and portfolio choice theory are explained.

- Chapter 3. Literature Review
  Provides an overview of the literature affiliated with portfolio optimization where topics such as portfolio allocation, parameter estimation, the ARCH methodologies, and external regressors are discussed before the motivation of the chosen model is presented. The review aims to motivate the chosen model and methodology employed in this thesis.

- Chapter 4. Methodology
  Presents the data and methodology used to perform the portfolio optimization in the attempt to answer the research question. Firstly, the input parameters are presented, followed by the data included in the portfolios. Then, the specification of the model and the employed allocation methods are described. The chapter ends with a detailed description of our approach.

- Chapter 5. Results
  Presents the result of the model performance, out-of-sample performance, followed by risk-adjusted performance and terminal wealth of the portfolios as well as sub-period results.

- Chapter 6. Analysis
  Discusses the empirical findings.
• Chapter 7. *Summary and conclusion*
  Provides a summary and conclusion of the thesis and presents some propositions regarding further research.
Chapter 2

2 Theoretical Framework

This chapter introduces economic theories relevant to asset pricing and market behavior. All theories are not for us to evaluate, some merely serve as a background to market behavior.

2.1 Economic theories

2.1.1 The Efficient Market Hypothesis

The financial markets primary objective is to allocate capital. To efficiently allocate capital, investors need access to all relevant information, fast and free. That market participants are rational, profit-maximizing, actively competing, and have access to information creates an efficient market. The Efficient Market Hypothesis (EMH) states that constant competition among market participants results in assets reflecting their intrinsic value at all times, taking into account both past and future expected events (Fama & Malkiel, 1970). Hence, market participants are serving as a market mechanism keeping assets fairly priced. However, in a world of uncertainty, there is always room for disagreements regarding an assets intrinsic value, which cannot be perfectly estimated. In an ideal market, transaction cost would be non-existent, and all information would be available at zero expense and market participants would agree on the pricing of assets, as information would be interpreted the same (Fama & Malkiel, 1970). A frictionless market as previously described is, however, not a fair description of reality, making Fama & Malkiel (1970)’s conditions a simplification of the markets today.

According to Fama & Malkiel (1970), there are three states of market efficiency. The first is weak-form in which the information available only considers historical prices or returns. The second type, the semi-strong form, considers past prices and returns as well as publicly available information such as for instance stock splits, announcements of annual earnings, and information in quarterly and annual reports. Lastly, the strong form market includes monopolistic access to private information relevant for asset pricing, i.e.
inside information is reflected in the price (Fama & Malkiel, 1970). The categories are illustrated in Figure 1.

**Figure 1 – Market Efficiency**

This figure illustrates the three forms of market efficiency, where the strong form contains most information and weak form the least.

*Source: Own illustration*

### 2.1.2 Random walk behavior

Fama (1995) suggests that stock prices follow a random and unpredictable walk, which means that historical prices cannot be utilized to predict or forecast future stock prices. Thus, there is no way of knowing what direction future stock prices will be following. If previously stated holds true, the random walk suggests that technical analysis would be of no use since technical analysis is based on that previous stock-movement trends and patterns tends to repeat themselves in the future (Fama, 1995).

### 2.1.3 Behavioral finance

Behavioral science has been adopted into the field of finance and has created a new stem of research, namely behavioral finance (BF). De Bondt & Thaler (1985) concludes that individuals at times overreact and misjudge patterns and market movements. BF is considered to be in contrast to EMH. While EMH implies that all information is included in the current price, BF tries to explain market anomalies and assumes that investors are irrational, hence, the markets are at times over- or undervalued.

Like we mentioned earlier, the assumptions of the efficient market hypothesis are in some cases far-fetched from reality. BF suggests that psychological factors of investors such as market overreaction, overconfidence, and herding to name a few, are causing the financial markets to be less efficient. The rationality of investors has also been questioned since it
has been concluded that investors are more sensitive to a financial loss than gain of the same magnitude (Ang, Chen & Xing, 2006).

EMH assumes that information is instantaneously and constantly processed and available to markets and its actors to the same extent. In contrary to EMH, BF assumes that arbitrage possibilities exist due to market anomalies and misinterpreted information. Arbitrage is a risk-free investment where the investor discovers miss-priced asset and exploit the opportunity.

2.1.4 Adaptive market hypothesis

The Adaptive Market Hypothesis (AMH), can be explained as a reconciliation between the EMH and BF, where elements of evolution such as competition, adaptation, and natural selection are incorporated. In AMH, prices reflect the amount of available information which is derived from a combination of environmental conditions and similar-behaving market participants such as investors, hedge-fund managers, and pension funds. AMH claims that when several groups compete for scarce resources in one specific market, that particular market is probably highly efficient, reflecting most relevant information rapidly. However, if only a few groups compete for plentiful resources in a single market, that markets would not be as efficient. AMH states that market efficiency is dynamic and deeply reliant on context, where the ability to adapt to a constantly altering environment is of importance (Lo, 2004).

Some assumptions of the AMH opposes the EMH, which states that arbitrage opportunities exist from time to time (Lo, 2004). Profit opportunities will emerge and disappear at times, which requires investment strategies to fit the environment of the market accordingly (Hiremath & Kumari, 2014). Without arbitrage opportunities, the search for over- and undervalued assets would cease since there no longer would exist incentives for gathering information (Grossman & Stiglitz, 1980). Another implication is that the risk-reward relationship is likely to be unstable over time and is determined by factors such as preferences and the relative size of market participants (Hiremath & Kumari, 2014). A third implication is that investment strategies will generate higher returns in some environments and less in others. EMH states that arbitrage opportunities are eliminated by competition, meanwhile AMH suggests that market conditions
determine the effectiveness of arbitrage strategies. From time to time the market is more efficient and arbitrage exploiting strategies are not available to the same extent. When the market shifts to more conducive conditions, strategies for exploitation could be used (Lo, 2004). While EMH states that specific levels of expected returns can be attained by taking on enough risk levels, the AMH suggests that the relationship between risk and reward fluctuate throughout time. Lastly, AMH suggests that the principle which determines the development of financial markets is survival (Lo, 2004).

### 2.1.5 Portfolio choice theory

Every investment decision is associated with some level of risk. The risk can be approximated by modeling or other means and estimated with different measurements. The risk can be managed by diversify the assets in the portfolio. According to MPT assets are exposed to the market risk. Markowitz (1952) argues that investors can minimize risk by choosing assets that move in different directions of the market return. Another way to lower exposure to market risk is to invest in several different markets. However, as the literature suggests, the financial markets are intertwined and connected (Chou, Lin & Wu, 1999; Brooks & Henry, 2000; Li, 2007).

Portfolio construction begins with selecting an investment strategy which is either active or passive. The active strategy uses accessible information and different forecasting techniques to achieve high returns and/or to manage risk. Meanwhile, a passive strategy requires a low level of maintenance and relies on diversification to manage risk. The market risk can at best be estimated through historical experience because expected returns and risk are future values yet to be realized. Hence, to be able to predict future expected return and risk one must use past, already realized values to forecast expected returns (Bodie, Kane & Marcus, 2014).

A rational individual would when given a choice between two assets with similar return, select the asset with the lower associated risk. An investor who considers higher risk should be rewarded with a higher expected return. When constructing a portfolio, the investor seeks to find the efficient trade-off between risk and return of different assets with the constraint of the level of risk aversion. For any given risk level, efficient portfolios that maximize the returns can be constructed. Portfolio optimization can be
described as the pursuit of the “best” capital allocation. The optimization can be done through different methods and risk modeling, it expresses the problem in mathematical terms from which solutions can be derived. The optimization model is composed to fulfill a certain objective, like maximize returns or minimize risk, which is dependent on a constraint, like a given level of risk or return. There are several different optimization methods that can be employed, such as MVO, Global Minimum Variance, Certainty Equivalence Tangency, Minimum Conditional Value-at-Risk, and others.

Portfolios can be allocated to achieve the optimal risk-return trade-off. The expected return is assumed to be a non-linear function of risk and the set of these optimal portfolios form a curve, called the efficient frontier, and can be shown graphically in a risk-return graph (see Figure 2 in section 4.4.2). The optimal portfolio is the one on the efficient frontier that corresponds to the preferred risk level of the investor (Fabozzi & Markowitz, 2011).

Summary

The economic theories are trying to explain the markets- and asset price behavior. Asset prices such as stocks, are determined by the equilibrium between bid and ask prices. EMH suggests that markets can have different forms of efficiency. Semi-strong form suggests that asset prices reflect their intrinsic value based on publicly available information and past prices. This is contradicted by BF and AMH which suggests that the market and asset pricing are inefficient at times due to anomalies caused by irrational behavior of investors. AMH also suggests that competitive markets are more likely to be efficient as information is reflected in the price more rapidly. All these theories are opposing the Random Walk theory. Random Walk assumes that asset prices are not following trends and patterns driven by market participants’ behavior. It states that future prices are independent of past information and cannot be forecasted and predicted.

It is essential to understand asset prices behavior in order to allocate the portfolio efficiently. If prices are forecasted accurately, portfolio managers and investors could potentially reduce risk and increase the returns of their investments by reallocation. The economic theories explore possible underlying factors which are driving the prices. By observing and statistically test how asset prices behave we can fit a model which takes
the characteristics into consideration when predicting future values. If markets follow the
description of BF and AMH it is not perfectly efficient and inefficient pricing and market
anomalies will be present at times. Much of the theoretical side of this thesis is within the
framework of the econometric model employed, described 4.3, and to what extent it can
be utilized. The possible market inefficiencies will be captured by the model as it allows
for time-varying effects as well as being updated daily.
Chapter 3

3 Literature review

This chapter provides an overview of the extensive literature affiliated to portfolio optimization. Essentially, this review motivates the methodology employed in this thesis.

3.1 Portfolio Optimization

Since Markowitz (1952) contribution to MPT and portfolio management, the body of literature has grown enormously. Markowitz’s MVO method remains a prominent allocation method even today, 67 years later. By utilizing returns, volatility, and covariance he assessed the concept of diversification in the context of portfolio allocation. The portfolio selection process is divided into two stages according to Markowitz (1952) and can be summarized as:

1. Parameter estimation, beliefs about future returns and variances.

2. Portfolio selection, choosing assets and weights based on the estimations.

Markowitz argued that investors should only consider the efficient portfolios, i.e. portfolios which yields the highest return given the preferred level of risk, or the lowest variance for the targeted return. The basic assumptions about rational investors, who are utility maximizing and risk-averse support the mean-variance framework in the context of portfolio allocation (Markowitz, 1959). The assumptions were later also introduced as the pillars of Fama & Malkiel (1970)’s efficient market hypothesis. These are common underlying assumptions of MPT as well as many other models. However, their relevance and description of reality have been questioned by new theories such as Behavioral finance and Adaptive market hypothesis.

Although more sophisticated models and strategies have been developed over the years, MVO is still accepted and widely used by practitioners (Tu & Zhou, 2011). But divided opinions and arguments have left the literature and practitioners ambiguous over the relevance of MVO. It has shown to be outperformed by simpler strategies in out-of-sample performance. DeMiguel, Garlappi, & Uppal (2007) compared several extensions
of MVO to the equally weighted $1/n$ strategy, a portfolio which is weighted equally between all assets and found that none could yield higher out-of-sample Sharpe ratio.

MVO incorporates variance as a measurement of risk which includes both upside- and downside risk. Criticism has been directed towards MVO due to it assumes normally distributed returns and that this distribution is constant. When these two assumptions hold true, variance is a good proxy for risk (Hult et al., 2012). However, if they fail to hold, variance might not be sufficient and could yield misleading estimates of portfolio risk. Instead, downside risk measurements have become increasingly popular and some academics claim that it is preferable (Bodnar & Zabolotskyy, 2017). According to BF and supported by Ang, Chen & Xing (2006) among others, investors are loss averse i.e. they are more sensitive to losses than gains, which underpin the use of downside risk measurements as a substitute to variance.

One downside risk measurement is Value-at-Risk (VaR). VaR was popularized by JP Morgan’s RiskMetrics in 1996 and has since been utilized widely. Basel Committee on Banking Supervision (1996), recommended VaR as the international standard for evaluating market risk. VaR estimates the highest possible loss for a specified probability, typically 90, 95 or 99%. Unlike variance, which measures the deviations from typical outcomes, VaR tries to quantify the rare outcomes precisely and expresses the monetary loss (Litterman, 1996).

The accuracy of VaR estimation is highly dependent on the employed model's ability to capture the asset returns’ true tail behavior (Malz, 2011). VaR estimations based on GARCH models have been proven to provide accurate results (So & Yu, 2006; Kuester et al., 2006). Weiß & Supper (2013) show that their GARCH based model performs exceptionally well in forecasting losses. Although, like variance, VaR has its own shortcomings. Research has shown that it does not always reward diversification as it possesses unwanted properties such as lack of convexity and additivity of risks (Artzner et al., 1999). Basak & Shapiro (2001) finds that investors who are managing market risk using VaR are more exposed to risky assets than other investors and consequently experience larger losses when they occur.

Conditional Value-at-Risk (CVaR) was introduced in an attempt to fill the shortcomings of VaR. CVaR measures the conditional expectation of losses higher than the specified
probability, i.e. the mean of losses higher than VaR. CVaR can be used as an optimization approach which in theory is beneficial for reducing portfolio risk, as low CVaR also implies low VaR. Making it suitable for any firm or investor who focuses on evaluating and managing risk (Rockafellar & Uryasev, 2000). Krokhmal, Palmquist, & Uryasev (2002) confirmed the approach by Rockafellar & Uryasev (2000).

Sharpe (1966) introduced a new performance measure which considered both expected return and risk, the Sharpe ratio (SR). It measures reward-to-volatility, the ratio between expected return and standard deviation, and is commonly used within finance. SR can be used as an asset- and portfolio evaluation measure, where it characterizes how well the investor is compensated for the risk taken. Naturally, SR can be used as an optimizing strategy as well, where the investor seeks to maximize the SR of the portfolio. This heavily relies on that variance is a good proxy for risk. The illuminated criticism towards variance has led to several suggestions to modify the SR in the literature, such as Stable Tail Adjusted Return Ratio (STARR) and Sortino Ratio which only incorporates downside risk. Bodnar & Zabolotskyy (2013) modified the SR by replacing the standard deviation with VaR. Where they showed that if asset returns are assumed to be normally distributed, maximization of both the modified SR portfolio based on VaR and the traditional SR portfolio coincides. Similar results were found when replacing standard deviation with CVaR. Furthermore, it seems like portfolios are independent of the level of confidence when computing VaR and CVaR (Bodnar & Zabolotskyy, 2017). Alexander & Baptista (2002; 2004) showed that if asset returns are assumed to be normally distributed, the optimal minimum VaR portfolio is positioned on the efficient frontier. Hence, the problem with selecting the optimal SR maximizing portfolio could be described as the problem of minimizing VaR and CVaR. These results coincide with the thoughts of Hult et al. (2012) regarding variance being a good proxy for risk if the assumption of normally distributed returns holds true.

### 3.2 Parameter estimation

Estimation errors in the input parameters will impact the portfolio optimization output. Even though the emphasis has been to estimate the expected return, in recent times researchers like Michaud et al. (2012) have taken an opposing stance and put emphasis
on the covariance matrix estimation. In the following two sub-sections literature regarding estimations of the two input parameters are discussed briefly.

### 3.2.1 Expected returns

In the context of finance, predicting future returns have for a long time been seen as the primary task for portfolio optimization. The classic approach relies on the sample mean of returns to predict future returns. However, Fabozzi et al. (2007) argue that the sample mean is no longer valid as a predictor when the return distributions experience heavy-tails or are significantly asymmetrical. They continue to argue that financial time-series often does not display covariance stationarity, i.e. varies over time in terms of means and variance.

Many models have been developed to estimate the expected return and to cope with the drawback just mentioned. Factor models have gotten much attention, where factors and structures are imposed to the estimator. The models are attempting to quantify the risks associated with the assets. The Capital Asset Pricing Model is a single-index model introduced by Sharpe (1964), which assumes that the firm-specific risk can be eliminated by diversification. Thus, it assumes that all risk originates from the market. When estimating the expected return for assets it assumes that it is dependent on the assets’ covariation with the market and the expected market risk premium. Although it offers a simpler estimation and with less possible estimation errors it could be mis specified and yield biased results (Fabozzi et al., 2007). Developments of CAPM have been made by Merton (1973) among others, to cope with some of the critiques of MVO, which was mentioned above in section 3.1.

Several extensions of the single-index model have surfaced, referred to as multi-factor models. Fama and French (1993) three-factor model and later five-factor model Fama and French (2015) have arguably gotten the most attention. These multi-factor models offer the possibility to include more factors which could improve the accuracy of the expected return estimation. They are, however, exposed to more possible estimation errors compared to single factor models as more factors are needed to be estimated which also increases the complexity (Fabozzi et al., 2007).
3.2.2 The Covariance Matrix

The estimation of the covariance matrix has grown to be a more discussed topic nowadays. Some researchers claim that it is even more crucial than the expected return estimate for the portfolio optimization output, which is demonstrated by (Michaud et al., 2012). Nevertheless, less attention has led to the absence of literature regarding the covariance matrix estimator. In estimations of the covariance matrix, the classic technique uses the in-sample covariance matrix. However, this estimate relies on the quality of information and that all information is contained within the data set. Further, it relies on the variance of assets being constant. If either mentioned is inaccurate, the forecast accuracy could be poor (Fabozzi, et al., 2007). Including more data could arguably be a possible solution to this problem. However, it could come at the expense of including more noisy and outdated data with no explanatory forecasting power. While proven to be a sufficient estimate for in-sample, the sample covariance matrix could show poor out-of-sample performance due to noisy data (Bengtsson & Holst, 2002).

Likewise, as with the estimations of the expected return vector, imposing factors to the estimator of the covariance matrix have been suggested to atone for the shortcomings of the sample covariance matrix. Factor models have again been the answer but there is still no consensus of which factors that should be included, although a market factor seems reasonable and its inclusion is commonly accepted. In CAPM, assets co-movements are assumed to depend on their common response to the market movements. That there is no other factor affecting the assets co-movements, is a rather strong assumption. Using multi-factor models allows for more underlying correlations to be considered and captured. However, there is no way of saying which factors that should be included and it may not yield a better out-of-sample result (Ledoit & Wolf, 2003a).

3.3 ARCH methodologies

The literature exploring asset forecasting is relatively vast. The positive effect of this large set of literature has resulted in continually developing econometric models to accurately forecast returns and predict risks. The autoregressive conditional heteroskedasticity (ARCH) methodology was introduced by Engle (1982). It was developed due to that traditional econometric models assumes constant variance in one-period forecasts. The
assumption of constant variance is implausible, as time-series data such as asset returns varies over time, causing uncertainty over expected return and variance forecasts (Engle, 1982). ARCH models the squared residuals as an autoregressive process, which allows for modeling the conditional volatility of a non-constant time series. This enables the model to estimate means and at the same time not constrain the variance to be constant, which makes it a useful model for forecasting financial data (Engle, 1982). The variance is to some extent dependent on past values and residuals tends to be followed by residuals of similar magnitude, although the direction is uncertain (Mussa, 1979). ARCH makes it possible to estimate volatile and tranquil periods in the same empirical data set (Enders, 1995).

The ARCH model was later developed to a generalized version, GARCH by Bollerslev (1986), which allows for autoregressive (AR) and moving average (MA) components in the variance equation. This model allows all past lags of conditional variance to have an influence, making it a long memory model while the ARCH is a short memory model, where the number of influenceable lags are limited (Elyasiani & Mansur, 1998). When estimating the relationship between mean returns and variance of a portfolio, Baillie & DeGennaro (1990) argues that traditional two-index models provide insufficient results and call for more appropriate methods for measuring risk. Similarly, Joseph & Vezos (2006) claims that when volatility clustering of residuals is present in the empirical data set, Ordinary Least Square regressions provide insufficient estimates. Akgiray (1989) concluded that GARCH models were superior when forecasting out-of-sample indices variances in comparison to standard rolling regression methods. The GARCH model has as any model some issues and cannot completely capture fat tail distribution of financial high-frequency time series, which has resulted in the usage of non-normal distributions to better capture the occurring kurtosis (Peters, 2001). Hence, the student-t distribution, which has the properties of fatter tails and is symmetrical, is advocated by Kaiser (1996) and Bollerslev (1987).

Ever since the development of ARCH and the following GARCH, researchers have continuously been developing the models and extended them to new versions, in attempts to improve the modeling and forecasting of financial assets returns. From a univariate model to several multivariate GARCH (MGARCH) models, which can estimate multiple
assets at the same time. For the interested reader, Bollerslev (2008) provides an extensive review of the ARCH-models.

The constant conditional correlation (CCC)-GARCH was introduced by Bollerslev (1990). The model has the characteristics of an MGARCH model which allows for time-varying conditional variances and covariances, but it assumes a constant conditional correlation. This assumption allows for comparisons between periods and simplifies estimations. However, correlations between assets are an important input in pricing methods and portfolio optimizations, which makes the assumption of constant correlation less suitable for financial analysis and optimization (Engle, 2002). With this in mind, Engle (2002) proposed the dynamic conditional correlation (DCC)-GARCH model, which allows correlation to vary over time.

The DCC-GARCH model has had some trouble to capture spillover effects in volatility, however, it has shown to capture persistence in correlation and volatility as well as time-varying correlation (Basher & Sadorsky, 2016). It has been shown to forecast the covariance matrix well and Chou, Wu & Liu (2009) concluded that it outperforms other GARCH models when it comes to both in-sample and out-of-sample forecast of the covariance. The model has also performed well in very volatile periods, such as in the aftermath of the dot com bubble which was examined by Laurent, Rombouts & Violante (2012). They, however, also concluded that the model sometimes overestimates the conditional variance which could lead to inaccurate forecasts and ineffective allocations. This shortcoming was also concluded by Aielli (2013), who found estimation inconsistency of the covariance matrix and expressed concerns about the interpretation of the DCC correlation parameter. However, de Almeida et al. (2018) found that the DCC-GARCH model adequately forecasts the conditional covariance matrix. Ku et al. (2007) find similar result and concludes that frequent fluctuations in forecasts can be captured better with DCC.

There are many studies employing different GARCH models in an attempt to investigate the volatility of stock markets. Lim & Sek (2013) compare GARCH models’ performance of modeling the volatility of the Malaysian stock market and find that the GARCH models’ performance is dependent on the time frame and that symmetrical GARCH models are preferred in periods of tranquility, meanwhile an asymmetric model performs better in a period of crisis. The volatility of stock markets tends to affect each other. When
analyzing these effects, MGARCH models have been popular to employ. Mensi, Hammoudeh & Kang (2017) found that there are significant time-varying correlations between stock markets. Chou, Lin & Wu (1999); Brooks & Henry (2000); and Li (2007) all examined spillover effects and co-movements and found evidence of volatility transference over stock markets. Similarly, Karunanayake et al. (2009) find unidirectional mean-spillover effects running from larger to smaller markets, suggesting a positive relationship between market size and its volatility persistence.

### 3.3.1 External regressor - Volatility Index

There are certain types of risk which might never get realized but exists within the data of asset returns, such as bankruptcy-risk. To measure and estimate these types of risks and account for them can be difficult. To deal with this type of problem, external regressors which are believed to contain “more information” can be added to models in attempts to improve the forecast accuracy by capturing additional risk (Poon & Granger, 2003).

One advantage of the DCC model besides those mentioned before is that its specification with ease can be extended to include external factors (Engle & Sheppard, 2001). In combination with GARCH models, a Volatility Index (VIX) has been included as an external regressor before. External regressors can be added to both the mean and variance equation of GARCH models. Bollerslev et al. (1988) and Rangel & Engle (2012) included externals regressors in their models mean equation. Other researchers argued that the risk factors could be included in the variance equation (Cochran et al., 2012; Kambouroudis & McMillan, 2016; Koopman et al., 2005; Day & Lewis, 1992; Kanas, 2012; 2013). With the inclusion of VIX, one is attempting to capture volatility effects. However, the result of VIX is inconclusive as the effects vary with significant and insignificant results.

VIX serves as the market’s expectation of future volatility and it contains information about future volatility (Poon & Granger, 2003). This was confirmed by Kanas (2012; 2013) who concluded that additional information could be captured by including VIX in the conditional variance equation. Additionally, Cochran et al. (2012) found significant results during a volatile period and suggested that VIX should be considered when modeling return volatility. According to Kambouroudis & McMillan (2016), the inclusion
of VIX leads to marginally better forecasting performance, and they argue that VIX provides “additional explanatory power”. However, that VIX contain more information does not necessarily imply that more accurate forecasts can be made. Koopman et al. (2005) suggest that including VIX is not important when forecasting volatility, insignificant coefficients imply that there is no accuracy improvement. Similar result was found by Day & Lewis (1992) where implied volatility generated insignificant risk-return relationships when included in their models.

3.4 Model Motivation

ARCH and GARCH methodologies have long been employed successfully within the literature since its introduction by Engle (1982). It is popularly applied to time-series analysis and it has been particularly successful within finance, where it is often a question of the trade-off between risk and return (Engle, 2001).

Financial assets’ means and variances vary over time as well as the correlations between assets. Tsui & Yu (1999) argues that, as asset correlations are time-varying, a constant correlation model would not provide valid estimations. The DCC-GARCH takes this into consideration and allows for the correlations between assets to be interpreted as a stochastic process, therefore reflecting the current market conditions well (Sampid & Hasim, 2018).

The estimation of some multivariate GARCH models has been proven difficult for large data samples because the number of free parameters increases very fast as more variables are added to the model (Engle, 2002). However, the DCC-GARCH offers a more flexible manner. It uses a two-step process when estimating the conditional correlations. First using univariate GARCH specifications to estimate the standardized residuals for each asset series and then using these standardized residuals to estimate a time-varying correlation matrix directly, rather than estimating the covariance matrix and then calculate the conditional correlations (Engle, 2009). Thus, the parameters are independent of the number of series, which makes it possible to estimate very large correlation matrices. This is one of the great advantages of the model (Engle & Sheppard, 2001; Engle, 2002). The reason for using a multivariate GARCH model, like the DCC-GARCH, instead of a univariate GARCH model is that it calculates the whole covariance matrix instantly which
is essential for optimization, rather than the diagonal covariances. Another benefit of the model is that it can easily be extended by including exogenous variables to explain both the conditional mean and conditional variance (Engle & Sheppard, 2001). Including external regressors in multivariate GARCH models have been done by researchers in the past, see section 3.3.1.

Lim & Sek (2013) and Berger (2013) argues that it is difficult to compare results and performance of models between studies due to the use of different assets, time-periods and models for addressing risk since the performance of the models depends on the characteristics of the period. Instead, researchers have often used benchmark portfolios, such as an equally weighted portfolio (EQW), often called a naïve portfolio as well as historical returns as benchmarks. According to DeMiguel, Garlappi & Uppal (2007), the equally weighted portfolio provides a good benchmark as it does not rely on estimations of either asset return or variance and has frequently implemented by investors.
Chapter 4

4 Methodology

This chapter presents the methodology used to perform the portfolio optimization. First, the basic concept of input parameters is presented, followed by the data included in the portfolios and model. Secondly, the specification of the model and the adapted allocation methods are described. Lastly, a detailed description of our approach is given.

This thesis is conducted in a quantitative nature which is signified by quantification in collection and analysis of data (Bryman & Bell, 2017). Previously conducted studies within this field of research have to our knowledge exclusively been conducted with the use of quantitative approaches and when the large number of observations utilized in this thesis were taken into consideration, we assessed that a quantitative study is preferable. Further, a deductive approach has been taken which is the most common approach for our type of study i.e. a study departing in existing knowledge within a targeted area of research and present theoretical considerations, which from the researcher deduce hypothesis. The existing theory and chosen hypothesis lay the foundation for which data to be collected, subject to empirical testing (Bryman & Bell, 2017).

Validity is, fundamentally, the problem of whether a measurement of a concept actually measures that concept (Bryman & Bell, 2017). For validity reasons, our thesis has exclusively incorporated the use of scientific articles and textbooks. We have been critical in our selection of articles to establish a high level of validity, where we decided to utilize only those who have been cited at least 10 times. A few articles did not meet this requirement, however, if they were published in a renowned journal, we deemed them credible. Lastly, we have only made use of textbooks written from established and well-known authors within their field of expertise.

Some validity concerns were discovered throughout the process of writing this thesis. The lack of consensus regarding the size of the out-of-sample period and in-sample period propose that these are chosen at random without an established motivation to why a specific size is chosen.
Reliability is essentially about the occurring problems of measurement consistency (Bryman & Bell, 2017). A thesis should be replicable to the extent that the results of the original investigation should yield the exact same results when replicated (Bryman & Bell, 2017). Therefore, we have to the best of our ability given a detailed, step by step description of our approach.

4.1 Input Parameters

This section briefly discusses the basic parameters of portfolio allocation, optimization, returns and variance of assets. How these two fundamental notions are calculated are described in the next two sub-sections.

4.1.1 Returns

A financial asset’s return can be explained as the loss or gain during a specified window of time. The return of asset $i$ at time $t$ is denoted by $R_{it}$ and for assets without dividends, it is according to Berk and DeMarzo (2017) calculated as:

$$ R_{it} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t}} $$

(1)

Where $P_{i,t}$ is the price of asset $i$ at time $t$.

A portfolio can be comprised of investments in many different assets, the weights of each asset can be calculated as:

$$ w_i = \frac{\text{Investment in } i}{\text{Total portfolio value}} $$

(2)

The portfolio return is then calculated as the sum of asset weights multiplied by the asset return in the following matter:
\[ R_p = \sum_{i=1}^{n} w_{it} R_{it} = \mathbf{w}^{T} \mathbf{R} \]  

(3)

Where \( \mathbf{w}^{T} \) denotes a transposed vector of the weights and \( \mathbf{R} \) is a vector of returns.

### 4.1.2 Variance

In the context of portfolio optimization, variance is another essential part besides returns. The variance of a portfolio is derived from the covariance matrix of the assets returns. It can be estimated according to Berk and DeMarzo (2017) as follows:

\[
\text{Var}(R_p) = E \left[ \left( \sum_{i=1}^{n} w_i (R_i - E[R_i]) \right)^2 \right]
\]

\[
= E \left[ \left( \sum_{i=1}^{n} w_i (R_i - E[R_i]) \right) \left( \sum_{j=1}^{n} w_j (R_j - E[R_j]) \right) \right]
\]

\[
= \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j E[(R_i - E[R_i])(R_j - E[R_j])] = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} = \mathbf{w}^{T} \Sigma \mathbf{w}
\]

Where the portfolio variance, \( \text{Var}(R_p) \), is calculated as the expected squared deviation from the expected portfolio return. \( w_i \) is the weight and \( R_i \) is the return of asset \( i \). The covariance between assets is described by \( \sigma_{ij}, \forall i \in \{1, 2, ..., n\} \).

To sum up:

\[ \text{Var}(R_p) = \mathbf{w}^{T} \Sigma \mathbf{w} \]  

(4)

Where \( \mathbf{w}^{T} \) is a transpose vector of asset weights and \( \Sigma \) denotes the covariance matrix of asset returns.
Chapter 4. Methodology

4.2 Data

The data consists of daily prices of seven stock market indices, S&P 500 (US), OMXS (Sweden), TOPIX (Japan), NIFTY 500 (India), TA 125 (Israel), PSEi (Philippines), and IDX (Indonesia) between 01 January 2000 to 31 December 2018 and counts to 4956 observations per asset. The prices are recalculated into logarithmic returns. Ardia & Hoogerheide (2014) found that the update frequency of the parameter estimates of the GARCH model on a daily or weekly basis outperformed monthly and quarterly updates, even though it was only marginally. Hence, daily frequency has been used. It is beneficial to use indices in the sense that they could be considered to be diversified as they represent whole markets. When choosing the indices, we wanted some variation in size and geographical location, the indices also needed to have available data from the year 2000. By using several market indices, the portfolios will be more diversified compared to portfolios based on individual stocks. A volatility index, (VIX) is also collected and used in the GARCH variance equation, VIX is set in percentage. All data is retrieved from Thomson Reuters’ DataStream. Table 1 presents a descriptive statistic over the asset returns. The daily logarithmic returns are calculated as:

\[ r_{it} = \ln \left( \frac{p_{it}}{p_{it-1}} \right), \forall i \in \{1, 2, ..., n\} \]  

(5)

In finance, logarithmic returns are frequently used as it possesses qualities which is useful when working with time-series data sets. It is time additive, which implies that adding one-period log returns yield the same return as a two-period log return.

\[ \ln \left( \frac{p_2}{p_1} \right) + \ln \left( \frac{p_3}{p_2} \right) = \ln \left( \frac{p_3}{p_1} \right) \]  

(6)

The log returns are time consistent and they also possess another desirable property, where if one single period log return is normally distributed, a common assumption of financial asset returns, then the sum of log returns is also normally distributed. By using log returns we follow previous studies within the literature such as Kanas (2013).
Chapter 4. Methodology

Table 1 – Descriptive Statistics

<table>
<thead>
<tr>
<th>Assets</th>
<th>Mean</th>
<th>St. Deviation</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500</td>
<td>0.111</td>
<td>1.185</td>
<td>0.020</td>
<td>-0.470</td>
<td>10.957</td>
<td>-0.218</td>
<td>8.936</td>
<td>16490***</td>
<td>0.940***</td>
</tr>
<tr>
<td>OMXS</td>
<td>0.099</td>
<td>1.657</td>
<td>0.021</td>
<td>-10.098</td>
<td>12.532</td>
<td>-0.002</td>
<td>5.279</td>
<td>5740.7***</td>
<td>0.939***</td>
</tr>
<tr>
<td>TLOSEI</td>
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<td>1.247</td>
<td>0.000</td>
<td>-9.097</td>
<td>11.324</td>
<td>-0.223</td>
<td>4.321</td>
<td>3882.7***</td>
<td>0.962***</td>
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<tr>
<td>NIFTY 50</td>
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<td>1.604</td>
<td>0.096</td>
<td>-12.818</td>
<td>18.107</td>
<td>-0.387</td>
<td>8.009</td>
<td>13540***</td>
<td>0.925***</td>
</tr>
<tr>
<td>TA 125</td>
<td>0.022</td>
<td>1.333</td>
<td>0.046</td>
<td>-9.590</td>
<td>10.317</td>
<td>-0.370</td>
<td>4.528</td>
<td>4336.2***</td>
<td>0.949***</td>
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<tr>
<td>PSEi</td>
<td>0.030</td>
<td>1.398</td>
<td>0.000</td>
<td>-18.910</td>
<td>21.267</td>
<td>0.003</td>
<td>19.835</td>
<td>83133***</td>
<td>0.903***</td>
</tr>
<tr>
<td>IDX</td>
<td>0.030</td>
<td>1.644</td>
<td>0.000</td>
<td>-15.415</td>
<td>12.862</td>
<td>-0.578</td>
<td>8.169</td>
<td>14024***</td>
<td>0.903***</td>
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</table>

**External Reg**

<table>
<thead>
<tr>
<th>VIX</th>
<th>19.689</th>
<th>8.593</th>
<th>17.580</th>
<th>9.140</th>
<th>80.800</th>
<th>2.093</th>
<th>7.040</th>
<th>13838***</th>
<th>0.830***</th>
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<tbody>
<tr>
<td>DCC test</td>
<td>21.278***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

This table shows the descriptive statistic of the average daily log returns for each index in the whole sample period (defined as: 2000-01-01 to 2018-12-31). The total number of observations counts to 4956 for each index. The values are displayed in percentage (1.0=1%). JB show the result for the Jarque-Bera test of normality, SW show the result for the Shapiro-Wilks test of normality, BL show the result from the Box-Ljung test of serial correlation. *** represents significance level of 1%.

Reviewing the assets daily log returns we can see that there is both high losses and high gains for all indices. The high losses are occurring around the subprime crisis in and around 2008. Also, most of the large increases are occurring in the buildup to the financial crisis and as a response to the shock (see Figure 6 in section 6.3). PSEi (Philippines) experienced the largest increase in the late 2000s of 21.267% and IDX (Indonesia) accounted for the largest loss of -15.415% which was noted in October 2002.

Turning the attention to the characteristics of the return distributions. Skewness measures how symmetrical the distribution is. A normal distribution has a skewness of 0 and is symmetrical. As a rule of thumb, a distribution of skewness > |0.5| is considered to be largely skewed. PSEi has the only return distribution with positive skewness, with a value of 0.603, indicating that it is largely positively skewed. This implies that the tail is pulled out to the right. The distribution of IDX is also largely skewed but it is negative, -0.578, indicating a longer tail of the left side of the distribution. The kurtosis measures the pointiness of the distribution. A normal distribution has a kurtosis of 3 and most often what is calculated is the excess kurtosis, sometimes called leptokurtic, this is represented by a positive value larger than 3. Again, PSEi has the highest kurtosis of 19.835 which is very high. The kurtosis for the other assets is also relatively high and are far from being similar to a normal distribution.

The Jarque-Bera (JB) test is a joint skewness and kurtosis test of normality. The null hypothesis of this test is that the asset returns have a skewness of 0 and kurtosis of 3,
indicating a normal distribution. A p-value < 0.05 would reject the null hypothesis and conclude that the returns are not normally distributed. Additionally, the Shapiro-Wilk (SW) test of normality, which has the same null hypothesis, was also performed. The tests are two different types of normality tests. The JB test is based on moments and tests the goodness-of-fit, i.e. if it matches a normal distribution of skewness and kurtosis, while the SW test is based on regression and correlation (Yap & Sim, 2011). We can reject the null hypothesis at the 1% level for both tests for all assets indicating that the asset returns are not normally distributed during the whole sample period. Thus, we assume a student-t distribution in our modeling.

The Box-Ljung (BL) test is a test of independence and it tests if autocorrelation is present in the time-series. The null-hypothesis state that the time-series is independent, i.e. autocorrelation is not present. We tested for autocorrelation for lag one, and we can reject the null hypothesis for all assets except OMXS at the 1% level of significance, indicating that autocorrelation is present.

The JB, SW, and BL show signs of non-normality and autocorrelation, respectively, over the whole sample period. However, these coefficients can vary within the sample. As we are using a rolling window, updated daily, we account for these possible variations. Lastly, we performed a test of non-constant correlation, DCC-test, between the assets. The test was significant, and we could reject constant correlation and assume a dynamic correlation.

### 4.3 Model specification

This thesis employs a DCC-GARCH (1,1) model and due to the presence of autocorrelation we include an ARMA (1,1) process. The idea behind ARMA is that it assumes that past values influence the current values. It consists of two parts, autoregressive (AR) and moving average (MA). The process is specified to how many past values which should be included in the model, in this case (1,1), i.e. the immediate past value of both AR and MA is included. The model is specified as follows:
\[ r_t = \mu + \varphi_1 r_{t-1} + \theta_1 e_{t-1} + \varepsilon_t \]  
\[ \varepsilon_t = H_t^{1/2} z_t \]  
\[ H_t = D_t R_t D_t \]  
\[ D_t = diag(\sqrt{h_{1t}}, \sqrt{h_{2t}}, ..., \sqrt{h_{nt}}) \]

Where, \( r_t \) denotes a \( n \times 1 \) vector of returns of \( n \) assets at time \( t \). \( \mu \) is a \( n \times 1 \) vector of constants and \( \varepsilon_t \) is a \( n \times 1 \) vector of residuals at time \( t \). The standardized residuals of asset returns contain significant information and play an important part when estimating the correlations. The information set of their conditional and unconditional first and second moment is 0 and 1, respectively, and the cross product can have nonzero means which can be predicted. This enables the conditional correlations to be dynamic. The model uses these standardized residuals when estimating the covariance matrix. At each new period, additional information is included and updates the estimation (Engle, 2009).

By assuming that the conditional mean follows an ARMA (1,1) process we follow the line of Sahamkhadam et al. (2018) among others. The ARMA process is represented by \( \theta \) and \( \varphi \) which denotes \( n \times n \) diagonal matrices corresponding to the AR and MA process, respectively. The conditional variance of \( \varepsilon_t \) is determined by Cholesky decomposition of the \( n \times n \) positive definite covariance matrix \( H_t \), giving \( H_t^{1/2} \) and \( z_t \) is a vector of standardized residuals which are independently identically distributed (i.i.d.), \( E[z_t] = 0 \) and \( E[z_t^T] = 1 \). \( D_t \) is a diagonal matrix of conditional standard deviations of assets, modeled by univariate GARCH models where \( R_t \) is a conditional correlation matrix at time \( t \).

\[ h_{it} = \omega_i + \alpha_i \varepsilon_{it}^2 + \beta_i h_{i,t-1} + \delta VIX_{t-1} \]  

\[ (7.1) \]
\[ (7.2) \]
\[ (7.3) \]
\[ (7.4) \]
\[ (7.5) \]
Equation 7.5 models the conditional volatility $h_{it}$ for asset $i$ at time $t$, this equation is known as the variance equation of the GARCH model. The parameters are restricted to $\omega_i > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, and $\alpha_i + \beta_i + \delta < 1$. We set the GARCH order to 1, meaning that we use one lagged period of the ARCH and GARCH terms. $\varepsilon^2_{i,t-1}$ is the ARCH term and it represents the past periods squared residuals of the mean equation, i.e. the movement of the return occurred in the previous period is captured by this term. This means that large movement in the return will be shown as an increased volatility and consequently $\varepsilon^2$ will be large. This tendency of financial time series was noted by Mussa (1979), where residuals were followed by residuals of similar magnitude. $h_{i,t-1}$ is the GARCH term which represents the moving average component of the conditional variance, which implies that the previous estimated conditional variance is considered in the estimation.

We are not only considering realized volatility, but we also consider implied volatility by including VIX. The lagged value of VIX, denoted by $\delta$, is included as an external regressor in the variance equation in the attempt to improve the estimation of the conditional variance of returns.

The employed VIX is issued by the Chicago Board of Options Exchange. It is an implied volatility measurement constructed from different options based on the S&P 500, composed at-the-money, constantly with 30 calendar days until expiry. To reduce measurement errors and pricing bias, eight options are chosen whereof four are calls- and four are put-options. To avoid bid-ask bounces the average bid-ask price is used rather than the traded price (CBOE, 2019). Note that today's expectations about future prices affect the price of tomorrow, thereof we use a lagged value of VIX following Kanas (2012) and Kambouroudis & McMillan (2016). By doing so, yesterday's expectations of the price today are considered. Notably, representative VIX for each market modeled with dummy variables would have been preferable. However, we were not able to obtain VIX for each market. Instead, we chose to use a VIX based on S&P 500 as a proxy for all markets. This decision is based on previous literature’ findings of volatility transference and co-movements and that it is the largest market in our portfolio. Thus, believed that it will be a good volatility proxy for the other markets as well.
4.3.1 Simulation

Based on our DCC-GARCH specification, we forecast future values of returns and covariances of assets. We simulate step-ahead values based on a rolling window of 500 observations, meaning that each daily forecast uses the previous 500 observations of realized returns. Therefore, new and updated information is always included in the forecast.

From the mean equation, which estimates \( r_t \), returns, we forecast the step-ahead returns at time \( t + 1 \), \( \tilde{r}_{t+1} \).

\[
\tilde{r}_{t+1} = \tilde{\mu}_{t+1} + \tilde{H}_{t+1}^{1/2} \tilde{z}
\]

\( \tilde{z} \sim \text{std} \ (0,1, \xi) \)

In section 4.4 where the three allocation methods are described, the forecasted returns are used, \( R_t = \tilde{r}_{t+1} \) and the forecasted covariance matrix \( \Sigma = \tilde{H}_{t+1}^{1/2} \). \( \tilde{z} \) are the forecasted standardized residuals with a student-t distribution with 0 mean and variance of 1, \( \xi \) is the shape parameter.

4.4 Allocation methods

4.4.1 Benchmark

Instead of comparing the investment strategy relative to the market performance, a benchmark portfolio can be used which allows for more meaningful evaluations of the investment. Benchmarks can be fitted to “reflect” the investors strategy as well as represent an alternative strategy. Benchmark portfolios are useful tools when evaluating investment strategies as it provides valuable information regarding risk and return (Rennie & Cowhey, 1990).

An equally weighted portfolio (EQW) has been successfully used as a benchmark. Its appeal is the ease of use, frequent implementation, and its viability in most investment decision. It has also shown adequate out-of-sample performance (DeMiguel, Garlappi & Uppal, 2007). If correlation, mean and variance are the same for all assets then it would
maximize the diversification of risk. However, if this assumption is inaccurate and individual asset risk are different, the diversification of risk could be very limited (Maillard, Roncalli & Teiletche, 2008). Following (DeMiguel, Garlappi & Uppal, 2007) we hold the EQW portfolio weight constant according to:

\[ w_i = \frac{1}{n} \]  

(9)

The portfolio return is then calculated as follows:

\[ R_{p,t}^{EQW} = \frac{1}{n} \sum_{i=1}^{n} R_{i,t} \]  

(10)

Where \( w_i \) is the weight of the asset \( i \) and \( n \) is the number of assets. The portfolio return is given by the average return of the assets in the portfolio at time \( t \). A benefit of this portfolio is that it is not exposed to transaction costs as the portfolio is not in need of rebalancing.

The other benchmark portfolio used in this thesis is here referred to as historical portfolios (Hist) which is more complicated than the EQW portfolio. These benchmark portfolios are based on the same DCC-GARCH (1,1) employed in this thesis (see section 4.3, Eq. 7.1-7.5). This strategy reflects the different investment strategies and is based solely on historical- and not forecasted values, which creates the possibility to evaluate the forecasting itself.

The forecasted and optimized portfolios described in the following sections are compared to these benchmark portfolios, i.e. EQW and historical portfolios.

### 4.4.2 Markowitz Mean-Variance

A wide range of portfolio optimization methods has been developed since Markowitz’s publication of the fundamental framework on portfolio selection. The portfolio selection generally consists of two steps. The first involves estimations of expected return and risk and the second step involves portfolio selection. The common objective with optimizing a portfolio is for an investor to maximize expected return or to minimize risk, which is derived from the utility maximizing- and risk aversion assumptions. Markowitz (1952)
argues that the presumption of a portfolio which both maximize expected return and minimize overall risk cannot be accepted due to the intercorrelation between assets. The total variance of a portfolio is calculated as a covariance matrix of any composition of individual assets or indices in the portfolio.

Recalling the expressions from section 4.1.1 Eq. (3) and 4.1.2 Eq. (4), the random portfolio return is expressed as:

$$R_p = w^T R$$

and the total portfolio variance is expressed as:

$$Var(R_p) = w^T \Sigma w$$

The level of risk depends on the risk-aversion of the investor. It is commonly known that risk aversion can be expressed as a function of utility, which depends on expected return and risk (variance). Markowitz (1952) showed that by using quadratic utility function, the optimal portfolios could be obtained for investors with the risk-aversion coefficient \( \rho \). By imposing the risk-aversion coefficient, we arrive at the MVO problem:

$$\text{Maximize} \quad w^T R - \rho \times w^T \Sigma w \quad (11)$$

Remember that MVO can be expressed to maximize returns for the given risk (specified above) or to minimize the variance for a given expected return, which then is specified as:

$$\text{Minimize} \quad w^T \Sigma w - \rho \times w^T R \quad (12)$$

Markowitz (1952) claimed that an investor should only consider efficient portfolios. By solving the MVO problem for various risk-aversion levels, investors will obtain the set of efficient portfolios. The set of these efficient portfolios form the efficient frontier and are displayed in Figure 2.
Figure 2 – The Efficient Frontier

Portfolio optimization is based on allocating the capital efficiently to fulfill an objective and several methods have been proposed in the literature. The methods used in this thesis all incorporate basic notions of risk, however, only one is focusing on the expected return. The optimizations are based on the forecasted covariance matrix $\Sigma = \tilde{\Sigma}_t^{1/2}$ and the forecasted returns $R_t = \tilde{\tau}_t$. For the optimization, $w_t$ is either maximized or minimized to fulfill the objective of the allocation method.

The first optimization method used in this thesis uses the GMV portfolio. It is derived from Markowitz Mean-variance efficient portfolios and is located on the efficient frontier (see Figure 2). The GMV portfolio is focusing on minimizing the variance of the portfolio $w_t^T \Sigma w_t$ and disregards the expected return. This portfolio could be considered by risk-averse investors with the primary target to minimize risk. It can be expressed as:
4.4.3 Sharpe Ratio

The Sharpe Ratio (SR) was introduced by Sharpe (1966). It states the attractiveness of a portfolio as it measures the reward-to-volatility, i.e. how well the investor is compensated for the level of risk taken. SR is a commonly used performance measure within finance and it can also be used as an allocation method where the investor seeks to maximize the SR, a higher return for the given level of risk. The SR stems from MVO but offers a simpler way to estimate the efficient portfolios on the efficient frontier than the original MVO. SR is also reliant on the notion that variance is a good proxy for risk. The SR can be defined for a portfolio \( j \) as:

\[
SR_j = \frac{R_{p,j} - R_f}{\sigma_j}
\]

Where \((R_{p,j} - R_f)\) is the return risk premium for portfolio \( j \) divided by the standard deviation \( \sigma_j \) of the portfolio returns.

The second allocation method proposed in this thesis maximizes the SR, also called Certainty Equivalence Tangency (CET) portfolio. The CET portfolio is the portfolio which has the most efficient trade-off between risk and return and is located where the capital allocation line tangent the efficient frontier (see Figure 2, section 4.4.2). The portfolio maximizes the reward-to-volatility and could, therefore, be considered by risk-neutral investors. A more risk-averse investor would choose a portfolio further down on the efficient frontier, representative of lower variance. The portfolio weights are allocated by the portfolio expected return \( w_t^T E[R_t] \) divided by the square root of portfolio variance \( w_t^T \Sigma w_t \), which is defined as follows:
4.4.4 Value-at-Risk and Conditional Value-at-Risk

Downside risk measurements which quantify losses have gained popularity (Lechner & Ovaert, 2010) and have become a substitute to traditionally used volatility-based risk measurements. Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) are two of these downside risk measurements. VaR states the possible loss which could occur over a given time period within a specified confidence interval (Jorion, 1996). One of the reasons VaR has become a popular measurement is because it sums up downside risk into a single number (Jorion, 1996). According to Lechner & Ovaert (2010), VaR is preferred because it is “the best” and most simple method to use when predicting asset losses. Krokhmal, Palmquist, & Uryasev (2002) agrees and states that the reason for VaR becoming popular is because it is easy to comprehend.

VaR is used to determine the potential loss that could occur for a portfolio or investments for a given probability \(\alpha\), within a determined time frame \(T\). Consider a portfolio with a value of \(X\) at future \(T = 1\), VaR indicates the value for the specified quantile of the distribution, i.e. the potential loss of the portfolio for the specified probability at \(T = 1\). VaR is calculated in three steps:

1. Specify a probability, \(\alpha\)
2. Identify the return distribution
3. Identify the return for the specified probability

Maximize

\[
\begin{align*}
\mathbf{w}_t^T \mathbf{E}[\mathbf{R}_t] & \quad \text{Sharpe ratio} \\
\sqrt{\mathbf{w}_t^T \Sigma \mathbf{w}_t} & \quad \text{Full investment} \\
\mathbf{w}_t^T \mathbf{1} = 1, & \quad \forall i \in \{1,2,\ldots,n\}: \text{Long position only} \\
w_{i,t} \geq 0
\end{align*}
\]
CVaR, also known as Expected Shortfall is derived from VaR. It measures the conditional expectation of losses higher than the specified probability. Shortly, it measures the mean of losses higher than VaR. VaR and CVaR are illustrated in Figure 3.

**Figure 3 – Value-at-Risk and Conditional Value-at-Risk**

This figure illustrates a normal distribution of asset returns. VaR is the return at the specified probability, \( \alpha \). CVaR is the mean of returns in the blue colored area lower than VaR. CVaR could also be specified as the median of the lower tail returns, then CVaR would be located slightly more to the left.

**Source:** Own illustration

CVaR can be obtained using the integral proposed by Rockafellar & Uryasev (2002) and is expressed as follows:

\[
CVaR_{\alpha}(w_t) = (1 - \alpha)^{-1} \int_{f(w_t,r_t) \geq \eta_\alpha(w_t)} f(w_t,r_t)p(r_t)dr_t
\]

Where \( \alpha \) is the specified probability and \( f(w_t,r_t) \) is the loss function for \( w_t \) asset weights and \( p(r_t) \) probability of \( r_t \) the return at time \( t \). \( \eta_\alpha(w_t) \) gives the loss random variable of VaR.

The first two allocation methods rely on that variance is a good proxy for risk. Minimizing CVaR is the third allocation method used in this thesis which instead considers the downside risk. It aims to minimize the expected losses exceeding VaR and can be considered by risk-averse investors. Allocating this portfolio and solving the integral for CVaR, Eq. (16) is rather complex and can be solved using linear programming. Due to the complexity, we refer to Rockafellar & Uryasev (2002) for the interested reader for
more details. We follow Rockafellar & Uryasev (2002) and Sahamkhadam et al. (2018) in its definition, which is as follows:

\[
\begin{align*}
\text{Minimize} & \quad f_\eta(w_t, \alpha) = \beta + \frac{1}{q(1-\alpha)} \\
\text{Subject to} & \quad w_t^T 1 = 1, \\
& \quad w_{i,t} \geq 0 \\
& \quad \mu(w_t) \leq -R,
\end{align*}
\]

Where \(\beta\) is the value of VaR at the specified probability \(\alpha\), here 0.95. \(w_t^T\) is the transposed vector of asset weights at time \(t\) and \(R_{kt}\) is the \(k\)th vector of simulated return at time \(t\) and is the number of Monte Carlo draws.

### 4.4.5 Constraint

The first constraint to the optimization is that the portfolio weights must sum to 1. Recall that the asset weights, \(w_i\), are defined as a function of the total amount invested. This condition is referred to as a fully invested portfolio, which is a common constraint to impose for portfolio optimization. Additionally, a short-selling constraint is imposed which is also common in the context of portfolio optimization, it was also considered by Markowitz (1952) in the development of MVO. Mathematically it limits the asset weights to be in the range of 0 and 1, i.e. they cannot take on a negative value. One might wonder if imposing this constraint would lead to that the optimal solution found might actually be a suboptimal solution since the weights are not allowed to be allocated freely in all the possible combinations. This would be true if there were no estimation errors. However, in practice, many funds and institutional investors are not allowed to short sell assets, making the constraint not a limitation but rather an adjustment to match a real-world setting.
Another real-world setting is the inclusion of transaction costs. Since the portfolios are rebalanced daily, the transaction costs are far from insignificant. One thing to remember is that even small changes in the estimates of expected return vector and covariance matrix are causing re-allocation of the weights. Gains provided by redistributions could be eaten up by the transaction costs if they are too small. To account for this, complicated penalty functions of transaction costs could be implemented into the allocation functions which limits the investor not to trade when transaction costs exceed the gain from rebalancing (Balduzzi & Lynch, 1999). These penalty functions are complex and another way to account for transaction costs is to deduct proportional amounts to the reallocation afterward. Due to the complexity of penalty functions, the portfolio returns were subjected to transaction costs after the forecast of 1 basis point (0.01%) following Low et al. (2013) and 5 basis points (0.05%) per transaction to mimic a closer real-world performance.

4.5 Hypotheses

In order to ensure if the forecasted portfolios can outperform the benchmarks, we set up 3 main hypotheses ($H1 - H3$), each is tested against 2 benchmarks. In line with DeMiguel, Garlappi, & Uppal (2007) we use a $1/n$ equally weighted portfolio as a comparison to our forecasted portfolios. We also use another benchmark portfolio in order to evaluate the forecast ability of the model. This benchmark allocates the weights of the assets according to the same criteria as the forecasted portfolios but is based solely on historical data.

$H1.1$: The forecasted portfolio $j$ yield on average higher return compared to the equally weighted portfolios: $\mu_j > \mu_{EQW}$

$H1.2$: The forecasted portfolio $j$ yield on average higher return compared to the historical portfolio: $\mu_j > \mu_{j;Hist}$
**Chapter 4. Methodology**

**H2.1:** The forecasted portfolio \( j \) yield a lower variance than the equally weighted portfolio: \( \sigma_j^2 < \sigma_{EQW}^2 \)

**H2.2:** The forecasted portfolio \( j \) yield a lower variance than the historical portfolio: \( \sigma_j^2 < \sigma_{j,\text{Hist}}^2 \)

**H3.1:** The forecasted portfolio \( j \) yield a higher SR compared to the equally weighted portfolio: \( SR_j > SR_{EQW} \)

**H3.2:** The forecasted portfolio \( j \) yield a higher SR than the historical portfolio: \( SR_j > SR_{j,\text{Hist}} \)

**External regressor**

The following hypotheses are evaluating the effects of adding VIX as an explanatory factor of the conditional variance. The first hypothesis (\( H4 \)) is set to evaluate the effects on the model, and the rest (\( H5 - H7 \)) evaluates the effects on the forecast. Here, VIX represents both VIX and \( \text{VIX}^2 \), the hypotheses are tested for both versions against the forecasted portfolios without VIX.

**H4:** VIX has a significant effect on the conditional variance: \( (\delta \neq 0) \). Implying that the market’s expectation of future volatility has an effect on the conditional variance of asset returns.

**H5.1:** Including VIX in the forecast yield a higher return on average: \( \mu_{VIX} > \mu_{CET} \)

**H5.2:** Including VIX in the forecast yield a higher return on average: \( \mu_{VIX} > \mu_{GMV} \)

**H5.3:** Including VIX in the forecast yield a higher return on average: \( \mu_{VIX} > \mu_{\text{MinCVaR}} \)

**H6.1:** Including VIX in the forecast yield a lower variance on average: \( \mu_{VIX} < \mu_{CET} \)

**H6.2:** Including VIX in the forecast yield a lower variance on average: \( \mu_{VIX} < \mu_{GMV} \)

**H6.3:** Including VIX in the forecast yield a lower variance on average: \( \mu_{VIX} < \mu_{\text{MinCVaR}} \)

**H7.1:** Including VIX in the forecast yield a higher SR on average: \( \mu_{VIX} > \mu_{CET} \)

**H7.2:** Including VIX in the forecast yield a higher SR on average: \( \mu_{VIX} > \mu_{GMV} \)

**H7.3:** Including VIX in the forecast yield a higher SR on average: \( \mu_{VIX} > \mu_{\text{MinCVaR}} \)
4.6 Approach

The estimations and simulations in this thesis are done with the programming software R. First, the data is recalculated to logarithmic returns according to Eq. (5) then it is imported into the program. Once in R, the definition and classification of the data can be done. The number of assets \( n = 7 \) is the stock market indices which are defined as returns and classified as a matrix. VIX is also classified as a matrix but defined separately as it will be included as an external regressor in the variance equation of the GARCH model. Next, we define the following parameters; \( T = \) number of observations of each asset returns, which counts to 4956; \( EWL = \) estimation window length, which is set to 500 observations; \( OSL = \) Out-of-Sample length, counting to 4456 (4956 – 500); lastly, we define \( Sim = \) number of simulated returns for each iteration, which are set to 1000.

The next step consists of defining the univariate GARCH specification which is used to estimate the standardized residuals for each asset series using the student-t distribution, as the data shows signs of leptokurtosis and non-normally distribution, which Bollerslev (1987) and Kaiser (1996) deemed appropriate. The univariate GARCH is specified with ARMA (1,1). By replicating the univariate specification \( n = 7 \) times we define the multivariate GARCH specification which we then characterize as a DCC-GARCH. It is specified according to Eq. (7.1-7.5) in section 4.3, which is a standard GARCH (1,1) with an ARMA process of order (1,1) and including VIX as an external regressor in the variance equation. Which then is used to estimate the conditional correlation matrices, using a multivariate student-t distribution.

Lastly, the parameters and coefficients of DCC-GARCH are estimated using maximum likelihood estimation. In accordance with Berger (2013); Basher & Sadorsky (2016) among others, we implement a rolling window for the out-of-sample estimations. The estimation window length in previous studies has ranged between 300 to 3000, see, Weiß & Supper (2013); Berger (2013); Basher & Sadorsky (2016); Sahamkhadam et al. (2018). Using a longer window are more time consuming as more data is considered in the forecasts. Thereof, we use a window length of 500 observations and as we use the lagged value of VIX we drop one observation and the total out-of-sample length counts to 4455 iterations. By using a rolling window of realized returns and step-ahead simulation following Kambouroudis & McMillan (2016) and Koopman et al. (2005), the software then simulates 1000 step-ahead returns for each iteration. On each day \( \forall t \in T \) we fit a
new DCC-GARCH to simulate the step-ahead return, meaning that it is always the previous 500 days of data that is included in the forecast, which enables us to account for the time-varying means and variances of asset returns. From the return distribution of the 1000 simulated returns, the optimal weights for each asset are obtained based on the portfolio criteria. To estimate the out-of-sample performance, each asset weight is multiplied with its respective realized return and then added together to form the portfolio return.

Previous results regarding VIX as an external regressor have been ambiguous. Therefore, we have run the model on three separate occasions where we examine the effects of external risk factor VIX. First, we run the model and the simulation without an external regressor, we then run the model with VIX in percentage form, and lastly, we square VIX following Kanas (2013) and run the model and simulations a third time.
Chapter 5

5 Results

This chapter presents the result of the model performance, out-of-sample performance, followed by risk-adjusted performance and terminal wealth of the portfolios as well as sub-period results.

5.1 Model performance

The result from the maximum likelihood estimation and the estimated coefficients over the whole sample period for each asset are presented in Table 2-4, based on the specified model in section 4.3 Eq. (7.1-7.5) with different variations of the external regressor, VIX. Table 2 presents the result of the estimations excluding VIX. Table 3 presents the result from the model with VIX in percentage form and lastly, Table 4 presents the results of the model with VIX². The results for the variance equations are different and will be discussed separately after each table but the result for the mean equations for each model are similar and will be discussed jointly.

Table 2 - Coefficients without VIX

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Assets</th>
<th>S&amp;P 500 (US)</th>
<th>OMXS (Sweden)</th>
<th>TOPIX (Japan)</th>
<th>NIFTY 500 (India)</th>
<th>TA 125 (Israel)</th>
<th>PSEI (Philippines)</th>
<th>IDX (Indonesia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>μ (Constant)</td>
<td></td>
<td>0.065***</td>
<td>0.059***</td>
<td>0.021**</td>
<td>0.120***</td>
<td>0.057***</td>
<td>0.041***</td>
<td>0.009***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.008)</td>
<td>(0.015)</td>
<td>(0.013)</td>
<td>(0.018)</td>
<td>(0.014)</td>
<td>(0.0156)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>ω₁ (Mean)</td>
<td></td>
<td>-0.817***</td>
<td>-0.860***</td>
<td>-0.027</td>
<td>-0.284</td>
<td>0.035</td>
<td>0.116</td>
<td>0.865***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.078)</td>
<td>(0.041)</td>
<td>(0.477)</td>
<td>(0.240)</td>
<td>(0.216)</td>
<td>(0.164)</td>
<td>(0.126)</td>
</tr>
<tr>
<td>θ₁ (α)</td>
<td></td>
<td>0.765***</td>
<td>0.845***</td>
<td>0.556</td>
<td>0.371</td>
<td>0.099</td>
<td>-0.068</td>
<td>-0.849***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.082)</td>
<td>(0.042)</td>
<td>(0.206)</td>
<td>(0.223)</td>
<td>(0.216)</td>
<td>(0.168)</td>
<td>(0.127)</td>
</tr>
<tr>
<td>Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>σ² (Constant)</td>
<td></td>
<td>0.009***</td>
<td>0.014**</td>
<td>0.011***</td>
<td>0.047***</td>
<td>0.099**</td>
<td>0.106***</td>
<td>0.041***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.002)</td>
<td>(0.006)</td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.016)</td>
<td>(0.020)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>α₁ (dARCH)</td>
<td></td>
<td>0.097***</td>
<td>0.069***</td>
<td>0.043***</td>
<td>0.115***</td>
<td>0.064***</td>
<td>0.136***</td>
<td>0.107***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.011)</td>
<td>(0.014)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>β₁ (GARCH)</td>
<td></td>
<td>0.902***</td>
<td>0.927***</td>
<td>0.901***</td>
<td>0.870***</td>
<td>0.983***</td>
<td>0.899***</td>
<td>0.892***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.012)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.022)</td>
<td>(0.022)</td>
</tr>
</tbody>
</table>

This table presents the maximum likelihood estimation of the ARMA DCC-GARCH (1,1), without the external regressor VIX, for the different assets over the whole sample period. Standard error is in the parentheses. *, **, and *** represents p < 0.1, 0.05, and 0.01, respectively.
The variance equation of the DCC-GARCH model without VIX perform well. Each parameter has significant coefficients at 1% level except for the constant of OMXS and TA125 which are significant at the 5% level. The ARCH and GARCH terms are significant and imply that one lagged component of the squared residuals and the conditional variance affect the conditional variance of asset returns positively.

Table 3 - Coefficients with VIX

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Assets</th>
<th>S&amp;P 500 (US)</th>
<th>OMXS (Sweden)</th>
<th>TOPIX (Japan)</th>
<th>NIFTY 500 (India)</th>
<th>TA 125 (Israel)</th>
<th>PSEI (Philippines)</th>
<th>IDX (Indonesia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$ (Constant)</td>
<td></td>
<td>0.065***</td>
<td>0.059***</td>
<td>0.034***</td>
<td>0.120***</td>
<td>0.058***</td>
<td>0.045***</td>
<td>0.089***</td>
</tr>
<tr>
<td>$\varphi_1$ (MA)</td>
<td></td>
<td>-0.814***</td>
<td>-0.860***</td>
<td>-0.610</td>
<td>-0.284</td>
<td>0.035</td>
<td>0.118</td>
<td>0.866***</td>
</tr>
<tr>
<td>$\theta_1$ (AR)</td>
<td></td>
<td>0.763***</td>
<td>0.845***</td>
<td>0.538</td>
<td>0.372</td>
<td>0.009</td>
<td>-0.010</td>
<td>-0.850***</td>
</tr>
<tr>
<td>Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$ (Constant)</td>
<td></td>
<td>0.000</td>
<td>0.014</td>
<td>0.000</td>
<td>0.047***</td>
<td>0.009</td>
<td>0.032</td>
<td>0.040</td>
</tr>
<tr>
<td>$\alpha_1$ (ARCH)</td>
<td></td>
<td>0.106***</td>
<td>0.069***</td>
<td>0.094***</td>
<td>0.115***</td>
<td>0.064***</td>
<td>0.135***</td>
<td>0.109***</td>
</tr>
<tr>
<td>$\beta_1$ (GARCH)</td>
<td></td>
<td>0.873***</td>
<td>0.927***</td>
<td>0.823***</td>
<td>0.870***</td>
<td>0.933***</td>
<td>0.774***</td>
<td>0.891***</td>
</tr>
<tr>
<td>$\delta$ (VIX)</td>
<td></td>
<td>0.002***</td>
<td>0.000</td>
<td>0.097***</td>
<td>0.000</td>
<td>0.000</td>
<td>0.907***</td>
<td>0.000</td>
</tr>
</tbody>
</table>

This table presents the maximum likelihood estimation of the ARMA DCC-GARCH (1,1), including VIX, for the different assets over the whole sample period. Standard error is in the parentheses. *, **, and *** represents $p < 0.1, 0.05,$ and 0.01, respectively.

The inclusion of VIX altered some coefficients of the model. The constant parameter is no longer significant except for NIFTY 500. ARCH and GARCH terms are, however, still significant on 1% level. The results for VIX are varying among the assets. We find significant results for 3 out of the 7 assets and we accept $H4$ and confirm that $\delta \neq 0$ for those.
CHAPTER 5. RESULT

Table 4 - Coefficients with VIX²

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Equation</td>
<td>S&amp;P 500 (US)        OMXS (Sweden)        TOPIX (Japan)        NIFTY 500 (India)        TA 125 (Israel)        PSEi (Philippines)        IDX (Indonesia)</td>
</tr>
<tr>
<td>μ (Constant)</td>
<td>0.077*** (0.008)    0.078*** (0.015)    0.036*** (0.013)    0.123*** (0.018)    0.063*** (0.015)    0.046*** (0.016)    0.690*** (0.015)</td>
</tr>
<tr>
<td>ϕ₁ (MA)</td>
<td>-0.839*** (0.062)   -0.864*** (0.035)   -0.621 (0.421)      -0.285 (0.236)      0.924 (0.196)      0.119 (0.106)      0.849*** (0.164)</td>
</tr>
<tr>
<td>θ₁ (AR)</td>
<td>0.780*** (0.069)    0.841*** (0.037)    0.551 (0.436)       0.373 (0.229)       0.027 (0.196)      -0.011 (0.108)     -0.831*** (0.173)</td>
</tr>
</tbody>
</table>

Variance Equation

|                               | S&P 500 (US)        OMXS (Sweden)        TOPIX (Japan)        NIFTY 500 (India)        TA 125 (Israel)        PSEi (Philippines)        IDX (Indonesia)       |
|-------------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| α₀ (Constant)                 | 0.000 (0.000)       0.000 (0.000)       0.042*** (0.016)    0.040*** (0.010)    0.002 (0.005)      0.000 (0.005)      0.000 (0.005)      |
| α₁ (ARCH)                     | 0.000 (0.013)       0.063*** (0.024)    0.097*** (0.014)    0.118*** (0.014)    0.080*** (0.016)    0.136*** (0.016)    0.129*** (0.016)    |
| β₁ (GARCH)                    | 0.000 (0.099)       0.333* (0.187)      0.803*** (0.042)    0.856*** (0.019)    0.860*** (0.051)    0.769*** (0.025)    0.833*** (0.048)    |
| δ (VIX² )                     | 0.002*** (0.000)    0.003*** (0.000)    0.000*** (0.000)    0.000* (0.000)      0.000* (0.000)      0.000*** (0.000)    0.000* (0.000)      |

This table presents the maximum likelihood estimation of the ARMA DCC-GARCH (1,1), including VIX², for the different assets over the whole sample period. Standard error is in the parentheses. *, **, and *** represents p < 0.1, 0.05, and 0.01, respectively.

The coefficients for the last model with VIX² improved the coefficients for VIX and was significant for 6 assets, 4 of those at the 1% level. We accept H4 for 6 of the 7 assets in the model and we can see that VIX² have a positive effect on the conditional variance of asset returns. However, the coefficients for ARCH and GARCH parameters of S&P500 are not significant anymore. Even though the coefficients are significant, the estimates are very low and the effects of VIX for TOPIX, NIFTY500, PSEi, and IDX are less than 0.000.

The mean equations for all models are similar to each other and the estimated coefficients do not vary much between the models. This is expected since VIX is added as an external regressor to explain the conditional variance and not in the mean equation. Mean returns are significant for all assets over the three models and the MA and AR terms are significant for 3 of the 7 assets on the 1% level. The significant MA and AR terms imply that they are important when estimating the conditional mean.

The inclusion of the external regressor VIX did not improve the variance equation of the model. Instead, it impaired the result of the other parameters where the constant, ARCH, and GARCH terms became less significant. In the next step of determining the value of adding the external regressor VIX, we evaluate the portfolio return and risk.
5.2 Out-of-sample performance

In this section, the results from the out-of-sample performance of the optimized portfolios are presented. The data used is presented in section 4.2, Table 1 and the out-of-sample performance of the portfolios is described in Table 5. At the end of section 5.2.1, figure 4 and 5 shows the relative performance between the portfolios which can be helpful when reading the results.

Table 5 – Out-of-Sample Performance

<table>
<thead>
<tr>
<th>Forecasted Portfolio</th>
<th>Mean</th>
<th>St. Deviation</th>
<th>Variance</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>0.127</td>
<td>1.050</td>
<td>1.102</td>
<td>0.121</td>
<td>-7.398</td>
<td>9.198</td>
<td>7.236</td>
<td>0.054</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>0.112</td>
<td>1.088</td>
<td>1.185</td>
<td>0.102</td>
<td>-6.802</td>
<td>9.445</td>
<td>6.453</td>
<td>0.098</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>0.134</td>
<td>1.153</td>
<td>1.423</td>
<td>0.112</td>
<td>-9.584</td>
<td>12.532</td>
<td>10.377</td>
<td>0.387</td>
</tr>
<tr>
<td>GMV</td>
<td>0.007</td>
<td>0.791</td>
<td>0.625</td>
<td>0.043</td>
<td>-9.172</td>
<td>8.042</td>
<td>14.720</td>
<td>-1.060</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>0.006</td>
<td>0.902</td>
<td>0.814</td>
<td>0.046</td>
<td>-8.970</td>
<td>8.089</td>
<td>10.189</td>
<td>-0.841</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>0.002</td>
<td>1.128</td>
<td>1.273</td>
<td>0.048</td>
<td>-9.594</td>
<td>8.477</td>
<td>8.443</td>
<td>-0.457</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>0.022</td>
<td>0.804</td>
<td>0.647</td>
<td>0.057</td>
<td>-9.023</td>
<td>8.728</td>
<td>14.516</td>
<td>-0.829</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>0.019</td>
<td>0.909</td>
<td>0.825</td>
<td>0.054</td>
<td>-8.685</td>
<td>8.814</td>
<td>9.621</td>
<td>-0.627</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>0.018</td>
<td>1.153</td>
<td>1.330</td>
<td>0.049</td>
<td>-9.354</td>
<td>12.532</td>
<td>12.719</td>
<td>-0.097</td>
</tr>
</tbody>
</table>

This table provides descriptive statistics for the out-of-sample performance of the optimized and benchmark portfolios average daily returns for the out-of-sample period (defined as: 2001-12-01 to 2018-12-31). The portfolios are based on simulated returns using a 500 observations EWL of the 7 market indices. The values are displayed in percentage (1.0=1%). The CET portfolios are maximizing SR, GMV minimizes the variance of the portfolio and MinCVaR minimizes the expected loss beyond VaR.

When comparing the optimized portfolios with benchmarks, we encounter some expected results. The CET portfolios were expected to have the highest returns as it is the only optimization which focuses on returns. This was also the case and they have the highest daily mean returns, slightly above 0.1% as well as the highest median returns with similar values. They also exceed the two benchmark portfolios in both cases. The GMV and MinCVaR portfolios aim to lower risk and were not expected to yield high mean returns.

To be able to determine with certainty if the forecasted portfolios yield higher mean returns, tested by \( H_1 \), we performed a paired t-tests on the mean returns between the portfolios. The results are presented in Table 11–13, see appendix. There is significant result for the CET portfolios as all three forecasted portfolios outperform the two benchmark portfolios by yielding significantly higher mean return. For testing the result of VIX, we tested the performance including VIX and VIX² against the portfolio without,
within the different allocation methods. Although the mean return was higher for CET+VIX^2 than CET, it does not seem to yield significantly greater mean returns. The other two optimization methods do not yield significant result. It is important to remember that these two methods do not put emphasis on returns but rather on minimizing the risk. To conclude, we accept \( H1.1 \) and \( H1.2 \) for all the forecasted CET portfolios and conclude with certainty that they outperform the benchmarks in terms of returns over the whole sample. However, we cannot reject the null hypotheses and accept \( H1.1 \) and \( H1.2 \) for the other forecasted portfolios.

The GMV portfolios are allocated to minimize variance and we can observe that the GMV portfolio yields low variance but does not beat GMV Hist. Adding VIX or VIX^2 seems to impair the objective of lowering variance, both GMV+VIX and GMV+VIX^2 fail to beat the benchmarks and GMV+VIX^2 yields even higher variance than the CET portfolios which is contradictive to respective optimization criteria. Although, the forecasted MinCVaR portfolios objective is to minimize the downside risk, to examine the variance could give some insights. We can see that the MinCVaR portfolios perform similarly to the GMV portfolios and MinCVaR lowers the variance compared to EQW but not lower than MinCVaR Hist.

To test if the differences between the variance of the portfolios are significant, F-tests were performed. The result is shown in appendix tables 14-16 and some significant result can be observed. The forecasted CET portfolios perform better and have significantly lower variance than CET Hist. This implies that by forecasting we can lower the variance for the SR maximizing strategies, however, the CET portfolios do not yield a lower variance than the EQW portfolio.

We observe that the GMV and MinCVaR portfolio outperforms the EQW portfolio and yield a significantly lower variance. They do not, however, outperform their historical benchmarks. Although looking at Table 5, we see that the variances of these portfolios are similar. Like we observed before, adding VIX or VIX^2 to risk minimizing strategies seems to impair the optimization. We do not find any significant result for the GMV and MinCVaR portfolios with VIX or VIX^2 when compared to any benchmark portfolio. To summarize, we accept \( H2.2 \) at 1 % level significance and conclude that the forecasted CET portfolios yields a lower variance than CET Hist. Further, we accept \( H2.1 \) for the
GMV and MinCVaR portfolio, that they have significantly lower variance than the EQW portfolio.

To sum up, VIX does not seem to yield much improvement. It impairs to lower variance in every case, however, the inclusion of VIX$^2$ improves the average return for the CET portfolio. Another interesting thing to observe is that the MinCVaR+VIX$^2$ yields the highest maximum daily return together with CET+VIX$^2$. Some benefits of including VIX$^2$ seems to exist when focusing on returns. Even though VIX$^2$ increase the mean returns of the CET portfolio we cannot reject the null hypothesis and accept $H_{5.1}$. We can not accept any of the $H_{5.1} - H_{6.3}$ implying that we cannot determine the effects of VIX on the mean returns and variance.

### 5.2.1 Risk-adjusted performance

The risk-adjusted performance of the forecasted portfolio strategies is presented in Table 6. The downside risk is evaluated in terms of VaR and CVaR and three risk-adjusted performance measures are presented to give a valuable insight into the portfolio strategies. This thesis does not take the risk-free rate into account when calculating the ratios. With a very low return of a few portfolios, we feared that the risk-free rate of return would exceed the average returns and we would end up with different signs of the ratio which would complicate the interpretation.
A trend which can be observed is that the inclusion of VIX and VIX$^2$ impairs the reduction of downside risk. The GMV and MinCVaR portfolio outperforms the benchmark portfolios for both VaR and CVaR. We can conclude that the forecasted GMV and MinCVaR portfolio fulfills their purpose and provides investors with allocation methods which lower the risk, yielding a less negative return for the specified probability.

The SR for the CET portfolios is higher than all the other portfolios, implying that they fulfill their objective to maximize the reward to volatility. The three CET portfolios show a difference compared to the other forecasted portfolios and benchmarks and attain a SR three times higher than EQW which has the fourth highest SR.

A difference in Sharpe ratio test is utilized to test for significant differences in the SR between portfolios. The test is a paired t-test of two vectors of data and the result is presented in appendix tables 17-19 and shows significantly higher SR for all forecasted CET portfolios compared to the benchmarks. The other portfolios did not yield a higher SR than the benchmarks on average and this is confirmed by the t-test. Consequently, we can only accept $H3.1$ and $H3.2$ for the forecasted CET portfolios and conclude that they

<table>
<thead>
<tr>
<th>Forecasted Portfolio</th>
<th>VaR</th>
<th>CVaR</th>
<th>Sharpe ratio</th>
<th>STARR Ratio</th>
<th>Sortino Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>-1.453</td>
<td>-2.381</td>
<td>0.121</td>
<td>0.053</td>
<td>0.160</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>-1.559</td>
<td>-2.474</td>
<td>0.103</td>
<td>0.045</td>
<td>0.139</td>
</tr>
<tr>
<td>CET+VIX$^2$</td>
<td>-1.740</td>
<td>-2.659</td>
<td>0.112</td>
<td>0.050</td>
<td>0.151</td>
</tr>
<tr>
<td>GMV</td>
<td>-1.228</td>
<td>-1.982</td>
<td>0.008</td>
<td>0.003</td>
<td>0.010</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>-1.434</td>
<td>-2.265</td>
<td>0.006</td>
<td>0.003</td>
<td>0.008</td>
</tr>
<tr>
<td>GMV+VIX$^2$</td>
<td>-1.796</td>
<td>-2.844</td>
<td>0.002</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>-1.205</td>
<td>-1.977</td>
<td>0.027</td>
<td>0.011</td>
<td>0.033</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>-1.425</td>
<td>-2.251</td>
<td>0.020</td>
<td>0.008</td>
<td>0.025</td>
</tr>
<tr>
<td>MinCVaR+VIX$^2$</td>
<td>-1.765</td>
<td>-2.821</td>
<td>0.016</td>
<td>0.006</td>
<td>0.020</td>
</tr>
<tr>
<td>Benchmark</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EQW</td>
<td>-1.370</td>
<td>-2.220</td>
<td>0.037</td>
<td>0.015</td>
<td>0.045</td>
</tr>
<tr>
<td>CET Hist</td>
<td>-1.980</td>
<td>-3.314</td>
<td>0.015</td>
<td>0.006</td>
<td>0.016</td>
</tr>
<tr>
<td>GMV Hist</td>
<td>-1.198</td>
<td>-1.936</td>
<td>0.031</td>
<td>0.012</td>
<td>0.039</td>
</tr>
<tr>
<td>MinCVaR Hist</td>
<td>-1.235</td>
<td>-1.964</td>
<td>0.025</td>
<td>0.010</td>
<td>0.031</td>
</tr>
</tbody>
</table>

This table shows the risk-adjusted performance for the out-of-sample period (defined as 2001-12-01 to 2018-12-31). Value-at-Risk and Conditional Value-at-Risk are specified at probability $\alpha = 0.95$. The values are displayed in percentage (1.0=1%). Sharpe-ratio is defined as the portfolio return over standard deviation. STARR-ratio is defined as the portfolio return over CVaR at a 5% confidence level. Sortino-ratio is defined as the portfolio return over downside standard deviation, with the minimum accepted return of zero.
yield a higher SR than the benchmarks. Adding VIX or VIX\(^2\) did not increase the SR for any portfolio and we can therefore not accept \(H7.1 - H7.3\).

Both STARR-ratio and Sortino-ratio accounts for the major criticism towards SR as it uses standard deviation as a risk measurement and does not differentiate between positive or negative standard deviation. These two ratios do not penalize for upside volatility which is considered a desirable attribute and the upside volatility can, therefore, be excluded in the risk assessment.

Interpreting STARR-ratio must be done with caution since a higher value is not always desirable. Depending on the signs of the numerator and the denominator, the ratio could take on negative or positive values. The numerator, returns, can be negative if a risk-free rate is considered and exceeds the portfolio return or simply if the returns are negative. The denominator, CVaR, is in most cases negative but could potentially be positive. To simplify the interpretation of the STARR ratio, the absolute value of CVaR could be used to give a positive ratio. In this case, we have done just that, and we excluded the risk-free rate to keep the numerator positive for all portfolios. In this case, a higher value is desirable. Once again, we observe improvements in the forecasted CET portfolios which outperforms the benchmarks. We can also observe a marginal improvement of the MinCVaR portfolio when compared to MinCVaR Hist, however, it does not outperform the EQW portfolio.

The Sortino-ratio differentiates itself by using the standard deviation under the minimum acceptable return, which could be specified according to the investor's preferences. We set the minimum acceptable return to 0, which implies that the denominator consists of standard deviation of negative returns. The interpretation of the Sortino-ratio is the same as for both SR and STARR-ratio, that larger values are desirable. The results show the same trend, that the CET portfolios outperform the benchmarks while the others fail to do so. Yet again, the MinCVaR portfolio marginally outperforms MinCVaR Hist.
This table shows the Sharpe Ratio and mean returns for the portfolios. The values are displayed in percentage (1.0=1%).

Figure 5 – Risk

This table shows the variance and conditional Value-at-Risk for the portfolios. The values are displayed in percentage (1.0=1%).
5.2.2 Economic performance

The economic performance of the portfolios is evaluated by the terminal wealth of each portfolio. The terminal wealth of the portfolios is given by an initial investment of 100 USD at the start of the out-of-sample period. The result is presented in Table 7.

Table 7 – Economic Performance

<table>
<thead>
<tr>
<th>Forecasted Portfolio</th>
<th>Terminal Wealth (Without Transaction Cost)</th>
<th>Terminal Wealth (1 Basis Point)</th>
<th>Terminal Wealth (5 Basis Points)</th>
<th>Average Turnover</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>22571.469</td>
<td>13726.078</td>
<td>1875.850</td>
<td>1.116</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>11197.196</td>
<td>6847.360</td>
<td>956.878</td>
<td>1.104</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>28668.943</td>
<td>17995.895</td>
<td>2791.907</td>
<td>1.045</td>
</tr>
<tr>
<td>GMV</td>
<td>117.230</td>
<td>106.523</td>
<td>72.619</td>
<td>0.215</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>108.203</td>
<td>93.127</td>
<td>51.092</td>
<td>0.337</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>81.306</td>
<td>71.934</td>
<td>44.067</td>
<td>0.275</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>231.034</td>
<td>195.760</td>
<td>100.898</td>
<td>0.372</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>190.503</td>
<td>155.494</td>
<td>69.005</td>
<td>0.456</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>166.988</td>
<td>140.277</td>
<td>69.841</td>
<td>0.391</td>
</tr>
</tbody>
</table>

This table shows the economic performance of the different portfolios for the out-of-sample period (defined as: 2001-12-01 to 2018-12-31). The portfolio wealth is displayed in USD at different transaction costs with an initial investment of 100 USD, where VIX is in percentage form and VIX² is the squared value of VIX.

Table 7 offers an interesting insight into the economic performance of the portfolios. The CET portfolios offer a high return over the out-of-sample period compared to the benchmark portfolios. The CET portfolios outperform CET Hist with about 14000% and exceed the EQW portfolio with about 6250%. When adding VIX² to the CET portfolio, the terminal wealth gets even higher and exceeds the CET Hist with 18000% and EQW with 8000%. However, when accounting for transaction costs the returns quickly decrease for the CET portfolios. We can see this based on the average turnover which is much higher for the CET portfolios than the other forecasted portfolios. The average turnover shows the average daily trading volume of rebalancing the weights in the portfolio, a higher average turnover means more transaction costs. When using a transaction costs of 5 basis points, the return decrease to 1875 for CET portfolio and 2791 for CET+VIX². When transaction costs are taken into consideration, the differences become much smaller compared to the EQW portfolio. The reason is that it is not in need of rebalancing and
therefore not subjected to transaction costs. However, CET and CET+VIX\(^2\) still yield 300% and 770% greater return, respectively. Using historical portfolios seems to decrease the average turnover, although it would not be beneficial as the terminal wealth is lower than the alternatives, EQW and forecasted portfolio.

The GMV and MinCVaR portfolios do not yield high terminal wealth. However, remember that their objective is to minimize risk and not maximize returns. The GMV portfolio does not surpass the benchmark portfolios return but the MinCVaR portfolio beats MinCVaR Hist, even though only marginally. We also observe that the inclusion of VIX or VIX\(^2\) seems to decrease the returns for the optimizing strategies which focuses on minimizing risk.

### 5.3 Sub-Periods

A robustness check was performed by estimating the model on three sub-sample periods, namely pre- (defined as 2001-12-01 to 2007-07-31), during- (defined as 2007-08-01 to 2009-05-31) and post- (defined as 2009-06-01 to 2018-12-31) subprime crisis. The period during the subprime crisis was defined according to Phillips & Yu (2011) who tried to pinpoint the starting point of the subprime crisis. Further, this allowed us to compare the accuracy of the model during tranquil and volatile periods. Figure 6 shows the accumulated log returns for each index over the out-of-sample period.
Chapter 5. Result

Figure 6 – Accumulated Log Returns

This figure displays the accumulated log returns (in USD) with a base of 100 USD for the different indices from the 1st of December 2001 to the 31st of December 2018. The figure is further divided into three sub-periods: pre-, during-, and post-period of the subprime crisis where the pre-period is dated from the 1st of December 2001 to 31st July 2007, the during-period from the 1st August 2007 to 31st May 2009 and the post-period to the 1st June 2009 to 31st December 2018.

The first period is signified by a long boom period of about six years in the buildup to the subprime crisis. The start of the crisis is also the start of our second sub-period. Unlike the first period, the second is experiencing a huge bust period which lasts for almost a year. But at the end of this period the assets accumulated returns have almost fully recovered. The last period is more tranquil than the other two, where the returns are more centered around their mean. Table 8-10 presents sub-periods performance. These will be compared to the whole out-of-sample performance as well as between each other. To avoid making another full-scale review of the results we have limited the robustness check to returns, risk, and risk-adjusted returns.
Table 8 – Performance pre-subprime

<table>
<thead>
<tr>
<th>Forecasted Portfolio</th>
<th>Mean</th>
<th>Variance</th>
<th>VaR</th>
<th>CVaR</th>
<th>Sharpe ratio</th>
<th>STARR Ratio</th>
<th>Sortino Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>0.165</td>
<td>0.912</td>
<td>-1.453</td>
<td>-2.111</td>
<td>0.173</td>
<td>0.078</td>
<td>0.240</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>0.160</td>
<td>0.919</td>
<td>-1.453</td>
<td>-1.985</td>
<td>0.167</td>
<td>0.081</td>
<td>0.251</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>0.159</td>
<td>1.029</td>
<td>-1.490</td>
<td>-2.226</td>
<td>0.157</td>
<td>0.072</td>
<td>0.220</td>
</tr>
<tr>
<td>GMV</td>
<td>0.045</td>
<td>0.360</td>
<td>-0.961</td>
<td>-1.315</td>
<td>0.075</td>
<td>0.034</td>
<td>0.111</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>0.035</td>
<td>0.467</td>
<td>-1.092</td>
<td>-1.527</td>
<td>0.051</td>
<td>0.023</td>
<td>0.076</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>0.025</td>
<td>0.717</td>
<td>-1.382</td>
<td>-2.011</td>
<td>0.030</td>
<td>0.012</td>
<td>0.040</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>0.060</td>
<td>0.371</td>
<td>-0.991</td>
<td>-1.313</td>
<td>0.099</td>
<td>0.046</td>
<td>0.149</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>0.047</td>
<td>0.474</td>
<td>-1.100</td>
<td>-1.521</td>
<td>0.068</td>
<td>0.031</td>
<td>0.101</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>0.036</td>
<td>0.715</td>
<td>-1.361</td>
<td>-2.005</td>
<td>0.043</td>
<td>0.018</td>
<td>0.057</td>
</tr>
</tbody>
</table>

This table shows a selection of performance measures for robustness and period evaluation purposes for the period pre the subprime crisis, defined as 2001-12-01 to 2007-07-31. VaR and CVaR at the specified probability $\alpha = 0.95$. The values are displayed in percentage (1.0=1%).

Table 9 – Performance during the subprime

<table>
<thead>
<tr>
<th>Forecasted Portfolio</th>
<th>Mean</th>
<th>Variance</th>
<th>VaR</th>
<th>CVaR</th>
<th>Sharpe ratio</th>
<th>STARR Ratio</th>
<th>Sortino Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>0.172</td>
<td>3.252</td>
<td>-2.504</td>
<td>-3.950</td>
<td>0.095</td>
<td>0.043</td>
<td>0.145</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>0.149</td>
<td>3.384</td>
<td>-2.745</td>
<td>-4.097</td>
<td>0.081</td>
<td>0.036</td>
<td>0.121</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>0.276</td>
<td>4.325</td>
<td>-2.637</td>
<td>-3.855</td>
<td>0.133</td>
<td>0.072</td>
<td>0.223</td>
</tr>
<tr>
<td>GMV</td>
<td>-0.127</td>
<td>2.398</td>
<td>-2.559</td>
<td>-4.046</td>
<td>-0.082</td>
<td>-0.031</td>
<td>-0.100</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>-0.122</td>
<td>2.421</td>
<td>-2.563</td>
<td>-4.137</td>
<td>-0.079</td>
<td>-0.030</td>
<td>-0.096</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>-0.121</td>
<td>3.986</td>
<td>-3.230</td>
<td>-4.762</td>
<td>-0.061</td>
<td>-0.025</td>
<td>-0.086</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>-0.088</td>
<td>2.515</td>
<td>-2.547</td>
<td>-4.053</td>
<td>-0.025</td>
<td>-0.022</td>
<td>-0.070</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>-0.090</td>
<td>2.508</td>
<td>-2.545</td>
<td>-4.038</td>
<td>-0.057</td>
<td>-0.022</td>
<td>-0.073</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>-0.032</td>
<td>4.865</td>
<td>-3.228</td>
<td>-5.012</td>
<td>-0.014</td>
<td>-0.006</td>
<td>-0.021</td>
</tr>
</tbody>
</table>

This table shows a selection of performance measures for robustness and period evaluation purposes for the period during the subprime crisis, defined as 2007-08-01 to 2009-05-31. VaR and CVaR at the specified probability $\alpha = 0.95$. The values are displayed in percentage (1.0=1%).
CHAPTER 5. RESULT

Table 10 – Performance post-subprime

<table>
<thead>
<tr>
<th>Forecasted Portfolio</th>
<th>Mean</th>
<th>Variance</th>
<th>VaR</th>
<th>CVaR</th>
<th>Sharpe ratio</th>
<th>STARR Ratio</th>
<th>Sortino Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>0.096</td>
<td>0.803</td>
<td>-1.291</td>
<td>-2.094</td>
<td>0.108</td>
<td>0.046</td>
<td>0.136</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>0.077</td>
<td>0.920</td>
<td>-1.453</td>
<td>-2.312</td>
<td>0.080</td>
<td>0.033</td>
<td>0.102</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>0.092</td>
<td>1.097</td>
<td>-1.643</td>
<td>-2.547</td>
<td>0.088</td>
<td>0.030</td>
<td>0.108</td>
</tr>
<tr>
<td>GMV</td>
<td>0.010</td>
<td>0.440</td>
<td>-1.140</td>
<td>-1.678</td>
<td>0.015</td>
<td>0.006</td>
<td>0.018</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>0.013</td>
<td>0.710</td>
<td>-1.415</td>
<td>-2.123</td>
<td>0.016</td>
<td>0.006</td>
<td>0.018</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>0.012</td>
<td>1.080</td>
<td>-1.731</td>
<td>-2.677</td>
<td>0.011</td>
<td>0.004</td>
<td>0.013</td>
</tr>
<tr>
<td>MinCVaR</td>
<td>0.020</td>
<td>0.451</td>
<td>-1.127</td>
<td>-1.681</td>
<td>0.031</td>
<td>0.012</td>
<td>0.038</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>0.023</td>
<td>0.710</td>
<td>-1.404</td>
<td>-2.122</td>
<td>0.027</td>
<td>0.011</td>
<td>0.034</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>0.017</td>
<td>1.019</td>
<td>-1.657</td>
<td>-2.590</td>
<td>0.017</td>
<td>0.007</td>
<td>0.020</td>
</tr>
</tbody>
</table>

This table shows a selection of performance measures for robustness and period evaluation purposes for the period after the subprime crisis, defined as 2009-06-01 to 2018-12-31. VaR and CVaR at the specified probability $\alpha = 0.95$. The values are displayed in percentage (1.0=1%).

The result of the pre-subprime period shows similar result as the whole out-of-sample performance in terms of which portfolios perform best and worst in the respective aspects. We can see that the risk-minimizing portfolios, GMV and MinCVaR perform well over this boom period. The variance of the portfolios is 0.360 and 0.371 and a VaR at 95% of -0.961 and -0.991, respectively.

The period during the subprime crisis shows an interesting result. The forecasted CET portfolios are the only ones which yield a positive return on average and they also yield the highest risk-adjusted returns. The result also shows that they perform well in terms of downside risk, where they provide similar VaR and even a better CVaR than the other forecasted portfolios, which is an unexpected result in regards of the optimization criteria. Due to negative returns, we find negative risk-adjusted ratios. The last period shows similar results as the first period. The GMV and MinCVaR portfolio perform well in terms of variance and downside risk and the CET portfolios show higher return and higher risk-adjusted returns like in the other periods.
Chapter 6

6 Analysis

This chapter discusses the empirical findings.

Initially, we analyze the effects of VIX on the optimization. Unlike Day & Lewis (1992) and Koopman et al. (2005), we found significant results of including VIX in the model. Our results are similar to Kambouroudis & McMillan (2016) who also found a marginal positive effect of including VIX in the variance equation. The employed VIX$^2$ which is based on S&P 500 option prices were found to be significant for most markets, indicating that it serves as a good proxy of volatility for most other markets as well. However, it would be interesting to see the effects of market specific volatility indices which possibly could improve the model. Like Kanas (2013) we find that VIX$^2$ can easier detect the risk-return relationship than VIX as it yields more significant results in comparison. However, even though VIX is significant, it impairs the result of the overall variance equation.

VIX$^2$ benefits the out-of-sample performance of the portfolios regarding one aspect. When the allocation method aims to allocate weights in a way that maximizes returns over risk, the inclusion of VIX$^2$ improve the optimization and yields a higher return. However, when the allocation method aims to minimize risk, VIX fails to improve the optimization. This is the case for both volatile and tranquil periods which we observed from the subperiods, tables 8-10. Our suggestion as to whether if VIX should be included in the model are highly dependent on the intended allocation method. If the investor aims to minimize risk, we suggest that VIX in either form should be excluded. While if the investor aims to maximize return over risk, VIX$^2$ could be included in the forecasting as it in our case yields similar risk-adjusted returns and unquestionably higher terminal value. The remaining part of the analysis will exclude the models and portfolios containing VIX.

The DCC-GARCH model shows high significance for modeling the conditional mean and especially for modeling the conditional variance for the assets. The significant AR and MA terms show high values, indicating that these are important when estimating the
conditional mean. However, we were concerned about the high values and found the
cause to be partly due to the use of the student-t distribution in the modeling. When testing
an ARMA process assuming normal distribution we got some lower but less significant
coefficients. However, coefficients with similar magnitude have been found in recent
studies by Sahamkhadam et al. (2018) and Mensi, Hammoudeh & Kang (2017), which
have also used student-t distribution.

Overall, we found highly significant ARCH and GARCH effects. The significant ARCH
effects indicate that the squared residual of past returns affect the current conditional
volatility of asset returns. It implies that if the past periods squared residuals are high, it
will be shown as increased volatility of current asset returns. The GARCH effects are
significant and high, which implies that the previous conditional variance has a great
influence on the current conditional variance of asset returns. Both significant ARCH and
GARCH effects suggest that the volatility and squared residuals of asset series are
affecting its own future conditional variance. It could be believed that low-frequency data
like monthly or quarterly data smooths out the volatility and the clustering effects are not
captured. Our result indicates that these effects do exist and like Ardia & Hoogerheide
(2014) concluded, high-frequency data is needed to capture the volatility clustering
effects with the GARCH model.

Hypotheses 1-3 allows us to evaluate the forecast performance of the DCC-GARCH
model. Our results indicate that we can achieve higher returns by the use of forecasting.
This is underpinned by acceptance of hypothesis 1.1 and 1.2 for the CET portfolio
implying that the maximizing SR strategy outperforms the benchmarks. This gives an
indication that the market is not characterized by random walk behavior because if assets
were moving in random patterns, forecasting would not be as beneficial. Further, this
means that technical analysis still can be beneficial. As the forecasting seems to work
well, the model seems to detect and capture possible market patterns and inefficiencies.
This indicates that assets might not reflect their intrinsic value at all times, which in that
case suggests that the EMH does not hold. The market conditions seem more in line with
BF and AMH which states that assets at times are mispriced.

We find the DCC-GARCH to be adequate in terms of modeling risk. When comparing
the risk-minimizing portfolios, GMV and MinCVaR, to an equally weighted portfolio,
significant improvement can be distinguished. However, the forecasting of these does not
yield additional improvements, they do not outperform GMV Hist and MinCVaR Hist. We find that it is only the forecasted CET portfolios which outperforms the benchmarks in terms of risk-adjusted return. In accordance with de Almeida et al. (2018) and Ku et al. (2007), our results suggest that forecasting using DCC-GARCH is efficient. We can also see indications that the model is efficient in both volatile periods in accordance with what Laurent, Rombouts & Violante (2012) found, as well as in tranquil periods, implying that it can capture time-varying correlations and covariances between assets.

Interestingly, we find that the risk-minimizing strategies perform well during tranquil periods. But during periods of high volatility when investors might be more interested in minimizing risk, these strategies do not function with the same efficiency. The CET portfolios outperform the GMV and MinCVaR portfolios both in terms of risk and return during volatile periods which lead us to the conclusion that our return maximizing strategy with DCC-GARCH is more beneficial than the risk-minimizing strategies. During the subprime crisis period, the result reflects the difference in the use of risk measurements, where the CET portfolio shows a lower risk when using the downside risk measures VaR and CVaR. However, it shows a higher variance. Here we see a clear example that including the upside volatility in the measure of risk can be misleading. This is in line with Bodnar & Zabolotskyy (2017) who argues that downside risk measures are preferable over variance. During the short volatile period of the subprime crisis, we find that one strategy (CET) both minimize risk and maximize return, which was deemed unrealistic by Markowitz (1952). However, as Markowitz are considering variance as a proxy for risk, his statement still holds true.

It would have been interesting to make comparisons of the portfolio optimization strategies with similar studies. However, this would be difficult because as Lim & Sek (2013) and Berger (2013) argues, studies use different models to address risk, choice of assets, time-periods and the models depend on these characteristics. Therefore, we have been limited to make comparison with benchmarks such as the EQW portfolio used by DeMiguel, Garlappi & Uppal (2007) and the historical portfolio as used by Sahamkhadam et al. (2018).
Chapter 7

7 Summary and Conclusion

This chapter concludes the thesis and suggests new approaches for future research.

This thesis aimed to evaluate one estimation and forecasting technique during whole- and sub-sample periods. By the application of the DCC-GARCH model, we forecasted returns and covariances and constructed portfolios out of 7 market indices based on 3 different allocation methods. Further, we evaluated the effects of a volatility index in the estimation of the conditional variance.

To answer the research question whether the forecasted optimized portfolios can outperform benchmark portfolios in terms of risk and expected return we can conclude that it was possible with the application of a DCC-GARCH model. The model can estimate and forecast risk and return sufficiently and can detect and capture possible market patterns and inefficient pricing. The SR maximizing portfolio outperforms the benchmarks in terms of average mean return, risk-adjusted returns, and terminal wealth, while the two risk-minimizing strategies reduce risk. VIX has only a marginal effect on the conditional variance of returns. If a volatility index is considered, we suggest using a squared version, as it seems to be beneficial when employing allocation strategies which focus on expected return. However, when considering risk-minimizing strategies it fails to improve the optimization. Lastly, we conclude that over the whole sample Markowitz was partly right, we could not find a portfolio which both minimizes risk and maximizes returns at the same time. But during one short volatile period, we can.

We conclude that the CET, SR maximizing optimization, works best overall and we show an example that including upside volatility can be misleading. An interesting take on this allocation method would be for future research to add constraints to mirror the risk-adjusted ratios, STARR, and Sortino which only incorporates downside risk and standard deviation of negative returns. We also suggest researchers to expand the knowledge of the effects of VIX on return maximizing versus risk-minimizing strategies.
The results could be of interest for researchers and financial agents who aim to employ a DCC-GARCH model within portfolio optimization. The optimization method we proposed can advantageously be used in computerized trading but can be less practical to utilize with manual trading.
References


APPENDIX

Paired t-Test - Mean Returns

Table 11 - (CET)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>CET Hist</th>
<th>CET</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>8.412***</td>
<td>6.316***</td>
<td>-</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>6.538***</td>
<td>5.233***</td>
<td>-1.575</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>7.271***</td>
<td>6.107***</td>
<td>0.522</td>
</tr>
</tbody>
</table>

This table shows results from a paired t-test on mean returns for the CET portfolios in comparison to benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.

Table 12 - (GMV)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>GMV Hist</th>
<th>GMV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMV</td>
<td>-0.367</td>
<td>-2.965</td>
<td>-</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>-2.800</td>
<td>-2.118</td>
<td>-0.127</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>-2.237</td>
<td>-1.782</td>
<td>-0.365</td>
</tr>
</tbody>
</table>

This table shows results from a paired t-test on mean returns for the GMV portfolios in comparison to their benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.

Table 13 - (MinCVaR)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>MinCVaR Hist</th>
<th>MinCVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinCVaR</td>
<td>-1.478</td>
<td>0.347</td>
<td>-</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>-1.478</td>
<td>-0.140</td>
<td>-0.499</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>-1.039</td>
<td>-0.129</td>
<td>-0.276</td>
</tr>
</tbody>
</table>

This table shows results from a paired t-test on mean returns for the MinCVaR portfolios in comparison to its benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.

F-Test - Variance

Table 14 – (CET)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>CET Hist</th>
<th>CET</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>1.406</td>
<td>0.701***</td>
<td>-</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>1.512</td>
<td>0.753***</td>
<td>1.075</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>1.816</td>
<td>0.905***</td>
<td>1.291</td>
</tr>
</tbody>
</table>

This table shows results for difference in variance for the CET portfolios in comparison to their benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.
Table 15 – (GMV)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>GMV Hist</th>
<th>GMV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMV</td>
<td>0.798***</td>
<td>1.027</td>
<td>-</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>1.039</td>
<td>1.337</td>
<td>1.302</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>1.624</td>
<td>2.090</td>
<td>2.036</td>
</tr>
</tbody>
</table>

This table shows results for difference in variance for the GMV portfolios in comparison to their benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.

Table 16 – (MinCVaR)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>MinCVaR Hist</th>
<th>MinCVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinCVaR</td>
<td>0.826***</td>
<td>1.029</td>
<td>-</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>1.053</td>
<td>1.313</td>
<td>1.276</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>1.697</td>
<td>2.115</td>
<td>2.055</td>
</tr>
</tbody>
</table>

This table shows results for difference in variance for the MinCVaR portfolios in comparison to their benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.

Paired t-Test - Sharpe Ratio

Table 17 - (CET)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>CET Hist</th>
<th>CET</th>
</tr>
</thead>
<tbody>
<tr>
<td>CET</td>
<td>7.386***</td>
<td>7.228***</td>
<td>-</td>
</tr>
<tr>
<td>CET+VIX</td>
<td>5.486***</td>
<td>5.825***</td>
<td>-2.029</td>
</tr>
<tr>
<td>CET+VIX²</td>
<td>5.835***</td>
<td>6.351***</td>
<td>-0.717</td>
</tr>
</tbody>
</table>

This table shows results for difference in Sharpe ratio for the CET portfolios in comparison to their benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.

Table 18 - (GMV)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>GMV Hist</th>
<th>GMV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMV</td>
<td>-3.424</td>
<td>-2.987</td>
<td>-</td>
</tr>
<tr>
<td>GMV+VIX</td>
<td>-2.826</td>
<td>-2.415</td>
<td>-0.260</td>
</tr>
<tr>
<td>GMV+VIX²</td>
<td>-2.623</td>
<td>-2.386</td>
<td>-0.517</td>
</tr>
</tbody>
</table>

This table shows results for difference in Sharpe ratio for the GMV portfolios in comparison to their benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.
Table 19 - (MinCVaR)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EQW</th>
<th>MinCVaR Hist</th>
<th>MinCVaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MinCVaR</td>
<td>-1.131</td>
<td>0.300</td>
<td>-</td>
</tr>
<tr>
<td>MinCVaR+VIX</td>
<td>-1.540</td>
<td>-0.445</td>
<td>-0.873</td>
</tr>
<tr>
<td>MinCVaR+VIX²</td>
<td>-1.584</td>
<td>-0.749</td>
<td>-0.856</td>
</tr>
</tbody>
</table>

This table shows results for difference in Sharpe ratio for the MinCVaR portfolios in comparison to their benchmarks. *, **, and *** represents significance level of 10, 5, and 1 percent, respectively.