Essays on Consumption: Aggregation, Asymmetry and Asset Distributions

Acta Wexionensia
No 68/2005
Economics

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Mårten Bjellerup

Växjö University Press

Essays on Consumption : Aggregation, Asymmetry and Asset Distributions. Thesis for the degree of Doctor of Philosophy, Växjö University, Sweden 2005

Series editors: Tommy Book and Kerstin Brodén
ISSN: 1404-4307
ISBN: 91-7636-465-8
Printed by: Intellecta Docusys, Göteborg 2005

Abstract

Bjellerup, Mårten (2005). *Essays on Consumption: Aggregation, Asymmetry and Asset Distributions.* Acta Wexionensia No. 68/2005. ISSN: 1404-4307, ISBN: 91-7636-465-8. Written in English.

The dissertation consists of four self-contained essays on consumption. Essays 1 and 2 consider different measures of aggregate consumption, and Essays 3 and 4 consider how the distributions of income and wealth affect consumption from a macro and micro perspective, respectively.

Essay 1 considers the empirical practice of seemingly interchangeable use of two measures of consumption; total consumption expenditure and consumption expenditure on nondurable goods and services. Using data from Sweden and the US in an error correction model, it is shown that consumption functions based on the two measures exhibit significant differences in several aspects of econometric modelling.

Essay 2, coauthored with Thomas Holgersson, considers derivation of a univariate and a multivariate version of a test for asymmetry, based on the third central moment. The logic behind the test is that the dependent variable should correspond to the specification of the econometric model; symmetric with linear models and asymmetric with non-linear models. The main result in the empirical application of the test is that orthodox theory seems to be supported for consumption of both nondurable and durable consumption. The consumption of durables shows little deviation from symmetry in the four-country sample, while the consumption of nondurables is shown to be asymmetric in two out of four cases, the UK and the US.

Essay 3 departs from the observation that introducing income uncertainty makes the consumption function concave, implying that the distributions of wealth and income are omitted variables in aggregate Euler equations. This implication is tested through estimation of the distributions over time and augmentation of consumption functions, using Swedish data for 1963-2000. The results show that only the dispersion of wealth is significant, the explanation of which is found in the marked changes of the group of households with negative wealth; a group that according to a concave consumption function has the highest marginal propensity to consume.

Essay 4 attempts to empirically specify the nature of the alleged concavity of the consumption function. Using grouped household level Swedish data for 1999-2001, it is shown that the marginal propensity to consume out of current resources, i.e. current income and net wealth, is strictly decreasing in current resources and net wealth, but approximately constant in income. Also, an empirical reciprocal to the stylized theoretical consumption function is estimated, and shown to bear a close resemblance to the theoretical version.

Keywords:
Aggregate consumption, Aggregation, Asymmetry, Wealth distribution, Income distribution, Concavity, Permanent Income Hypothesis, Buffer stock saving

Preface

Several years ago, as I pursued my undergraduate studies at Lund University, I began thinking (as I guess most students at that (st)age) about my future and what to do with my studies. One day my mother phoned as I was standing outside the Economics Department. She had been speaking to her colleague, Siv Berglund, who was working as an economist and who suggested that I opt for a Ph.D. After having spent some 18 years in the educational system, I was more inclined to try finding a way out - not a way to stay in. Consequently, I laughed at my mother's ridiculous suggestion. Didn't she know how much work that would be and how many years it would take?

As it happened, less than two years later I found myself in Växjö, having joined three other students in the inaugural class of the Ph.D. program in economics at Växjö University. The first couple of years were spent muddling through the mandatory and optional courses. Here, the Ph.D. student network created and organized by Jan Ekberg played a pivotal role as it gave us the possibility of attending courses at different universities, often of our own choosing. Actually, we were not merely given the possibility; we were actively encouraged and supported, financially as well as academically.[1] A fond memory from the courses in micro- and macroeconomics in Göteborg are the numerous, seemingly

[1] As in most cases, the bottom line tells the story. Totaling 80 credits, the Ph.D. courses that I've passed are distributed over Göteborg University (35 credits), Lund University (25), Växjö University (15) and the University of Copenhagen (5).

endless, train rides. More than once, an entire journey was spent trying to solve hand-ins; the collaborative efforts of Mikael Ohlson, Henrik Andersson and I improved my understanding (as well as my grades), for which I'm always thankful. What's more, I want to express my gratitude to Jan Ekberg for always standing up for us, the Ph.D. students. No matter what the circumstances, you've always been there for us and have made sure that we've had the best conditions possible.

My work at ABN AMRO Bank, parallel to the courses, provided me with ideas for my first paper as well as inspiration and encouragement. Discussions on economics, the financial markets and life in general with Michael Grahn, Leif Lindahl, Brian Cordischi, Andrew Marsh and several others, provided both the answers and the questions that influenced my choice then, not to abandon academia for international finance.

As for writing the papers included in this dissertation, the road ahead was at times invisible, especially at the beginning. Writing several papers? I could barely muster ideas for one, let alone three or four! Through the whole process, I've treasured the continuous support and encouragement of Håkan Locking. Suggesting topics, discussing ideas, putting out psychological fires and answering my endless flow of questions; these are some of the aspects of your supervision that I have enjoyed most, Håkan. Thank you. Of course, there has also been valuable support from other members of the department and Ghazi Shukur deserves special mention. Besides your support and advice, I'm pleased that you introduced me to Thomas Holgersson, coauthor of one of the essays. During the emotional roller-coaster I've experienced on a yearly, monthly, weekly and often daily basis, I've very much appreciated the support coming from my fellow Ph.D. students: Ali, Henrik, Jonas, Maria, Mikael, Monika and Susanna. I'm not sure whether anyone has ever thought of us as a team, but in my opinion I've benefited from our team spirit and I'd especially like to thank my roomies over the years. Besides the internal seminars, I have benefited from valuable comments and criticism at external seminars at the Swedish Central Bank, the Swedish National Institute for Economic Research and the South Swedish Graduate Program in Economics 2004 Workshop; special thanks go to Bengt Assarsson for the feedback I got at the final seminar. I like to flatter myself (sometimes) that I write decent English, but Mimi Möllers proofreading undoubtedly improved the language, for which I'm thankful.

Oh, I almost forgot. Besides mentioning the positive climate of the department for which all colleagues deserves credit, a special thank-you goes to Innebandygänget (the floor ball players) at the university. The matches have more than once been a biweekly high point that provided me much needed rejuvenation of mind and body.

My supportive relatives have always meant a great deal to me. Having my mother and father tell me how proud they are and that they believe in me, over and over, is invaluable. My thank you's are as endless as your support.

My vocabulary doesn't do justice to my love for you, but I'll give it a try. You mean the world to me. Since a couple of weeks we are a family of three and I can't imagine anything better. I love you, Jessica and Amanda.

Mårten Bjellerup

Växjö

May 2005

Contents

Introduction

The background to the dissertation consists of two topics, leading to two essays per topic, thus yielding a natural division of the dissertation. The two first essays are concerned with the different properties of the subcomponents of total consumption expenditure, as measured in the National Accounts. The last two essays take a micro and macro approach respectively, on the subject of the concavity of the consumption function.

1 Aggregation, asymmetry and consumer behavior

The theoretical definition of consumption, pure consumption, as found in mainstream as well as orthodox theory, is troublesome from an empirical perspective given the lack of a reciprocal in the statistics. Theory, focusing on the flow of utility from goods purchased in the current and previous periods, is not easily reconciled with the National Accounts where only expenditure in the current period is accounted for. From a theoretical perspective, it is of course desirable to try to calculate the appropriate measure of consumption. However, this approach is infrequently chosen, given the difficulties concerning such calculations.[2] Instead, the choice is usually between expenditure on nondurable goods and services and total consumption expenditure, the latter is a subset of pure consumption while the former is interesting from a policy perspective. The difference between the two measures is the expenditure on durable goods, usually considered as an investment rather than a consumption good. The use of total consumption expenditure, often used in combination with consumption expenditure on nondurable goods and services, thus carries implicit assumptions on its two components, assumptions that are rarely commented on. This observation leads to two, related ideas. First, a natural question is whether the practice of interchangeable use of the broad measures of consumption as found in the National Accounts, is statistically acceptable. Second, it is appealing to undertake a study to analyze the differing theoretical views on durable and nondurable goods, through the creation of a statistical test. Several theories yield testable hypotheses on the symmetry, or lack thereof, for both goods. In mainstream theory on the consumption of nondurable goods, consumption is symmetrically behaved, while, in contrast, mainstream theory on durable goods suggests that it is asymmetrically behaved.[3] Essay 1 is a test of the first, and Essay 2 is a test of the second of these ideas.

2 Asset distributions and consumer behavior

The optimal intertemporal consumption problem has since the influential paper of Hall (1978), to a very large extent been addressed using linear or approximately linear consumption functions, based on the Euler equation.[4] The standard perfect certainty and certainty equivalent versions of the consumption

[2] A prominent example of this view is Hall (1978).

[3] Hall (1978) and Campbell and Mankiw (1989) are prominent examples of the former and Gregorio et al (1998) and Leahy and Zeira (2000) of the latter.

[4] As Carroll and Kimball (1996) notes: "Of the 25 household-level studies summarized in the recent survey by Browning and Lusardi (1996), only two allow for a nonlinear consumption function..." (p.982, note 5).

decision, usually using representative agent models, imply a marginal propensity to consume that is unrelated to the level of household wealth. However, this class of models, usually labelled the permanent income hypothesis with rational expectations (PIH), has suffered from notable discrepancies between the model's predictions and aggregate data. In response to these anomalies, works by Zeldes (1989), Deaton (1991) and Carroll and Kimball (1996), among others, have pointed out that the introduction of labor income uncertainty makes the consumption function concave. In turn, this implies that the marginal propensity to consume out of current resources (MPC_{CR}) is strictly decreasing in current resources (defined as current income plus non-human wealth). In short, this class of models, known as precautionary saving or buffer stock saving models (BSH), have been suggested as a remedy for the above mentioned anomalies related to the PIH.

Essays 3 and 4 address the fundamental prediction of the BSH, this "workhorse of modern-day consumer theory" (Ludvigson and Michaelides (2001), p.632), namely the concavity of the consumption function and its implications. Against the backdrop of the claim in Attanasio (1999), that "[t]he relevance of the precautionary saving motive is ultimately an empirical matter" (p.772), the former of the two essays uses a macro approach while the latter uses a micro approach. The conclusion in Carroll and Kimball (1996), that "[a]t the aggregate level, concavity means that the entire wealth distribution is an omitted variable when estimating aggregate consumption Euler equations..." (p.982), serves as the puzzle to be investigated in Essay 3. At the household level, the BSH suggests, as mentioned above, that the MPC_{CR} is strictly diminishing in current resources. However, given the possibility to disaggregate current resources into current income and net wealth, Essay 4 is concerned with the question of how the alleged concavity relates to the components of current resources.

3 A summary of the included essays

Essay 1

Do the Measures of Consumption Measure Up?, considers the empirical practice of seemingly interchangeable use of two measures of consumption as found in the National Accounts; total consumption expenditure and consumption expenditure on nondurable goods and services. For estimation of aggregate consumption functions, the original problem is the discrepancy between the theoretical definition of pure consumption and the available statistics in the National Accounts. The early and influential papers of Hall (1978) and Davidson et al (1978) both used nondurable consumption, after which we have seen a growing number of papers using both nondurable and total consumption. Given that the difference between the two measures consists of consumer expenditure on durable goods, to which mainstream consumption theory is not applicable, there are theoretical reasons for questioning this practice. From an empirical perspective, stylized facts of the series show significant differences, reinforcing the theoretical standpoint. Using data from Sweden (1970:2-2001:4) and the US (1959:1-2002:4) in an error correction model, the practice of interchangeability is tested.

The main result of the paper is that, as expected, the two measures cannot be used interchangeably. For the US, the ECM specification is shown to exhibit significant differences in several aspect of the modelling of the cointegrating relation, while for Sweden it is shown that there are significant differences in both the long and the short-run modeling. The results suggest that it is not possible to reconcile theoretical requirements with those of policy, given that nondurable consumption, being a subset of the theoretical definition of pure consumption, is not interchangeable with the variable of policy interest, total consumption.

Essay 2

A Simple Multivariate Test for Asymmetry with Applications to Aggregate Consumption, is concerned with deriving a univariate and a multivariate version of a test for asymmetry, based on the third central moment.[5] The logic behind the test is that the dependent variable should correspond to the specification of the econometric model; symmetric with linear models and asymmetric with non-linear models. Through the use of the series to be tested in levels or first differences, tests for deepness and steepness can be conducted. Deepness considers peaks and troughs distance above and below trend, while steepness considers the speed at which the peaks and troughs are approached.

In the application of the test, we depart from the observation that mainstream and orthodox theory on the consumption of nondurable and durable goods respectively, yield different implications as to the symmetry of the series. Typically, mainstream theory sees the consumption of nondurables as symmetric while it sees the consumption of durables as asymmetric. Although the theory itself should produce a verdict on the magnitude of the alleged asymmetry, it is of course an empirical matter. The test could be viewed as a model specification test given the current application but being robust against serial correlation, autoregressive conditional heteroscedasticity and non-normality, this test can be applied to any stationary series.

The main result of the paper is that orthodox theory seems to be supported for both nondurable and durable consumption in our four-country sample; Canada, Sweden, the UK and the US. In the case of durables, deviation from symmetry could only be detected for one of the four countries in the sample. For nondurable consumption, the UK and the US both exhibited positive deepness, i.e. the peaks were taller than the troughs were deep. In a test of

[5] This paper is coauthored with Thomas Holgersson.

the difference between the univariate and the multivariate versions of the test, it is concluded that, given the current data set, the choice between the two is important as they yield different verdicts on rejection in 4 out of 8 cases.

Essay 3

Is the Consumption Function Concave?, departs from the controversy on the linearity of the consumption function. If recent theories such as the buffer stock saving model (BSH) are correct, then at the aggregate level, concavity means that the distributions of income and wealth, are omitted variables when estimating aggregate consumption Euler equations. This hypothesis is tested through augmentation of the rule of thumb model (following Carroll et al (1994)) and the error correction model (following Lettau and Ludvigson (2004)), using Swedish data for 1963-2000. Solving the problem on the scarcity of data on wealth involves the use of yearly wealth tax data as well as infrequent but more detailed register data, allowing fitting of the skew-t and gamma distributions to the yearly wealth tax data. For both the distribution of income and the distribution of wealth, simple measures of dispersion are calculated, based on the second central moment, or the corresponding dispersion parameter of the distribution.

The main result of the paper is that the dispersion of wealth has a significant and positive impact on consumption while the dispersion of income is shown to be insignificant. Relating to the underlying theory, the positive impact from the increasing dispersion of wealth is seen as coming from the increase in the number of households with negative net wealth. This group, according to the BSH, has the highest marginal propensity to consume and given the relative growth of the group, it would imply a positive boost to consumption. The results do not seem to be dependent on the method employed as they are stable across models and across distributions.

Essay 4

Does Consumption Function Concavity Vary with Asset Type?, attempts to empirically specify the nature of the alleged concavity of the consumption function. The BSH states that consumption is concave in current resources, defined as current income and non-human wealth, also implying a strictly decreasing MPC_{CR}. However, it is not clear how concavity relates to the components of current resources. Using grouped household level data, consumption is imputed after which GMM estimation is employed to get consumption function estimates. The hypothesis tested is then whether the estimated parameters vary with income, wealth and current resources, respectively, and also whether MPC_{CR} varies in each case.

The main result of the paper is that the concavity relates to wealth and also current resources, but not income. Furthermore, MPC_{CR} is found to be strictly decreasing in wealth and current resources but approximately constant in income. Also, an empirical reciprocal to the stylized theoretical consumption function in Carroll (2000) is estimated, and shown to bear a close resemblance to the theoretical version.

Considering the joint implications of the results in Essay 3 and Essay 4, a few comments can be made. Interestingly enough, an analysis in Essay 4 of the group of households with negative net wealth (suggested in Essay 3 to play an important role) shows that the group exhibits a significantly higher average study debt than the other groups of households. Against this backdrop, a speculative explanation for the results in Essay 3, would be the dramatic increase in the attendance of higher education, as witnessed during the 1980's and 1990's. A less speculative explanation, that is also consistent with theory, would be the deregulation of the credit market in the early and mid-1980's, allowing households to adjust their portfolios and in effect take on more debt on an aggregate level.

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If you press the data hard enough, it will confess

Essay 1

Do the Measures of Consumption Measure Up?*

Mårten Bjellerup†

May 9, 2005

Abstract

In empirical research on the aggregate consumption function, the definition of the dependent variable - without distinct consensus - is either total consumption expenditure or consumption expenditure on nondurable goods and services. Through estimation of an error correction model (ECM) using quarterly data for Sweden and the US, this paper shows that these two definitions cannot be used interchangeably, contrary to what has often been done. For the US, the ECM specification is shown to exhibit significant differences in several aspects of the modeling of the cointegrating relation, while the specification for Sweden shows that there are significant differences in both the long- and the short-run modeling. The results suggest that it is not possible to reconcile theoretical requirements with those of policy, given that nondurable consumption - being a subset of the theoretical definition of pure consumption - is not interchangeable with total consumption, the variable of policy interest.

*I am very grateful for Håkan Locking's invaluable support throughout the process of writing this paper and I also want to thank Ghazi Shukur for guidance with the econometric part. I also wish to especially thank the participants of the seminars at Växjö University and at the National Institute for Economic Research (NIER) for their constructive criticism. The generosity of Jesper Hansson and the NIER in supplying the Swedish data set, is much appreciated. Finally, I want to thank Bengt Assarsson for helping me straighten out my thoughts when I tried to formulate the original idea and for the feedback at the final seminar.

†School of Management and Economics, Växjö University, SE-351 95, Växjö, Sweden; *e-mail*: marten.bjellerup@ehv.vxu.se.

1 Introduction

Consumer expenditure is by far the largest component of the gross domestic product, accounting for between 50% and 75% in most economies. The swings in aggregate consumer behavior are thus of great importance for economic growth and welfare. As a consequence, there is considerable interest, not the least from a policy perspective, to be able to explain and predict these swings. Muellbauer and Lattimore (1995) go as far as saying that "[n]ot surprisingly, the consumption function has been the most studied of the aggregate expenditure relationships and has been a key element of all the macroeconometric model building efforts since the seminal work of Klein and Goldberger (1955)" (p.223).

Against this backdrop it is noteworthy that there seems to be a lack of unanimity regarding the definition of the dependent variable in the aggregate consumption function. The theoretical foundation, as laid out by Modigliani and Brumberg (1954) and Friedman (1957), departs from the notion of pure consumption; i.e., the stream of utility that comes from goods and services purchased in the current period or previous periods. The influential papers of Hall (1978) and Davidson et al. (1978) contained empirical analyses, based on the definition of consumption as consumer expenditure on nondurable goods and services (c^{nd} hereafter). Among the many papers written since then, a growing number have used the wider definition of total consumer expenditure (c^{tot} hereafter), or both definitions.[1] As for the discussion on the dependent variable, it takes different forms in different papers, ranging from theoretical to empirical and from brief to extensive (very seldom). The seemingly interchangeable use of the definitions of consumption and its impact on results is the focus of this paper.

If the two measures of consumption, c^{nd} and c^{tot}, are to be interchangeable, the two components (c^{nd} and c^{d}) of the wider measure c^{tot} have to be identical. This assumption, far from always addressed, is troublesome from a theoretical as well as a practical perspective. Theoretically, c^{nd} and c^{d} are usually viewed

[1] The wider definiton of total consumer expenditure (c^{tot}) equals consumer expenditure on nondurable goods and services (c^{nd}) plus consumer expenditure on durable goods (c^{nd}).

as inherently different, with the latter being considered an investment good. Preferably, it should be treated as a stock rather than a flow variable; see e.g. Leahy and Zeira (2000). From a statistical point of view, the stylized facts of the two series yield little support for interchangeability, as both growth rates and volatility measures differ significantly; see e.g. Attanasio (1999). Thus through estimating consumption functions with c^{tot} and c^{nd} as the dependent variables, it will be possible to test if the interchangeable use of the measures is correct. The specification is an error correction model (ECM hereafter), chosen for its empirical as well as theoretical merits.

Previewing the results, I find that the two measures are not interchangeable, mainly because of significantly different cointegrating relations, but in the Swedish case, also because of significantly different short-run adjustment. The results suggest that it is not possible to reconcile theoretical requirements with those of policy, given that nondurable consumption, being a subset of the theoretical definition of pure consumption, is not interchangeable with the variable of policy interest - total consumption.

The outline of the paper is as follows: Section 2 reviews different aspects of previous research, while section 3 contains a description of the data and a discussion on seasonality. Section 4 investigates whether the error correction model finds empirical support in all cases and then proceeds to test for equality between the consumption functions. Section 5 comments and concludes.

2 Background

The objective of this paper builds to a large extent upon the approaches used in previous papers. Furthermore, since the focus of this paper concerns the definition of the dependent variable, the following description of previous research will have two aspects; definition of variables and model specification. The review of previous model specifications will serve as an introduction to the model chosen in this paper as well as into the discussion on the variables entering the consumption function. There, clear motivation for the question addressed in this paper will be offered as the dependent variable is discussed first, followed by the independent variables, primarily income and wealth.

2.1 Model specification in previous research

The starting point of modern empirical literature on the aggregate consumption function is usually traced back to Spiro (1962), Ando and Modigliani (1963), Ball and Drake (1964) and Stone (1964), and their work on the relationship between consumer spending, income, wealth and the interest rate. Subsequent research suffered from a number of shortcomings, however. The most notable of these is that the properties of non-stationary time series were not well understood. Furthermore, there was a lack of consistency with economic theory (not the least concerning expectations), and the availability of household assets was very limited. Two papers, Hall (1978) and Davidson et al. (1978) (DHSY, henceforth), overcame most of these shortcomings and are now regarded as "a milestone for research on the aggregate consumption function" (Muellbauer and Lattimore (1995), p.222). Hall, combining the life cycle hypothesis with rational expectations, showed, using a Euler equation consumption function, that the best forecast of next period's consumption is this period's consumption; i.e. consumption should be random walk. A simple version of a Hall (1978) type Euler equation is

$$c_{t+1} = \gamma c_t + \varepsilon_{t+1}$$

where c_t denotes consumption and ε_{t+1} is a "true regression disturbance; that is, $E_t \varepsilon_{t+1} = 0$." (Hall (1978), p.974). Shortly after, his results were rejected by a number of papers and much of the research on consumption has since been occupied with relaxing one or more of Hall's (1978) assumptions.

The other strand of research emanates from the other prominent paper of 1978. DHSY followed the earlier tradition using a "solved out" or "structural" consumption function, although their econometric specification now also contained an error correction mechanism. The concept of such a mechanism was not an innovation in itself, but it was the first time it was embodied in an aggregate consumption function.[2] The basic DHSY model is

$$\Delta_4 c_t = \beta_0 + \beta_1 \Delta_4 y_t + \beta_2 (c_{t-4} - y_{t-4}) + \varepsilon_t \tag{1}$$

where the third term on the RHS is the ECM; Δ_4 denotes the fourth difference (e.g. $\Delta_4 c_t = c_t - c_{t-4}$), c_t denotes consumption, y_t denotes disposable income and ε_t is white noise. As DHSY point out in the first paragraph of their paper, the specification of their model was a product of a development through extensive and heterogeneous research and publication within the field. This "plethora of substantially different quarterly regression equations" (DHSY, p.661) was addressed and the models improved and augmented via rigorous econometric testing.

In this context it is essential emphasize that the theory of time series econometrics had yet to develop an understanding of nonstationarity and cointegration, today's cornerstones of error correction models. The use of ECM-type models thus preceded the theoretical understanding by a decade as the theoretical discoveries were not made until the late 1980's. Here the paper of Engle and Granger (1987) in which their "2-step model" was presented, is an obvious milestone. The theory (which has since been developed further) means that we can test for the statistical presence of cointegration, i.e. two or more series that share the same long-run trend. In turn, it means that we can validate the use of an ECM-type model and that we do not solely have to rely on economic theory or econometric trial and error. Further improvement came with Johansen (1988) and the introduction of a test for cointegration based on maximum likelihood estimation of a VAR model. The result was that many of the problems concerning the residual based tests were overcome; for instance, the possibility of testing for multiple cointegrating vectors was introduced. The papers that came after Engle and Granger (1987) and Johansen (1988) of course drew on their results. Regarding the specification of the models, the progress that was

[2] The terminology of ECM was first introduced into economics by Phelps (1957) and a similar lag structure is found, for instance, in the aforementioned work by Stone (1964).

made is described by the discussion on the dependent and independent variables in the following sections. However, despite substantial progress in econometric theory, there is a striking resemblance between the DHSY model, eq.(1), and many of today's models.

One of the latest additions to empirical research on the consumption function is Lettau and Ludvigson (2004), which also serves as the methodological foundation of this paper. Following Lettau and Ludvigson (2004), who in turn build upon the work of Campbell and Mankiw (1989), consider a representative agent economy in which all wealth is tradable where the accumulation equation for aggregate wealth is

$$W_{t+1} = (1 + R_{W,t+1})(W_t - C_t), \tag{2}$$

where W_t is beginning of period aggregate wealth (defined as the sum of human capital, H_t, and nonhuman, or asset wealth, A_t) in period t, $R_{W,t+1}$ is the net return on aggregate wealth and C_t is consumption in period t.[3] Through taking a first-order Taylor expansion of eq.(2), solving the resulting first-difference equation for log wealth forward, imposing a transversality condition and taking expectations, Campbell and Mankiw (1989) derive an expression for the log consumption-aggregate wealth ratio:

$$c_t - w_t = E_t \sum_{i=1}^{\infty} \rho_w^i (r_{w,t+i} - \Delta c_{t+i}), \tag{3}$$

where $r \equiv \log(1 + R)$ and $\rho_w \equiv 1 - \exp(\overline{c - w})$.[4] Regrettably for empirical work, this expression contains a non-observable variable as w_t includes human capital. Lettau and Ludvigson (2001) solve this problem by transforming the current cointegrating relationship into a trivariate, including c_t, a_t and labor income y_t.

[3] None of the derivations below, and especially then the resulting eq.(4), are dependent on the implicit assumption of human capital being tradeable.

[4] Following Lettau and Ludvigson (2004), linearization constants of no importance are omitted in the derivation of the model.

Assuming that H_t in logs (h_t) is a linear function of y_t with a random component given by $v_t = E_t \sum_{j=1}^{\infty} \rho_h^j (\Delta y_{t+j} - r_{h,t+j})$, and that log aggregate wealth is a linear function of its elements a_t and h_t with respective steady-state weights of $(1-v)$ and v, yields an approximation of eq.(3) using only the observable variables on the left hand side:

$$c_t - \beta_a a_t - \beta_y y_t \approx E_t \sum_{i=1}^{\infty} \rho_w^i \left((1-\nu) r_{a,t+i} - \Delta c_{t+i} + \nu \Delta y_{t+i+1}\right). \qquad (4)$$

Among several points made regarding eq.(4) above, Lettau and Ludvigson (2004) point out that under the maintained hypothesis that $r_{w,t}, \Delta c_t$ and Δy_t are stationary, eq.(4) implies that c_t, a_t and y_t are cointegrated. The parameters β_a and β_y should in principle equal the shares $(1-\nu)$ and ν respectively, but may in practice sum to a number other than one depending on what measure of consumption is used. The implication of cointegration means that it is possible to construct an econometric model building upon the theoretical derivation above; the presentation of such a model follows in Section 4.1.1.

Given the applied nature of the addressed question in this paper, the reason for choosing an ECM type of model and not a Euler type of model, is threefold. First, following the paper of DHSY, the ECM has been popular not the least for its good empirical fit for research undertaken using both definitions of consumption. Second, the specification does not impose any consumer preferences, thereby being "applicable to a wide variety of theoretical structures." (Lettau and Ludvigson (2004), p.280). Third, it is widely used among practitioners, often directly responsible for economic policy.[5] Next follows a discussion on variable definitions in previous research.

[5] To name but a few, see Johnsson and Kaplan (1999) for Sweden, Mehra (2001) and the FRB/US model for the US, Macklem (1994) for Canada, Downing and Goh (2002) for New Zealand and Fernandez-Corugedo et al (2002) and Byrne and Davis (2003) for the UK.

2.2 Definitions of variables in previous research

2.2.1 The dependent variable

When discussing what definition of the dependent variable is used in different papers, it is important to stress that the papers do not differ significantly in other important aspects, such as would influence the choice of definition. Although this is not a survey paper, I believe it fair to say that a vast majority of the papers on aggregate consumption relate to or stem from the two influential publications of 1978; Hall (1978) and Davidson et al. (1978). Whether the model chosen is a Euler-type model (Hall (1978)) or a solved out error correction model (Davidson et al. (1978)), there is no obvious reason for the definition of the dependent variable to differ between the two, meaning that it is possible to treat the two approaches as one in this respect.[6]

Multiple definitions better serve the purpose in more than one case and several prominent papers on aggregate consumption over the last decade(s) have chosen to use both the c^{tot} and the c^{nd} definition.[7] Nonetheless, several other papers have opted for only the former definition.[8] Although not decisive for the aim of this paper, we can note that the motivations behind the choice of definition vary substantially. Most common is not to comment on the choice of definition or simply to state that the differences in estimation are negligible.[9] Another solution is to acknowledge the problem of finding an empirical reciprocal of theory's pure consumption (as e.g. in Hall (1978) and Lettau and Ludvigson (2004)), therefore choosing a definition, c^{nd}, that is a subset of the theoretical definition. Yet another approach is to discuss the practical matters of the calculation of measures in the National Accounts, as in Carroll et al (1994), thus choosing several measures, including c^{nd} and c^{tot}. An approach that is

[6] However, this is not necessary as the papers adopting an ECM approach are sufficient in terms of providing examples of varying definitions. The inclusion of papers using Euler type models, is done to show that the varying definitions not is an issue for papers using ECM specifications, but rather for the field as a whole.

[7] Examples are Campbell (1987), Carroll et al (1994), and Lettau and Ludvigson (2004).

[8] See for instance Berg and Bergström (1995), Case et al. (2001) and Byrne and Davis (2003).

[9] Davidson et al. (1978) is an example of the former and Ludvigson and Steindel (1999) of the latter.

somewhere in-between that of Carroll et al (1994) and this paper is Slesnick (1998); he is critical of the measures as found in the personal consumption expenditures in the National Accounts due to their (in his view) lack of quality and definitional inconsistency with theory. He therefore also chooses multiple measures.

The problem of seemingly interchangeable use of c^{tot} and c^{nd} as definitions of consumption, is that it carries the implicit assumption of c^d behaving exactly as c^{nd}. The case that c^{nd} corresponds to pure consumption can perhaps be made against the backdrop of the former being a subset of the latter. As for c^d, the view is usually somewhat different given that it is usually viewed as an investment good and studied accordingly.[10] In the words of Muellbauer and Lattimore (1995), the "conventional treatment of durable goods is to assume that they are proportional to the stock S: $S_t = (1-d)S_{t-1} + cd_t$, where d is the rate at which the stock wears out or depreciates in real terms and cd_t is the flow of purchases." However, this theoretical difference is usually not commented on. One exception is Johnsson and Kaplan (1999), who argue that "if purchases of durables are spread out evenly over time and in the population, there is reason to believe" (p.8) that the difference between consumption expenditure as measured by the National Accounts and the flow from the stock of durables may in fact not be large at an aggregated level. An argument against this line of reasoning is the apparent dissimilarities from a statistical, stylized facts point of view. As Attanasio (1999) convincingly points out, the c^{nd} and c^d series exhibit quite different properties. Over the last four decades, the average growth rate of consumption expenditure on durable goods has been at least twice that of nondurable goods in the UK and the US. Turning to volatility, the difference is even larger. In the US the standard deviation of the consumption expenditure on durable goods is more than three times that of consumption expenditure on nondurable goods, while for the UK it is more than five times as large. Finally, advocating interchangeable definitions, it could be pointed out that consumption

[10] A recent paper in this line of research with several references to earlier work, is Leahy and Zeira (2000).

expenditure on nondurable goods and services usually constitutes 85-90% of total consumption expenditure, rendering possible differences between c^{nd} and c^{d} most likely insignificant. Whether or not this is the case is an empirical matter, one that this paper tries to settle. Before turning to the empirical part, we take a brief look at the independent variables.

2.2.2 The independent variables

The methodology, using an error correction model, is the same for both countries in this study. However, the choice of using the same specification as two recent papers on Sweden (Johnsson and Kaplan (1999)) and the US (Lettau and Ludvigson (2004)) respectively, means that the independent variables in the two models are not identical. Thus, a brief look at the independent variables in previous research seems warranted.

Besides the theoretical advances in time series econometrics and their effect on empirical modeling, different model specifications have been tried on the basis of economic theory and policy. Although the scarcity of data, e.g. on personal assets, has been gradually overcome, other related problems have lingered. A vast majority of empirical works on consumption assume some sort of life-cycle perspective, meaning that not only current income and wealth are needed, but also future income. The obviously weak correspondence between theoretical demands and empirical supply has, less surprisingly, proven rather resilient and therefore proxy variables have been used.

The theoretically appropriate variables just mentioned, have usually been replaced by the more accessible present income and present wealth.[11] A representative motivation for this choice of approximation is the assumption that future income is a function of present income, current wealth and the expected real interest rate; the latter being assumed to be constant. The definition of income is usually disposable income or disposable labor income, where the argument in favor of the latter is its closer correspondence to the theoretical

[11] Often the real interest rate is included, as well as a measure of uncertainty, e.g. unemployment, but a common problem is that the associated parameters come out insignificant.

definition. Another issue concerns wealth, namely the level of aggregation. It is theoretically appealing, for a number of reasons, to group assets according to their liquidity.[12] Since a high degree of disaggregation can render problems with significance, usually only the distinction between liquid and illiquid assets is made; often labeled "financial wealth" and "non-financial wealth" or "housing wealth" respectively.[13] Furthermore, debt has to be taken into account. Here, no clear consensus is established on which of the two wealth series it is to be deducted from. An argument in favor of grouping it with financial wealth is that it is liquid, while the opposite choice is supported by the fact that most debt is associated with the acquisition of a new home and thus should be grouped with non-financial wealth.

Finally, it is worth mentioning that the disparity concerning the independent variables in previous research almost never is explained by the definition of the dependent variable. Given the highlighted discrepancies between c^{nd} and c^{tot}, a natural idea would be to augment one or both models so as to better reflect the differences.[14] However, given that previous research has not argued along these lines, i.e. coupling the discussions on the dependent and independent variables to one another, doing so here would not adhere to the background and aim of the paper.

3 Data

The US data used in this study, equivalent in definition to the data used in Lettau and Ludvigson (2004), has been obtained from the US Department of

[12] First, increases or decreases in different forms of wealth may be viewed as temporary and/or uncertain. Second, households might have incentives, such as taxes, or motives, such as accumulation as an end in itself, that differs between types of assets. Third, people separate different kinds of wealth into different "mental accounts", meaning that their willingness to consume out of these accounts vary (Shefrin and Thaler (1988)). Fourth, the possibility to accurately measure wealth vary across asset types; the more liquid the market is, the more accurate the valuation is.

[13] Examples are, for Sweden, Berg (1990), Berg and Bergström (1995) and Johnsson and Kaplan (1999) and, for the US, Mehra (2001) and Case et al. (2001).

[14] Using the relative price of durables/nondurables for augmentation for the US does not improve the models as the associated parameter comes out insignificant in estimations.

Commerce (consumption and income series) and the Federal Reserve Board (wealth series). The data set is comprised of quarterly seasonally adjusted per capita observations of total consumer expenditure (c_t^{tot}), consumer expenditure on nondurable goods and services (c_t^{nd}), after-tax labor income (y_t^{li}), and net total wealth (nw_t) from 1959 to 2002. All series are in logs and have been deflated by the implicit total consumption price deflator (1996 base year).

The Swedish data used in this study has been obtained from the National Institute of Economic Research (Konjunkturinstitutet) and Statistics Sweden (Statistiska Centralbyrån). The data set is comprised of quarterly observations of total consumer expenditure (c_t^{tot}) and consumer expenditure on nondurable goods and services (c_t^{nd}), households' disposable income (y_t), households' financial (w_t^f) and net non-financial wealth (w_t^{nnf}) from 1970 to 2001. All series are in logs and have been deflated by the implicit total consumption price deflator (1991 base year).

3.1 Handling seasonality

There are, at least, two reasons why a brief discussion on seasonality is in place. First, several countries only publish data that is seasonally adjusted which is why for comparative reasons it is advantageous to seasonally adjust the data series that are not adjusted already. Of course, this approach has drawbacks since we are tampering with original data series and can never be sure that it is only the seasonal fluctuations that we are getting rid of.

Second, the discussion on seasonality in this context also yields better insight into how the empirical modeling of aggregate consumption has developed during the last three to four decades. Early specifications of ECM type consumption functions, exemplified here by the DHSY model (eq.(1)), contains variables that are year-on-year changes on a quarterly frequency. The fourth differencing of these variables means that the problem of seasonality is avoided. As research on integration and cointegration has progressed, it has become clear that this practice implies assumptions of not only a unit root at the annual frequency, but also of unit roots at other frequencies. Put differently, using $\Delta_4 c_t = c_t - c_{t-4}$ means that more differencing is used than what is needed. If this is the case, there might be a problem as overdifferencing introduces spurious MA terms into the series. In the case of fourth differencing as mentioned above, there are thus unrealized implied assumptions of unit roots at the biannual and quarterly frequencies. All of these problems are avoided if we choose to use seasonally adjusted data, which has been the path most research has followed since DHSY, as mentioned in section 2, thereby making it the choice of this paper as well.[15] Next, we look at the empirical analysis.

4 Empirical analysis

The ECM model used in this section, as previously mentioned, draws on the theoretical model emanating in eq.(4), p.11, which suggests that the variables entering the consumption functions should be cointegrated. We next test for this.[16]

4.1 Testing the consumption functions for cointegration

In our consumption function application, the ECM is the single equation version of the vector error correction model (VECM), which in turn, making use of Granger's representation theorem, is a reparameterized version of a vector autoregressive model (VAR). In order for us to use a single equation ECM, several requirements must be fulfilled.

4.1.1 Method

The VECM can in our case be described by

$$\Delta \mathbf{x}_t = \mathbf{\Phi D}_t + \boldsymbol{\alpha\beta}' \mathbf{x}_{t-1} + \sum_{i=2}^{k} \boldsymbol{\gamma}_i \Delta \mathbf{x}_{t-(i-1)} + \boldsymbol{\varepsilon}_t, \tag{5}$$

[15] The method emplyed is the X-12 ARIMA.
[16] All estimations in this section have been done using PCGive 10.0.

where $\mathbf{x}_t$ is the vector containing the consumption, income and wealth variables, $\mathbf{D}_t$ is a vector of deterministic components and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are parameter matrices. Given our aim of being consistent with earlier papers, there are slight differences between the specification of the models for the two countries. For Sweden, following Johnsson and Kaplan (1999), we have $\mathbf{x}_t \equiv (c_t^i, y_t, w_t^f, w_t^{nnf})$, where c_t^i is consumption, y_t is disposable income and w_t^f and w_t^{nnf} are financial and net nonfinancial wealth respectively.[17] For the US, following Lettau and Ludvigson (2004), we have $\mathbf{x}_t \equiv (c_t^i, y_t^{li}, nw_t)$, where c_t^i is consumption, y_t^{li} is labor income and nw_t is net wealth.

The intuition behind the model is that variables in $\mathbf{x}_t$ share the same trend, i.e. they are cointegrated. Often the first differences are referred to as the short-run part of the model while the levels are referred to as the long-run part of the model. In the long-run part, the value of α determines at which speed the error, or disequilibrium, is corrected while β determines what the relationship between the variables looks like.

Various tests for cointegration exist but no test is uniformly better than the Johansen (1988) test. The reason for choosing the Johansen (1988) test over residual based tests such as Phillips and Ouliaris (1990) and Engle and Granger (1987), is twofold. First, the residual based tests have a disadvantage in that they are sensitive to model misspecification. Second, residual based tests can only test the hypothesis of H_0 : Cointegration versus H_1 : No cointegration. Thus, to be able to test for both the presence and number of such cointegrating relationships, we employ the Johansen (1988) test.

The procedure is as follows. Letting $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, we want to test for the number of independent rows in $\mathbf{\Pi}$, i.e. its rank, which in turn is equal to the number of stationary relations in $\mathbf{x}_t$ which in turn is equal to the number of cointegrating vectors. For the single equation specification to be possible, we have to

[17] The superscript i in c_t^i, denotes the measure of consumption; total consumer expenditure (c_t^{tot}) and consumer expenditure on nondurable goods and services (c_t^{nd}). Also, $\mathbf{D}_t$ contains a dummy, D9192, to take account for the large tax reform and the abondonment of the fixed exchange rate in the early 1990's. D9192 takes the value 0 in 1970-1990, 0.33 in 1991, 0.66 in 1992 and 1 thereafter.

have $r(\mathbf{\Pi}) = 1$. Next, we estimate the cointegrating parameters, $\boldsymbol{\beta}$. Following Lettau and Ludvigson (2004), we use a dynamic least squares procedure to get "superconsistent" estimates (Stock and Watson (1993)). Besides estimating $\boldsymbol{\beta}$, we have to make sure that within this cointegrating relationship there is only one error correction mechanism. This is really a test for weak exogeneity, and is carried out by testing the $\boldsymbol{\alpha}$ vector (which is now (4×1) since $r(\mathbf{\Pi}) = 1$) for significance. If only one α is significant, then and only then can we model the ECM as a single equation with one error correction mechanism. Furthermore, the variable that is associated with α is the endogenous variable, i.e. the one through which the error correction takes place.

4.1.2 Results

The first step is to find a specification of the unrestricted VAR, i.e. the central model design. Theory and practice focus on the specification of $\mathbf{D}_t$ and the lag length, i.e. the value of k, which in turn is evaluated through the behavior of the residuals and an information criterion such as Akaike's or Schwarz's. As for the deterministic component, the choice is usually between model 3 and model 4; the former includes a constant in both the short- and long-run part of the model and the latter adds a time trend in the long-run part. Using model 2 would mean that no time trend is present in the data while model 5 would mean that there is a quadratic trend, neither of which seems plausible in the current case.[18]

In the light of the differences in specifications of the models, given the different measures of consumption, it is not surprising that the unrestricted VARs differ marginally. Since we generally want to have few lags in the system, we sometimes have to include dummies since we do not want the residuals to deviate "too much from Gaussian white noise" (Johansen (1995) p.20).[19] As for the choice of models 3 or 4, the picture is somewhat mixed, as can be seen

[18] Recent research supports the choice of either model 3 or 4; see for instance Johansson (2003).

[19] As will be seen below, this is the case for Sweden on one occasion, where we can reduce the number of lags significantly by introducing a dummy.

in Table 6 below.[20]

Table 6: Results from the trace test for cointegration

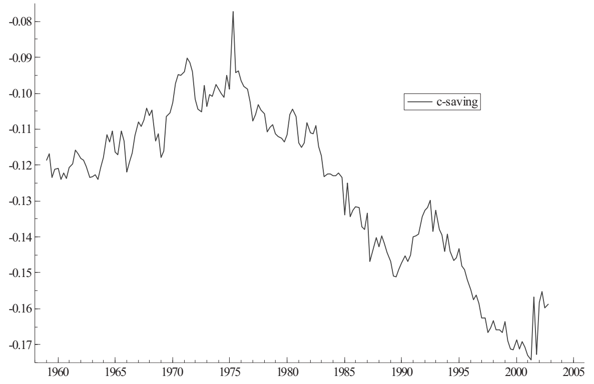
	Sweden		US		
	c^{tot}	c^{nd}	c^{tot}	c^{nd}	
k	3	5	4	2	
Dummy	D711	-	-	-	(6)
Model	3	3	4	4	
$H_0 : r(\Pi) = 0$	0.000**	0.002**	0.034*	0.048*	
$H_0 : r(\Pi) = 1$	0.198	0.592	0.193	0.504	

* denotes significance at the 5% level

** denotes significance at the 1% level

That is, in order to find statistical evidence of cointegration for the US in the c^{tot} and c^{nd} models, we have to allow for a time trend during the sample period in the long-run part of the model; the economic interpretation being a time trend in saving. Given the rather dramatic decline in the saving rate, defined as $y_t^{li} - c_t^{tot}$, this is perhaps not surprising; see Figure 1 below.

Figure 1: US saving rate, calculated as $y_t^{li} - c_t^{tot}$.



We can conclude that we have found cointegration in all four cases, and

[20] A lag length of 4 or 5 might indicate there are problems with model specification. Although this is aknowledged, it would not be consistent with the aim of the paper to augment the existing models, as discussed earlier in section 2.2.2.

For all models, a longer lag length is not supported by the information criterias and a shorter lag length is not supported by the tests for autocorrelation, ARCH and normality.

also that there is only one cointegrating vector in each case. Next, in order to determine how many error correction mechanisms are present, we follow Lettau and Ludvigson (2004) and start by getting "superconsistent" estimates of $\boldsymbol{\beta}$, through using dynamic least squares (DLS). The results from estimation of

$$c_t^i = \alpha + \boldsymbol{\beta}'\mathbf{z}_t + \sum_{j=t-k}^{t+k} \boldsymbol{\varphi}\boldsymbol{\Delta}\mathbf{z}_j + \varepsilon_t, \tag{7}$$

where $\mathbf{z}$ is a vector containing the independent variables, are found in Table 8 below.[21]

Table 8: Results from OLS estimation of eq.(7)

	Sweden			US	
	c^{tot}	c^{nd}		c^{tot}	c^{nd}
β_1	0.507 [0.079]	0.535 [0.051]	β_1	0.576 [0.022]	0.858 [0.045]
β_2	0.189 [0.026]	0.136 [0.017]	β_2	0.139 [0.029]	0.132 [0.057]
β_3	−0.009 [0.012]	−0.007 [0.007]			

(8)

Standard errors in brackets

The one conspicuous result is that for Sweden, β_3,the parameter associated with net nonfinancial wealth, is negative in both cases. An explanation for this could be that the restraining effect from taking on new loans and the positive effect coming from the acquisition of a new home vary and thus the total effect would be inconclusive *a priori*. In a survey of several macro studies, Muellbauer and Lattimore (1995) make the observation that "[e]vidence is accumulating that house prices have these dual effects implied by economic theory: a positive wealth effect,..., and a negative relative price effect" (p.271).

Next, we use these values in a constrained VAR estimation and test for significance of $\boldsymbol{\alpha}$, which is equivalent to testing for weak exogeneity. As expected, only one α is significant in each case, see Table 9 below.

[21] The number of leads and lags, i.e. k in eq.(7), is 3 and 8 for Sweden and the US respectively. However, the lead/lag length does not matter for the conclusions in this section, nor for the conclusions later in the paper. Also, the c^{tot} and c^{nd} equations for the US, in line with choice of model earlier, contain a time trend.

Table 9: Test of weak exogeneity in the cointegrated model

	Sweden		US	
	c^{tot}	c^{nd}	c^{tot}	c^{nd}
α	-0.197	-0.179	0.368	-0.058
Dep. var.	c^{tot}	c^{nd}	nw	c^{nd}

(9)

However, in the case of the US we find somewhat surprisingly, that it is not the same α in both models. In the c^{tot}-model the significant α is associated with nw_t, while in the other case it is associated with consumption, the result in both Swedish models as well. Also to be noted is that in the c^{nd}-model for the US, the value of α is surprisingly small. Even more troublesome in the case of the US c^{tot} equation, is that the estimated specification is actually error amplifying, not error correcting. As for the issue of which variable should be endogenous in the cointegrating relation, earlier papers are not unanimous. Lettau and Ludvigson (2004) argue that the error correction should take place through wealth while earlier papers such as Davis and Palumbo (2001) argue that it should take place through consumption. For the purpose of this paper, it suffices to observe that the results once again speak against the hypothesis that the measures of consumption are interchangeable.

To conclude, our results show that it is possible, although not without difficulty, to specify the model as a single equation error correction model for both measures of consumption for both economies. Next, the models are estimated without restrictions.

4.2 Estimating the unrestricted models

Before testing the hypothesis, it is of interest to estimate the equations without restrictions. The results from testing for cointegration in the previous section means that the model we want to estimate for both Sweden and the US, now looks like:

$$\boldsymbol{\gamma}\boldsymbol{\Delta}\mathbf{x}_t' = \Phi D_t + \alpha\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1} + \varepsilon_t, \tag{10}$$

where $\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1}$ is the previously estimated cointegrating relation. The results from OLS estimation of eq.(10) are found below in Table 11.

Table 11: OLS estimation of eq.(10)

	Sweden			US	
	c_t^{tot}	c_t^{nd}		c_t^{tot}	c_t^{nd}
γ_1	0.079	0.083	γ_1	0.221	0.208
	0.042	*0.030*		*0.041*	*0.063*
γ_2	0.095	0.033	γ_2	0.070	0.034
	0.054	*0.037*		*0.018*	*0.028*
γ_3	0.026	0.002			
	0.020	*0.014*			
α	-0.190	-0.169	α	-0.179	-0.087
	0.043	*0.041*		*0.039*	*0.027*

(11)

Standard errors in italics

As for the short-run part of the model, the γ's, we find that parameter values in the Swedish models are not always significant at conventional levels. In the US case, the picture is somewhat better with γ_2^{nd} being an exception. Next, the hypothesis is tested.

4.3 Are c^{nd} and c^{tot} interchangeable?

In line with Lettau and Ludvigson (2004), the estimation of eq.(5) consists of two steps as described in section 4.1.1. First, the long-run part of the model is tested for equality, after which the short-run part is tested. Before that, however, a brief discussion on the test used, the Rao (1973) test, is in order.

4.3.1 Why is Rao's test preferred?

Among the more common tests Wald's is perhaps most widely used, due in part to it being a standard feature of various econometric software packages. This χ^2 distributed test has a drawback in this context, a drawback that it shares with other such tests, for instance the Lagrange multiplier (LM) and likelihood ratio

(LR) tests: the discrepancy between large and small sample properties grows with the number of equations in the system. This does not mean that F-tests such as Rao's test do not exhibit this property, but they do so to a lesser degree. More importantly, Edgerton and Shukur (1999) show that when estimating a system of equations, Rao's F-test outperforms the others. Also reassuring is the fact that they find that the test works very well in the current setting, that is in a 2-equation system. Furthermore, they conclude "that the traditional Wald test [is] shown to perform *extremely* badly in all situations!" (p.376). Note that the better performance not generally is due to the test being exact, which it is only if the number of equations and number equations are equal.

What then does the Rao (1973) test look like? In order to improve the small sample properties it has been augmented when compared to the simpler tests, leaving the test statistic looking like:

$$RAO = \frac{hs - q}{r}\left[\left(\frac{|\hat{\Sigma}_R|}{|\hat{\Sigma}_U|}\right)^{\frac{1}{s}} - 1\right] \sim F(r, hs - q)$$

where

$$s = \sqrt{\frac{r^2 - 4}{m^2 + (r/m)^2 - 5}} \text{ and } h = T - k - \frac{1}{2}[m - (r/m) + 1] \text{ and } q = \frac{r}{2} - 1.$$

r and m is the number of restrictions and number of equations respectively, $\hat{\Sigma}_R$ and $\hat{\Sigma}_U$ is the residual sum of square for the restricted and unrestricted regressions respectively, and T is the number of observations. Next, follows the test for equality in the long-run part of the model.

4.3.2 Are the cointegrating relations the same?

The cointegrating relation among the variables, expressed by $\boldsymbol{\beta}'\mathbf{x}_{t-1}$ in eq.(5), is estimated by DLS, as described earlier.[22] Through using one equation with c_t^{tot} and another with c_t^{nd} as the dependent variable respectively, we can test the parameter vector $\boldsymbol{\beta}$ for equality, thereby testing our hypothesis. Results

[22] Alll estimations in this section are done using RATS 5.0.

from DLS estimation and hypothesis testing for the US and Sweden are found in Table 12 below.

Table 12: p-values from testing $\boldsymbol{\beta}^i$'s in eq.(7) for equality

H_0 :	Sweden	H_0 :	US
$\boldsymbol{\beta}^{nd} = \boldsymbol{\beta}^{tot}$	0.000	$\boldsymbol{\beta}^{nd} = \boldsymbol{\beta}^{tot}$	0.000
$\beta_1^{nd} = \beta_1^{tot}$	0.478	$\beta_1^{nd} = \beta_1^{tot}$	0.000
$\beta_2^{nd} = \beta_2^{tot}$	0.000	$\beta_2^{nd} = \beta_2^{tot}$	0.908
$\beta_3^{nd} = \beta_3^{tot}$	0.721		

(12)

Here, we see that vector equality is clearly rejected for both countries. More specifically, when testing for equality of individual parameters, we see that for Sweden, it is the financial wealth parameter, β_2, that is significantly different, while for the US it is the income parameter, β_1.

4.3.3 Are the short-run parameters the same?

Given the equation-specific error correction mechanism as estimated in the previous section, $\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1}$, the purpose here is to test the vector containing the short-run parameters and the error correction parameter, $\boldsymbol{\gamma}$ and α respectively in eq.(5), for equality across equations. The results from OLS estimation of the c_t^{tot} and the c_t^{nd} equations and the described restrictions are found in Table 13 below.

Table 13: p-values from testing γ and α in eq.(10) for equality

H_0 :	Sweden	H_0 :	US
all equal	0.048	all equal	0.475
$\alpha^{nd} = \alpha^{tot}$	0.089		
$\gamma_1^{nd} = \gamma_1^{tot}$	0.918		
$\gamma_2^{nd} = \gamma_2^{tot}$	0.060		
$\gamma_3^{nd} = \gamma_3^{tot}$	0.033		

(13)

Here, we see that it is not possible to reject equality for the US. For Sweden, vector equality is rejected, a rejection that is due to rejection or near rejection of all parameters but one; γ_1, the income parameter.

4.4 Results

Strong theoretical as well as empirical arguments can be put forward speaking against c^{tot} and c^{nd} being interchangeable. On the other hand, the widespread practice of interchangeability in previous research, coupled with the observation that c^{nd} constitutes about 85-90% of c^{tot}, leave us about where we started. Ultimately the discussion is an empirical matter, an insight that serves as the basis of this paper. Thus, the results in the above section merely suggest that the assumptions underlying interchangeability, as exemplified by the previous citation from Johnsson and Kaplan (1999) page 13, are incorrect.

5 Conclusions and comments

The results in the empirical analysis indicate that total consumption expenditure and consumption expenditure on nondurable goods and services in an ECM type consumption function cannot be used interchangeably. For the US, the problems concerns model specification, i.e. whether or not a trend should be included in the cointegrating relationship, the cointegrating relation itself, as well as the causality in this long-run relation. For Sweden, there is a significant difference in the cointegrating relation for the two models as well as in the short-run adjustment parameters. Given these findings, the importance of the choice and definition of the dependent variable in an aggregate empirical consumption function is underscored. Put differently, the results suggest that it is not possible to reconcile theoretical requirements with those of policy, given that nondurable consumption, being a subset of the theoretical definition of pure consumption, is not interchangeable with the variable of policy interest, total consumption.

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Essay 2

A Simple Multivariate Test for Asymmetry with Applications to Aggregate Consumption*

Mårten Bjellerup† Thomas Holgersson‡

May 9, 2005

Abstract

In this paper we formulate a univariate test for symmetry based on the third central moment and extend it to a multivariate test; the test does not require modeling and is robust against serial correlation, autoregressive conditional heteroscedasticity and non-normality. It is found in the empirical application of the test, perhaps in contrast to conventional wisdom, that consumption expenditure on durable goods show little deviation from symmetry while consumption expenditure on nondurable goods is asymmetric for the US and UK, with peaks being higher than troughs are deep. Also, the empirical importance of the choice between the univariate and the multivariate test for possibly correlated series is underscored; the results from the two approaches clearly differ.

*We wish to thank seminar participants at Växjö University for their constructive criticism, Bengt Assarsson for the feedback at Mårten Bjellerup's final seminar and Ghazi Shukur for introducing the authors to one another.

†School of Management and Economics, Växjö University, SE-351 95, Växjö, Sweden; *e-mail*: marten.bjellerup@ehv.vxu.se.

‡Department of Economics and Statistics, Jönköping International Business School, Box 1026, SE-551 11, Jönköping, Sweden; *e-mail*: thomas.holgersson@ihh.hj.se

1 Introduction

There has long been recognition that business cycles can be asymmetric, see e.g. Neftci (1984) and Potter (1995). Asymmetry of a macroeconomic times series is not a problem, though, unless it is modeled or treated as a symmetric series. However, since many macroeconomic models are linear, i.e. symmetric, we run the risk of trying to square the circle if we use them with an asymmetric dependent variable.

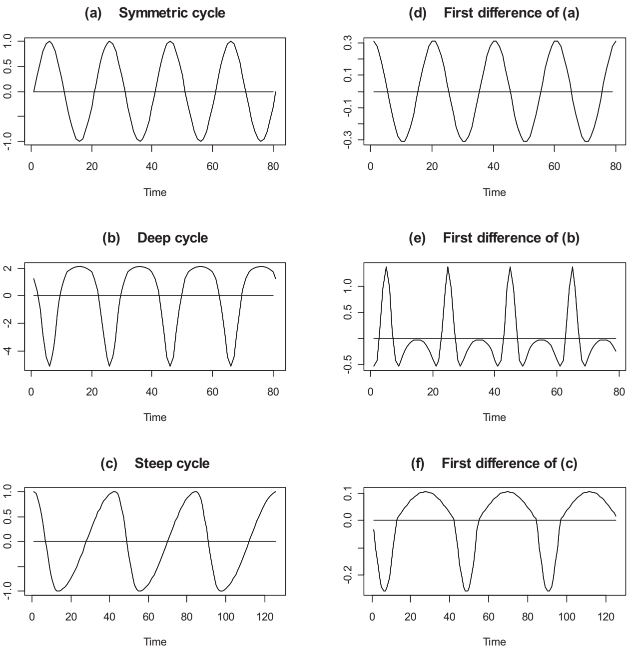
In this paper we propose a simple multivariate test for asymmetry based on the third central moment. The test allows us to test for two types of asymmetry using the terminology of Sichel (1993); *steepness* and *deepness*. Deepness considers distance of peaks and troughs above and below trend, while steepness considers the speed at which the peaks and troughs are approached. Positive deepness is present when peaks are taller than troughs are deep and negative deepness is present when the converse is true. Positive steepness is when the peaks are approached faster than the troughs and negative steepness is present when the converse is true. In order to obtain an intuitive idea of the meaning of these patterns, we present one symmetric cycle, one cycle with negative deepness and one cycle with negative steepness; see Figures 1(a)-1(c) below.[1]

In applications where we test the dependent variable in a symmetric model for symmetry, our test can be viewed as a model specification test. Although in an economic setting in this paper, the proposed test is general in the sense that it can be applied to any stationary series. Also, the test does not require modeling and is robust against serial correlation, autoregressive conditional heteroscedasticity and non-normality. There are numerous proposed tests in the literature (using STAR-models for instance), that to a greater extent determine the finer characteristics of the asymmetry while modeling it, treatments that is beyond the scope of this paper.[2]

[1] These stylized cycles are only intended to give an intuitive understanding of the two concepts; deepness and steepness. In reality, the series under investigation is usually much less well-behaved. For a more thorough presentation of the data series used in this paper, see the appendix p.51.

[2] Examples of such models are found in Bradley and Jansen (2000), for instance, where they

Figure 1: Three different cycles in levels and first differences.



Turning to empirical applications, one objection to this kind of diagnostic testing could be that such testing is superfluous since the question we are asking should already be answered by economic theory. However, it is not obvious to what degree a supposed asymmetry should be present. Moreover, the proposed diagnostic test could then be viewed as a way of testing theory regarding the behavior of the dependent variable. Speaking in favor of our approach is also the fact that not all economic theory is unanimous. Regarding consumption as well as business cycle theory, to mention but a few, competing views do not agree on whether growth and consumption is symmetric or not.

fit non-linear time series models to output data for 26 countries. The result is a great deal of heterogeneity concerning the different countries' business cycles as the characteristics of the series are not easily classified into common groups as there is little unanimity concerning the non-linearity. In our context, however, this kind of precision is not needed as we are interested in the overall question of whether or not there are asymmetries. While it is of interest in other circumstances to model the possible nonlinearity with great precision, the verdict of a significant rejection (or not) of symmetry, is sufficient for our purposes.

The theoretical aim of the paper is twofold. First, we want to create a univariate test for asymmetry for which the statistical properties are clearly derived. Second, we want to develop the univariate test into a multivariate, thus being able to take into account the possible interdependence between two or more of the series to be tested.

The empirical aim is to test theories of consumption suggesting symmetric or asymmetric behavior, as well as testing the validity of common practices in empirical studies on aggregate consumption. Lastly, the empirical section contains a test of the difference between the univariate and the multivariate test. In the empirical application we use data from the United States, the United Kingdom, Canada and Sweden.

The outline of the paper is as follows. Section 2 starts with a brief discussion on asymmetry, followed by the derivation of the univariate test as well as the multivariate test. Section 3 first looks at the data and the method of detrending, followed by the empirical testing and a summary of the results. Section 4 comments and concludes. A more exhaustive graphical presentation of the data is found in the appendix.

2 Asymmetry

There are several ways in which a cycle may deviate from symmetry. Sichel (1993) suggests two types of asymmetry of prime importance, as presented in the previous section, which he refers to as deepness and steepness, respectively. Our main objective here is to detect deviations from symmetry of business cycles (i.e. to detect deviations from the pattern exemplified in Figure 1(a)). The series displayed in Figure 1, p.35, suggest that possible steepness (Figure 1(b)) or deepness (Figure 1(c)) may be analyzed with trigonometric models. However, this is not to be recommended as the cycles are stylized, seldom occurring in economic data. We will therefore aim to find techniques for evaluation of asymmetry that do not rely on a parametric model.

2.1 The univariate test for asymmetry

Consider a variable $Y_t \equiv (X_t - \mu_X)^3$ and define $\mu_X \equiv E[X_t]$, $\mu_Y \equiv E[Y_t]$ and $\gamma_j \equiv E\left[(Y_t - \mu_Y)(Y_{t-j} - \mu_Y)\right]$.[3] Then consider a set of constants $\left\{\psi_j\right\}_{j=0}^{\infty}$ such that $\sum_{j=0}^{\infty} \left|\psi_j\right| < \infty$ by assumption. By Wold's decomposition theorem (Hamilton (1994) p. 108), the variable Y_t can be written in the form

$$Y_t = \mu_Y + \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} \tag{1}$$

where $\psi_0 = 1$ and $\{\varepsilon_t\}$ is a sequence of i.i.d. random variables. The linear model in eq.(1) is frequently referred to as the $MA(\infty)$ representation of Y_t. Our interest lies in assessing whether μ_Y is zero or not. As μ_Y is unknown, we need an estimate of it along with a known asymptotic null distribution. Assuming that $E[\varepsilon_t^2] = \sigma_\varepsilon^2 < \infty$, it may be shown that

$$\hat{\theta} = \frac{\sqrt{T}(\bar{Y} - \mu_Y)}{\sqrt{v^2}} \xrightarrow{l} N(0,1) \tag{2}$$

where $v \equiv \sum_{j=-\infty}^{\infty} \gamma_j$ (see e.g. Hamilton (1994), p. 195). Eq.(2) does not lead to a feasible test regarding μ_Y since we assumed that v is known, which is usually not true. However, we may estimate v by

$$\hat{v}^2 = \hat{\gamma}_0 + 2\sum_{i=1}^{q} \left[1 - \frac{i}{q+1}\right] \hat{\gamma}_i. \tag{3}$$

Then, if q is chosen such that $\left(q/T^{1/3}\right) \to 0$, it follows that $\hat{v} \xrightarrow{p} v$ (Hamilton (1994) p. 283). Hence,

$$\frac{\sqrt{T}(\bar{Y} - \mu_Y)}{\sqrt{\hat{v}^2}} \xrightarrow{l} N(0,1). \tag{4}$$

Definition 1 *A test of* $H_0 : E(X_t - \mu_X)^3 = 0$ *against* $H_A : E(X_t - \mu_X)^3 \neq 0$ *is defined by the rejection area* $|\hat{\theta}| > q_\alpha$ *where* q_α *is the* $(1-\alpha) \times 100^{th}$ *percentile of*

[3] Note that in our case, we know from our method of detrending that $\mu_X = 0$; see section 3.2, p.40.

the standard normal distribution. That is, H_0 *is rejected for values of* $|\hat{\theta}| > q_\alpha$.

2.2 The multivariate test for asymmetry

A multivariate version of the above may readily be constructed, the approach being analogous to the univariate case. Consider the P-dimensional random vector

$$\mathbf{Y}_t \equiv (\mathbf{X}_t - \boldsymbol{\mu}_{\mathbf{x}})^3$$

and let $\boldsymbol{\mu}_{\mathbf{X}_t} \equiv E[\mathbf{X}_t]$ and $\boldsymbol{\mu}_{\mathbf{Y}_t} \equiv E[\mathbf{Y}_t]$ and $\boldsymbol{\Gamma}_j \equiv E\left[(\mathbf{Y}_t - \boldsymbol{\mu}_{\mathbf{Y}})(\mathbf{Y}_{t-j} - \boldsymbol{\mu}_{\mathbf{Y}})'\right]$. Furthermore, consider a set of constants $\{\boldsymbol{\Psi}_j\}_{j=0}^{\infty}$ such that $\sum_{j=0}^{\infty} |\boldsymbol{\Psi}_j| < \infty$ by assumption and $\boldsymbol{\Psi}_0 = \mathbf{I}$. Then, by Wold's decomposition theorem (Lütkepohl (1991), p. 20), the variable $\mathbf{Y}_t$ can be written in the form

$$\mathbf{Y}_t = \boldsymbol{\mu}_{\mathbf{Y}} + \sum_{j=0}^{\infty} \boldsymbol{\Psi}_j \boldsymbol{\varepsilon}_{t-j}.$$

Again, our interest lies in assessing whether $\boldsymbol{\mu}_{\mathbf{Y}}$ is equal to zero or not. As $\boldsymbol{\mu}_{\mathbf{Y}}$ is unknown, we need an estimate of it along with an asymptotically known null distribution. Assuming that $E[\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'] = \boldsymbol{\Sigma}_\varepsilon < \infty$, it may be shown that

$$\sqrt{T}\mathbf{v}^{-1/2}(\bar{\mathbf{Y}} - \boldsymbol{\mu}_{\mathbf{Y}}) \xrightarrow{l} N(\mathbf{0}, \mathbf{I})$$

where $\mathbf{v} \equiv \boldsymbol{\Gamma}_0 + \sum_{j=1}^{\infty} \left(\boldsymbol{\Gamma}_j + \boldsymbol{\Gamma}_j'\right)$ (see, e.g., Brockwell and Davis (1991)). A consistent estimate of $\mathbf{v}$ may be obtained by

$$\hat{\mathbf{v}} = \hat{\boldsymbol{\Gamma}}_0 + \sum_{i=1}^{q} \left[1 - \frac{i}{q+1}\right] \left(\hat{\boldsymbol{\Gamma}} + \hat{\boldsymbol{\Gamma}}'\right) \tag{5}$$

and hence

$$\hat{\boldsymbol{\Theta}} = \sqrt{T}\hat{\mathbf{v}}^{-1/2} \left(\bar{\mathbf{Y}} - \boldsymbol{\mu}_{\mathbf{Y}}\right) \xrightarrow{l} N(\mathbf{0}, \mathbf{I}).$$

Definition 2 *A one-sided test of the hypothesis* $H_0 : E(\mathbf{X}_t - \boldsymbol{\mu}_{\mathbf{x}})^3 = \mathbf{0}$ *against* $H_A : E(\mathbf{X}_t - \boldsymbol{\mu}_{\mathbf{x}})^3 \neq \mathbf{0}$ *is defined by the rejection area* $\hat{\boldsymbol{\Theta}}'\hat{\boldsymbol{\Theta}} > Q_\alpha$ *where* Q_α *is the* $(1-\alpha) \times 100^{th}$ *percentile from the* $\chi^2_{(P)}$ *distribution. That is,* H_0 *is rejected* *for values of* $\hat{\mathbf{\Theta}}'\hat{\mathbf{\Theta}} > Q_\alpha$.

3 Empirical application

3.1 Data

The data set comprises quarterly aggregate consumption data for four countries; the US, the UK, Canada and Sweden.[4] For the US, the three series are total personal consumption expenditure, personal expenditure on non-durable goods and services and personal expenditure on durable goods for 1959:1-2002:4. All three series are seasonally adjusted, are in 1996 USD and have been obtained from the US Department of Commerce. For the UK, the three series are total domestic household consumption expenditure, households' domestic expenditure on non-durable goods and services and households' domestic expenditure on durable goods for 1963:1-2002:3. All three series are seasonally adjusted, are in 1995 GBP and have been obtained from the Office for National Statistics. For Canada, the three series are total personal consumption expenditure, personal expenditure on non-durable goods and services and personal expenditure on durable goods for 1961:1-2002:3. All three series are seasonally adjusted, are in 1997 CAD and have been obtained from Statistics Canada. For Sweden, the three series are total household final consumption expenditure, household final consumption expenditure on non-durable goods and services and household final consumption expenditure on durable goods for 1963:1-2001:4. All three series are in 1991 SEK and have been obtained from Statistics Sweden and the National Institute for Economic Research. The Swedish series have been seasonally adjusted using the X12 ARIMA program. Henceforth, we will use c_t^i for total consumption expenditure, cd_t^i for consumption expenditure on durable goods and cnd_t^i for consumption expenditure on non-durable goods and services where the superscript $i = US, UK, CAN$ or SWE for the US, the UK, Canada or Sweden respectively.

[4] For a graphical presentation of the data, see the appendix, p.51.

Since our empirical application of the test for asymmetry stems from an interest in whether the dependent variable in a linear model is linear, the starting point of our choice of data is the type of data used in such models. When estimating an aggregate consumption function, the dependent variable chosen is usually both seasonally adjusted and in natural logarithmic form. Thus, besides the seasonal adjustment, we have taken the natural logarithm of all series. From a statistical point of view, tampering with the original data as mentioned above might seem out of place. However, since the objective of, and motivation for, our application of the test is a macro consumption model and its empirical specification, such tampering is both motivated and necessary.

3.2 Detrending

A series that exhibits growth over time, as most macroeconomic time series, is asymmetric a priori which necessitates detrending of the variable of interest. This is obviously controversial as we are tampering with the original series. However, since we are only interested in one specific characteristic, we can adjust our requirements accordingly. As long as the choice of detrending process does not influence the proposed test for asymmetry, any possible concerns for undesired properties that might follow from the detrending process are a non-issue in the current context. Next, we consider a simple way of detrending a time series.

3.2.1 The Hodrick-Prescott filter

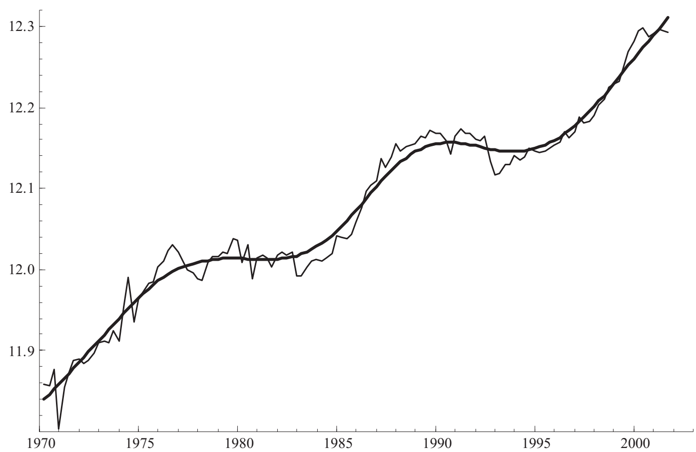
Formally, the Hodrick-Prescott (HP hereafter) filter proposes a trend τ_t for an original series Z_t which is the solution to the problem

$$\min \sum_{t=1}^{T} \{(Z_t - \tau_t)^2 + \lambda[(1-L)^2 \tau_t]^2\} \tag{6}$$

where L is the lag operator (Hodrick and Prescott (1997)). λ is known as the smoothing parameter and the choice of it will determine the characteristics of the filtered series. As an illustrative example, a choice of $\lambda = 0$ leads to $\tau_t = Z_t$

while a choice of $\lambda = \infty$ leads to a linear trend. Drawing upon earlier work in general, for instance Canova (1994 and 1998) and Ravn and Uhlig (2002), and the original in particular, we let $\lambda = 1600$ as we have quarterly data. A graphical illustration of the two variables, τ_t and Z_t, is found in Figure 2 below in an application to Swedish total private consumption for the period 1970:2-2001:4.

Figure 2: Swedish total private consumption 1970:2-2001:4, original series and estimated HP-trend with $\lambda = 1600$.



The HP filter has been criticized for amplifying fluctuations in the cycle under investigation, not a particularly desirable quality in several respects. This feature however, when combined with the test proposed here, is actually not negative but rather the contrary. If anything, it makes detection of asymmetry easier and is thus very much in line with the aim of the test.[5]

Thus, the variable X_t that is the starting point in Section 2.1 and 2.2 where

[5] The advantages and drawbacks of various methods of detrending have been an issue for quite some time, and Canova (1998), for instance, provides a good overview of the field. The HP filter is, for instance, found to perform best when it comes to identifying the turning point of business cycles (Canova (1994)). Also, none of the HP filter's drawbacks presented in Canova (1998) poses a problem in the current context. All calculations for the HP filter have been done using PCGive 10.0.

we derive the tests, is obtained in slightly different ways for the two tests.[6] For the deepness test, we acquire the desired variable by subtracting the HP estimated trend, τ_t, from the original series Z_t, where Z_t is in levels; e.g. $Z_t = c_t^i$. For the steepness test, Z_t is the original series in first differences; e.g. $Z_t = \Delta c_t^i$ where $\Delta c_t^i = c_t^i - c_{t-1}^i$. Consequently, $X_t = Z_t - \tau_t$ in both tests, but with Z_t in levels and first differences for the deepness and steepness tests, respectively.

3.3 Testing

Theories on consumption as well as empirical practices, imply symmetric or asymmetric properties of different measures of aggregate consumption, properties that will be tested for in this section.

3.3.1 Does cd_t exhibit asymmetry?

It is fair to say that there is an established consensus that durable goods are to be viewed as investment goods and modelled accordingly, see e.g. Leahy and Zeira (2000). Furthermore, several theories agree on that cd_t exhibit steepness: Leahy and Zeira (2000) propose a model in which consumption expenditure on durable goods exhibit negative steepness while Gregorio et al (1998) presents a model where the steepness is positive.[7] Using our univariate test, we can try these hypotheses using data for the four countries in question. Looking at Table 7 below, we find the results from the tests.

Table 7: p-values from tests of cd_t

	cd_t^{US}	cd_t^{UK}	cd_t^{CAN}	cd_t^{SWE}
Deepness	0.11 [−1.63]	0.13 [1.53]	0.04* [−2.11]	0.67 [0.43]
Steepness	0.25 [−1.16]	0.31 [−1.02]	0.08* [−1.79]	0.15 [−1.44]

(7)

* denotes significance at the 10% level
t-values are in brackets

[6] For a graphical presentation of all series, see the appendix p.51.

[7] Gale (1996) proposes a model for investment goods that share the characteristic of negative steepness with the model in Leahy and Zeira (2000).

For both tests, we see that deviation from symmetry only is significant for one country, Canada. Although the p-values for the deepness test indicate possible deviation from symmetry in three out of four cases, the corresponding t-values do not agree on the nature of the possible asymmetry. For the steepness test, the picture is somewhat clearer. Although the deviation from symmetry only is significant (at the 10% level) in one case, all four series have negative t-values. This implies that although not always statistically significant, the steepness is negative and not positive. Next, we test if the widespread practice of using cnd_t as the dependent variable in linear consumption functions is recommendable.

3.3.2 Is cnd_t asymmetric?

If consumption was shown to be asymmetric, it would be at odds with mainstream consumption theory, be it the Permanent Income Hypothesis or alternatives to it such as liquidity constraints, rule-of-thumb behavior or habit formation. A recent theoretical alternative is a loss aversion model in Bowman et al (1999), a model "...of consumption and saving based on Kahneman and Tversky's Prospect Theory that implies a fundamental asymmetry in consumption behavior inconsistent with other models of consumption." (Abstract). Applying the test for asymmetry to cnd_t is thus a test of mainstream theory, although rejection of symmetry not automatically equals support of the model proposed in Bowman et al (1999).[8] The results can be found in Table 8 below.

Table 8: p-values from univariate testing of cnd_t

	cnd_t^{US}	cnd_t^{UK}	cnd_t^{CAN}	cnd_t^{SWE}
Deepness	0.08*	0.09*	0.80	0.89
Steepness	0.69	0.73	0.39	0.32

(8)

* denotes significance at the 10% level

Here we see that for the two larger economies, the UK and the US, cnd_t displays deepness while we cannot reject the hypothesis of normality for Canada and Sweden. The well-known similarities between the two Anglo-Saxon business

[8] Bowman et al (1999) also use c_t, as mentioned in note 14, p.166. Their main measure of consumption, however, is cnd_t.

cycles, are further supported by our findings since the two t-values are both positive, meaning that we have positive deepness; i.e. peaks are higher than troughs are deep.[9]

3.3.3 Are all consumption series symmetric?

Using c_t as the dependent variable in a linear consumption function is a practice that has grown more common over time; see e.g. Bjellerup (2005). In our context, such use of c_t carries the implicit assumption of c_t, cd_t and cnd_t having the same symmetric properties. Using the univariate test we can test for the symmetry of c_t, and using the multivariate (or in this case bivariate) we can test for the symmetry of cd_t and cnd_t. The bivariate test is in contrast to the previous section where no interdependence was possible given the setting. Here, however, cd_t and cnd_t are of interest given the use of them as an aggregate; c_t.

In Table 9 below, we see the results from the univariate testing of c_t, that is the p-values from finite sample testing of eq.(4). The table reveals that, with a high probability, total consumption in the UK exhibits deepness.[10]

Table 9: p-values from univariate testing of c_t

	c_t^{US}	c_t^{UK}	c_t^{CAN}	c_t^{SWE}
Deepness	0.58	0.07*	0.17	0.87
Steepness	0.26	0.59	0.19	0.17

(9)

* denotes significance on the 10% level

Apart from the deepness test for the UK, all the other tests stay clear of conventional significance levels. Turning to the two components of c_t - cd_t and cnd_t - we can look at Table 10 where we find the p-values from the χ^2-distributed bivariate test.

[9] Although not as reliable as the test of course, this result can be viewed against the background of the number of positive and negative observations in the detrended series. In the two graphs on the left in Figure 9, p.54 in the appendix, we can see that the number of negative observations clearly outnumber the number of positive observations.

[10] Although this type of classification not is the main aim of the paper, we can conclude that that it is a positive deepness, by looking at the t-statistic which is positive; i.e. the peaks are higher than the troughs are deep for c_t^{UK}.

Table 10: p-values from bivariate testing of cd_t and cnd_t

	US	UK	CAN	SWE
Deepness	0.25	0.41	0.32	0.92
Steepness	0.44	0.31	0.13	0.20

(10)

Here we can see that we are not able to reject the null hypothesis of symmetry in any of the cases. Before commenting on the results, we also want to investigate whether it matters, in the current context, if the series are modeled as correlated or not.

3.3.4 The impact of correlation - the univariate vs. the multivariate test

From a theoretical point of view, given some correlation between the series, it is obvious that the univariate and the multivariate tests will yield different results. However, from an empirical point of view, these differences can of course vary in magnitude therefore a closer look at the two approaches is warranted. More specifically, we are interested in whether the verdict of rejection differs or not between approaches. Before proceeding, it is important to stress that this section is intended only to highlight the possible difference between the two approaches and in that way underscore the importance of having chosen the right approach. The choice itself, however, is of course made on a theoretical basis depending on the situation and the data at hand. Here, we merely demonstrate the difference between the two options given the current context.

The novelty of the multivariate approach is that $\mathbf{v}$ in eq.(5) captures the covariances of the series, something of course lacking in the univariate approach where the series are tested one at a time, independently of one another. In Table 11 below, the importance of the covariance is demonstrated in the current empirical application through a comparison between the two approaches.

Table 11: p-values from the bivariate ('Joint') and the univariate tests (cd_t and cnd_t)

		Deepness	Steepness
US	Joint	0.25	0.44
	cd_t	0.11	0.25
	cnd_t	0.08*	0.69
UK	Joint	0.41	0.31
	cd_t	0.13	0.31
	cnd_t	0.09*	0.79
CAN	Joint	0.32	0.13
	cd_t	0.04**	0.08*
	cnd_t	0.80	0.39
SWE	Joint	0.92	0.20
	cd_t	0.67	0.15
	cnd_t	0.89	0.32

** denotes significance on the 5% level

* denotes significance on the 10% level

(11)

As we know from the previous section, we are not able to reject the null hypothesis of symmetry in any of the eight bivariate tests. If we instead turn to the disaggregated approach in the univariate setting, we find that we are able to reject the null hypothesis in four cases; the deepness test for cnd_t^{UK} and cnd_t^{US}, and both tests for cd_t^{CAN}, corresponding to four out of the eight cases in the bivariate test.

3.4 Results

The models in Gregorio et al (1998) and Leahy and Zeira (2000) imply that cd_t exhibits positive and negative steepness, respectively. Although the deviation from symmetry only is significant in one case in our four-country sample, all four tests indicates negativity, thereby offering some support for the model in Leahy and Zeira (2000). As for the test for deepness, the picture is less clear with symmetry being rejected only once and with no clear pattern concerning positive/negative steepness.

As for the alleged symmetry of cnd_t, the results are different. For the two large Anglo-Saxon economies, the US and the UK, we can reject the null as both series exhibits positive deepness. Given the widespread use of cnd_t in linear consumption models, we believe our results should be a cause of some concern. Specifically, our results suggest that any linear consumption model of the UK or the US will consistently underestimate the height of the peaks in the business cycle. Thus, the results here are broadly in line with previous findings in model based tests of asymmetry in Shea (1995a), Shea (1995b) and Bowman et al (1999), yielding tentative evidence in favor of the loss-aversion model of Bowman et al (1999).

The next hypothesis, that the three consumption measures, c_t, cd_t and cnd_t have the same symmetric properties, is difficult to reject. Again, we stress that cd_t and cnd_t were tested jointly, in contrast to the previous hypothesis test where we test cd_t and cnd_t in a univariate setting. Here, we can only reject the null hypothesis at a 10% significance level in one out of the sixteen tests. Put differently, we can not reject it for three out of four countries and in the case of the UK, we can reject it based on a rejection of the deepness test of c_t^{UK} at a 7% significance level. The empirical practice of seemingly interchangeable use of c_t and cnd_t is thus not rejected from the point of the symmetries of the series.

Turning to the question of the importance of correlation between the series to be tested, we find it to be very important in the current context. In the bivariate case we can not reject the null hypothesis of symmetry in any of the eight cases. However, in 4 out of the 8 corresponding cases for the univariate test, we find that symmetry for one of the two series is rejected. Of course, the results for the difference between the two approaches will vary with the data, but the results here at least indicate that the potential for very different conclusions is not negligible. Again, we want to stress that this is only intended as a comment on the differences between the two alternatives, not on the choice between the two alternatives.

We also want to point out that the question of aggregation is highly non-trivial. In the current setting, we can take the example of the UK and the US. For both economies, cnd_t exhibits positive deepness while the aggregate c_t only does for the UK. To address the possible explanation for such issues, quite a different type of analysis would have to be employed, an analysis beyond the scope of this paper.

4 Comments and conclusions

In this paper we formulate a univariate test for symmetry based on the third central moment and extended it to a multivariate test. The test is robust in several aspects. First, it does not require modeling. Second, it is robust against serial correlation, autoregressive conditional heteroscedasticity as well as non-normality.

In the empirical application of the test we focus on aggregate consumption. Theory, as well as its empirical applications, have several implications which the test allows us to test for. First, significant deviations from symmetry for cd_t are difficult to find, although the test for deepness yields negative t-values in all cases, offering some support to the hypothesis of negative steepness in Leahy and Zeira (2000). Second, the alleged symmetry of the consumption of nondurable goods (cnd_t), does not find the expected support since the US and the UK series both exhibited positive deepness, i.e. the peaks are higher than the troughs are deep. For total consumption (c_t), no such asymmetry is found. Given the widespread use of cnd_t in linear models, we believe our results should be a cause of some concern. Specifically, our results suggest that any linear consumption model of the UK or the US will consistently underestimate the height of the peaks in the business cycle. Third, the univariate and multivariate tests are used to test the hypothesis that c_t, cd_t and cnd_t have the same symmetric properties. Out of the four countries in the sample (the US, the UK, Canada and Sweden), it is only for the UK that the hypothesis is rejected. Thus, from the point of symmetry, the use of c_t as the dependent variable in a consumption regression is given some support, although not unanimous. Fourth, the empirical importance of choosing whether to test possibly correlated series in a univariate or a multivariate setting is underscored. The results from the two approaches differs significantly, emphasizing why the importance of this choice should not be underestimated. However, such a choice is guided by theoretical considerations; the results here merely highlight the importance of that choice in the current empirical context.

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Appendix

Graphical illustration of the data

In the graphical presentation of the various series as used in the paper given below, we have chosen not to include the HP-trend in the diagrams in Figure 3, since the graphs have to be of considerable size to be meaningful. The example given in Figure 2, p.41, serves well as a general illustration of what the estimated HP-trend looks like. Since the test can be viewed as a test for the number of positive and negative observations we have included two numbers (#xx) in each of the graphs depicting the deepness and steepness series. The number above the zero line indicates the number of positive observations while the number below the zero line indicates the number of negative observations.

Figure 3: The original series in logs for the three measures of consumption for all four countries.

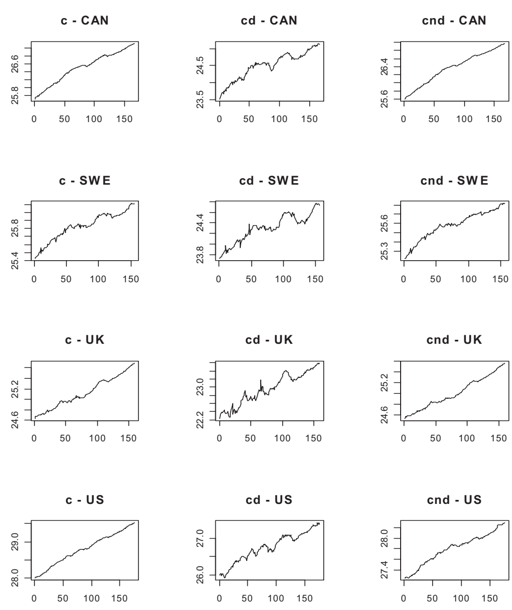


Figure 4: Deepness and steepness series for c_t^{SWE} and c_t^{CAN}.

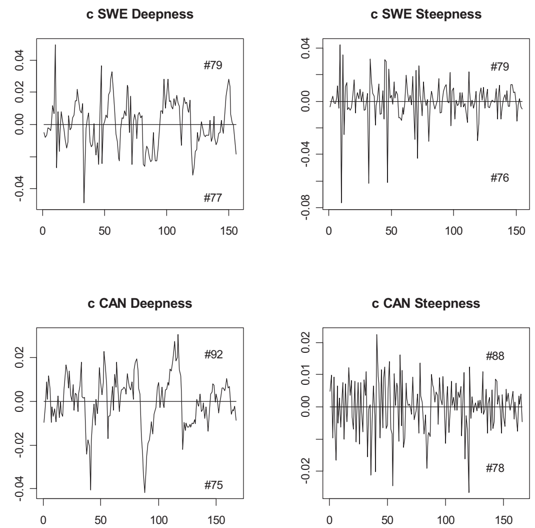


Figure 5: Deepness and steepness series for c_t^{US} and c_t^{UK}.

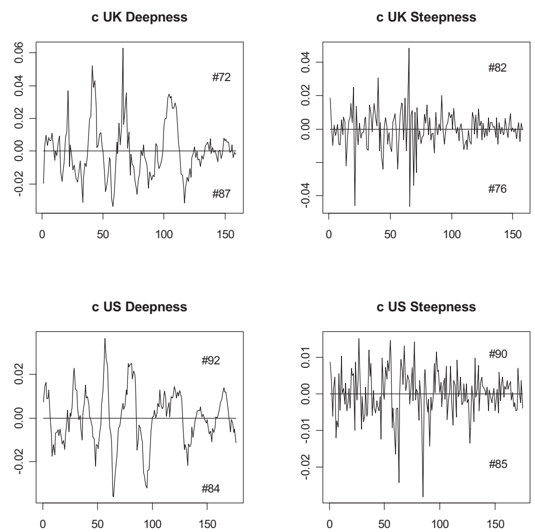


Figure 6: Deepness and steepness series for cd_t^{SWE} and cd_t^{CAN}.

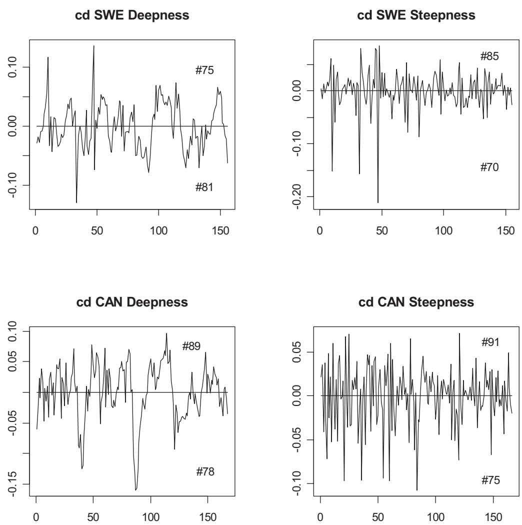


Figure 7: Deepness and steepness series for cd_t^{UK} and cd_t^{US}.

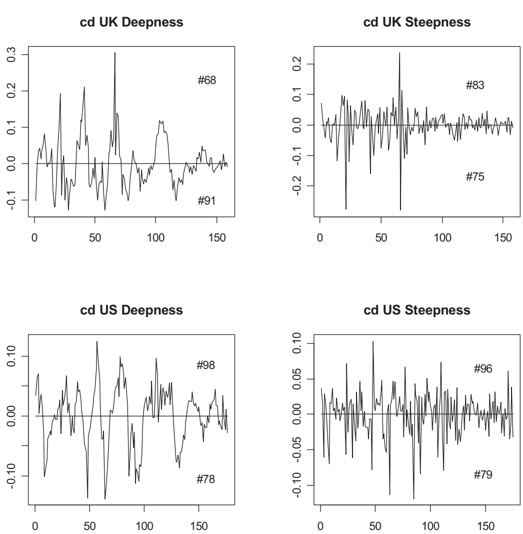


Figure 8: Deepness and steepness series for cnd_t^{SWE} and cnd_t^{CAN}.

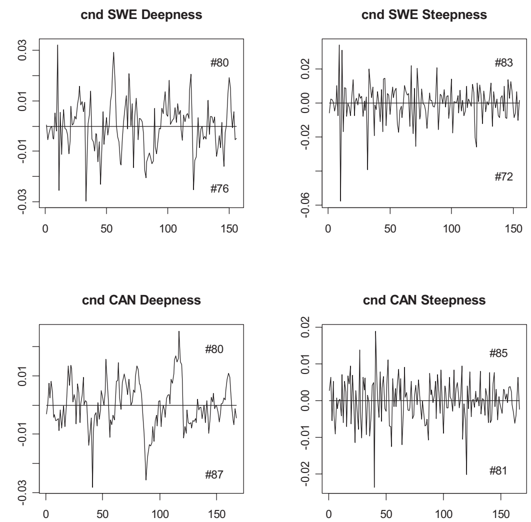
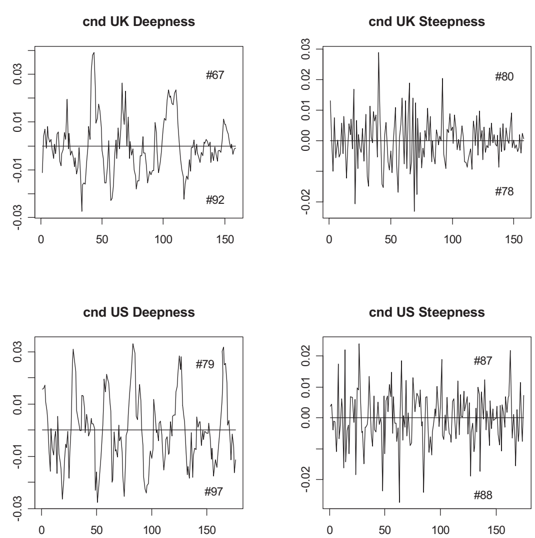


Figure 9: Deepness and steepness series for cnd_t^{UK} and cnd_t^{US}.



Essay 3

Is the Consumption Function Concave?*

Mårten Bjellerup†

May 9, 2005

Abstract

Introducing income uncertainty makes the consumption function concave, implying that the distributions of wealth and income are omitted variables in aggregate Euler equations. This implication is tested through estimation of the distributions over time and augmentation of consumption functions, using Swedish data for 1963-2000. The results show that only the dispersion of wealth is significant, the explanation of which is found in the marked changes of the group of households with negative wealth; a group that according to a concave consumption function has the highest marginal propensity to consume. The results agree with previous findings at the micro level.

*A special thank you goes to Adelchi Azzalini for generously sharing both knowledge and routines for R. Furthermore, I very much appreciate the help I received in my quest for data from Lena Bjerke and Kjell Jansson at Statistics Sweden. Also, I wish to thank seminar participants at the Swedish Central Bank, the Swedish National Institute for Economic Research and the South Swedish Graduate Programme in Economics for their constructive criticism as well as Bengt Assarsson for the feedback at the final seminar. The 2004 Summer School in Econometrics at the University of Copenhagen provided me with inspiration as well as valuable discussions. Finally, I want to thank Håkan Locking for suggesting this line of research, as well as for his continuous support and encouragement.

†School of Management and Economics, Växjö University, SE-351 95, Växjö, Sweden; *e-mail*: marten.bjellerup@ehv.vxu.se.

1 Introduction

"We need also to examine trends in the distribution of wealth, which, more fundamentally than earnings or income, represents a measure of the ability of households to consume."

(Greenspan (1998))

"The structure of the wealth distribution is of key importance for understanding macroeconomic behavior."

(Carroll (2000), p.111)

The two quotes are meant to illustrate, with respect to policy as well as theory, the interest in understanding the role of the wealth distribution in macroeconomics. As for consumption, empirical as well as theoretical work has a long tradition of assuming a linear consumption function, thereby implying that the marginal propensity to consume is unrelated to the level of household wealth.

After the seminal contributions of Keynes (1936) and Friedman (1957), the rational expectations version of the life-cycle hypothesis in Hall (1978) marked the starting point of a period of intensive work. The vast majority of the perfect-certainty and certainty-equivalent models with representative agents that followed, used linear consumption functions.[1] However, as noted by, e.g., Zeldes (1989) and proved by Carroll and Kimball (1996), introducing uninsurable labor income uncertainty makes the consumption function strictly concave for a wide class of models. Developing the permanent income hypothesis using a model in which there are patient and impatient consumers, implying buffer-stock (target saving) behavior, Carroll (1997) is able to account for the above-mentioned findings.[2]

Generally, the concavity of the consumption function implies that since consumption growth depends on level of wealth which is serially correlated, the

[1] As Carroll and Kimball (1996) notes: "Of the 25 household-level studies summarized in the recent survey by Browning and Lusardi (1996), only two allow for a nonlinear consumption function..." (p.982, note 5).

[2] Carroll (2001a) provides a good summary of this field of research; a more rigorous and detailed version of the paper is Carroll (2001b).

famous implication of Hall (1978) that consumption is a random walk at the household level, no longer holds true. Particularly, in the words of Carroll and Kimball (1996), "[a]t the aggregate level, concavity means that the entire wealth distribution is an omitted variable when estimating aggregate consumption Euler equations..." (p.982). This claim is what is tested in this paper.[3] However, since consumption is defined (see, e.g., Carroll (2000)) as being concave in current resources, i.e. nonhuman wealth plus current income, the distribution of income will also be considered.

Besides the direct test of whether the wealth distribution is an omitted variable, estimating the wealth distribution over time enables comparison of the empirical development with what the theories predict. First, the abolition of credit regulations in the mid 1980's, being approximately in the middle of our sample period of 1963-2000, serves as a natural experiment. Both the original permanent income hypothesis and the more recent buffer-stock version, predict that easing liquidity constraints will cause the dispersion of the wealth distribution to increase. Second, the buffer-stock hypothesis predicts that there will be a rapid convergence to a steady-state distribution of wealth, while the permanent income hypothesis makes no such prediction.

However, the scarcity of data on wealth as noted in Muellbauer and Lattimore (1995), is a widespread problem. In this paper, it is addressed through pooling two sources of information. The first source is Jansson (2000), containing detailed data on wealth covering the whole range of the distribution for seven years between 1978 and 1997. The second source is Statistics Sweden (various years), containing summaries of tax reports, from which a time series on the distribution of taxable wealth is obtained.[4] Using these two sources, the whole distribution for each year between 1963 and 2000 is estimated, using the

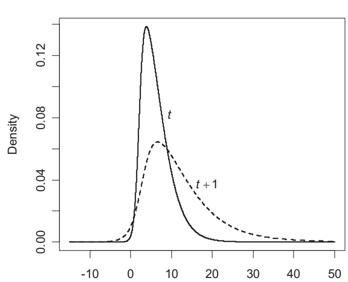
[3] There is a vast literature that examines and extends the implications of the standard buffer-stock model; see, e.g., McCarthy (1995), Lusardi (1996), Souleles (1999) and Parker (1999). However, I am not aware of any paper using the approach adopted in this paper.

[4] Although the data in the tax reports only concerns households (approximately 5-10% of all households) that are above a threshold value after which you are eligable to pay capital tax, it is enough given the estimations of the whole distribution for the seven years between 1978 and 1997.

gamma and the skew-t distributions.[5] There are several reasons for estimating the whole distribution. First, I believe it is of interest to try to determine how different distributions fit the data, especially for wealth. Second, it is a goal in itself to be able to describe the development of the distributions of income and wealth over time. Through estimation of the distributions for different years, comparison between years is facilitated. Third, I want to circumvent the restriction of the original data being in percentile form. Through the estimations of the parameters of the distributions, it is possible to extract any information for all years. Finally, the reason for using two different distributions is that, preferably, the results should not hinge upon a single distribution.

The question of how changes in the distributions affect consumption is solved through creating a simple measure of dispersion, based on the second moment. The reason for this choice is twofold. First, the knowledge of the estimated distribution is used, and not the original percentile information. Second, no *ex-ante* assumptions on MPC's for different levels of income and wealth are made. Despite its simplicity, the suggested measure of dispersion offers rather detailed information. An example illustrates the argument. Assume that we study a distribution at time t and time $t+1$, as depicted in Figure 1 below.

Figure 1: A hypothetical distribution at times t and $t+1$.



[5] Thus, it is possible to circumvent the claim in Jansson (2000), that "...the official records on taxable wealth cannot be used to describe the distribution of wealth for the whole population" (p.7, author's translation).

Here, we can note two things. First, the distribution at time $t+1$ has greater dispersion than the distribution at time t. Second, the distribution at time $t+1$ clearly has a greater mass above the value 10, than the distribution at time t. Now, if the value 10, in our case, could be interpreted as a threshold level at which the MPC changed, the increased dispersion would indeed carry valuable information.

Previewing the results, I find that the wealth distribution is an omitted variable in aggregate consumption functions based on representative-agent models. The result implies a rejection of the proposition in Hall (1978) that consumption should follow a random walk, and thus supports the prediction of the buffer-stock saving hypothesis (BSH). The explanation for the results can be found in the observation that increased wealth dispersion has meant a marked increase in the group of households with negative net wealth, a group that according to the BSH has the highest marginal propensity to consume. Furthermore, we observe that the estimated effect is not only statistically significant. On average, increased wealth dispersion has added approximately $.05 - .07$ percentage points to yearly consumption growth between 1980 and 2000; a figure economically significant as well as plausible. The results here indicate that merely considering the level and mean of aggregate net wealth is insufficient for understanding the effects on consumer spending; the changes in the distribution must also be taken into account.

The paper consists of two major parts. Section 2 starts with a presentation of the gamma and the skew-t distributions and their properties, as well as a presentation of the method of maximum likelihood estimation used for fitting the distributions to the grouped data on income and wealth. Sections 2.2 and 2.3 present the calculation of the income and wealth dispersion measures, respectively. The second part of the paper starts with section 3 and a motivation for the choice of consumption functions, followed by presentation, estimation and augmentation of the consumption functions; the rule-of-thumb model in section 3.1 and the error correction model in section 3.2. Section 3.3 reviews and explains the results while section 4 comments and concludes.

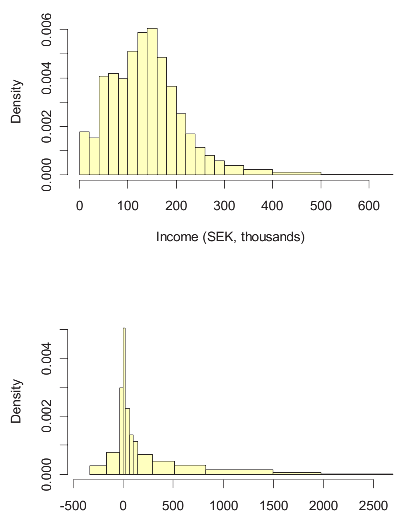
2 The dispersion of income and wealth

As mentioned in the introduction, the aim of this section is to calculate simple measures of dispersion.[6] However, before turning to the calculations, the question of which distributions best describe the data needs to be addressed.

2.1 The data and fitting of distributions

Graphing the distribution of income and wealth for an arbitrary year reveals that any symmetric distribution clearly is ruled out; see Figure 2 below.[7]

Figure 2: Distribution of income and wealth in 1992. (Source: Jansson (2000) for wealth and Statistics Sweden (various years) for income.)



When determining which distribution best describe the data, two distributions have been evaluated: the gamma distribution and the skew-t distribution.[8]

[6] See the appendix p.101 for a presentation of the data used in this section.

[7] For convinience, an upper and lower limit for wealth and an upper limit for income has been assumed in the graphical presentation. In the original data, no such limitations existed as the here considered classes had open boundaries.

[8] Recent research supports this choice; see for instance Azzalini, Dal Capello and Kotz

2.1.1 The gamma distribution

Following Johnson et al (1994), a random variable X has a gamma distribution if its probability density function is of form

$$p_X(x) = \frac{(x-\gamma)^{\alpha-1}\exp[-(x-\gamma)/\beta]}{\beta^\alpha \Gamma(\alpha)}, \quad \alpha > 0, \beta > 0; x > \gamma. \tag{1}$$

When estimating the gamma distribution, denoted gamma (α, β, γ), this three-parameter distribution is usually normalized. By letting the location parameter γ equal zero, the estimation is reduced to two parameters, α and β, which are known as the shape and scale parameters, respectively.[9] For the two-parameter gamma distribution, the two first moments are described by

$$E[X] = \alpha\beta$$

and

$$Var[X] = \alpha\beta^2.$$

2.1.2 The skew-normal and skew-t distribution

Since the skew-t distribution is somewhat less known than the gamma distribution, a slightly longer presentation is motivated. Following Azzalini (1985), a random variable Z has a 'standard' skew-normal distribution $SN(0, 1, \alpha)$ if its density function is of form

$$2\phi(z)\Phi(\alpha z), \qquad z \in \mathbb{R}$$

where ϕ and Φ are the $N(0, 1)$ density and distribution function respectively and the parameter α, taking on values in $(-\infty, \infty)$, controls the skewness of the distribution. Letting $\alpha = 0$, gives the $N(0, 1)$ distribution.

(2003).

[9] In our case this is necessary for the estimations on wealth distribution, since the data concerns net wealth which can take negative values, but not for the estimations on income distribution.

We obtain the skew t-distribution by considering the transformed variable

$$\tilde{Z} = \frac{Z}{\sqrt{V/\nu}}$$

where $V \sim \chi^2_\nu$, independent of Z. The density function is then

$$p_{X_\nu}(x) = 2t(x;\nu)T\left(\alpha x\left(\frac{1+\nu}{x^2+\nu}\right)^{\frac{1}{2}};\nu+1\right)$$

where

$$t(x;\nu) = \frac{\Gamma((\nu+1)/2)}{(\pi\nu)^{1/2}\Gamma(\nu/2)}\left(1+\frac{x^2}{\nu}\right)^{-\frac{(\nu+1)}{2}}$$

is the density function of a regular t variate with ν degrees of freedom and $T(x;\nu+1)$ denotes the t-distribution function with $\nu+1$ degrees of freedom.

In practical applications, we consider the transformed variable $\tilde{Y} = \xi + \omega\tilde{Z}$, whose distribution is determined by a location parameter ξ, a (positive) scale parameter ω, a skewness parameter α and the degrees of freedom ν; we say that $\tilde{Y}$ has a $ST(\xi,\omega,\alpha,\nu)$ distribution. The first two moments are

$$E\left[\tilde{Y}\right] = \xi + \omega\mu \quad (\text{if } \nu > 1) \tag{2}$$

and

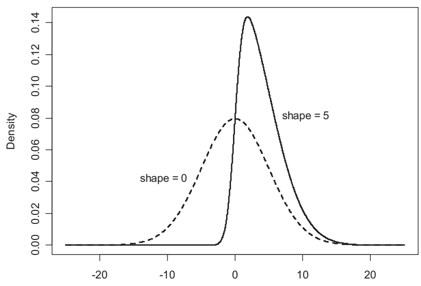
$$Var\left[\tilde{Y}\right] = \omega^2\left(\frac{\nu}{\nu-2} - \mu^2\right) \quad (\text{if } \nu > 2), \tag{3}$$

where

$$\mu = \delta\sqrt{\nu/\pi}\frac{\Gamma\left(\frac{1}{2}(\nu-1)\right)}{\Gamma\left(\frac{1}{2}\nu\right)}$$

where $\delta = \frac{\alpha}{\sqrt{1+\alpha^2}}$ and $\Gamma(.)$ is the gamma function. Setting the shape parameter α equal to zero, we obtain the normal t-density, as can be seen in Figure 3 below.

Figure 3: Examples of the skew-t distribution for different shapes (α).



2.1.3 Method of estimation

Since the data, as presented in the appendix, is in the form of grouped data, parameter estimation has been carried out using maximum likelihood (ML) estimation for grouped data which requires maximization of

$$L(\theta) = constant + \sum_j n_j p_j(\theta) \tag{4}$$

where $\theta = (\xi, \omega, \alpha, \nu)$ or (α, β, γ), depending on the selected distribution. Here n_j is the observed relative frequency for the j-th interval and $p_j(\theta)$ is the probability assigned by the given parameter to the appropriate interval on the scale.

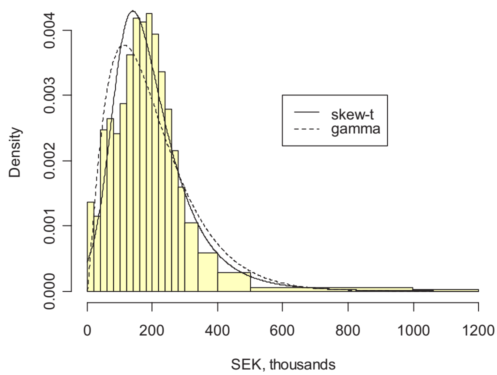
To conduct the ML estimation of eq.(4) and obtain the parameter estimates, the statistical computing environment R has been used (Ihaka and Gentleman (1996)). For the skew-t distribution a supplement consisting of an additional package named 'library sn', specifically developed for this distribution by Adelchi Azzalini, has been used. For the gamma distribution, the 'library sn' package has been modified slightly to allow for a gamma distribution.[10]

[10] The home web page for R is www.r-project, and the one for 'library sn' is http://azzalini.stat.unipd.it/SN.

2.1.4 The question of distributional form

From a graphical presentation of the results, we can see that the skew-t distribution outperforms the gamma distribution. Taking the data on income in 2000 and the respective estimation of the two distributional forms as an example, we can see the difference between them in Figure 4 below.

Figure 4: Fitting a gamma $(2.29, 87.5)$ and a $ST(80, 131, 2.3, 4.3)$ distribution to data on income in 2000 (Source: Statistics Sweden (various years)).

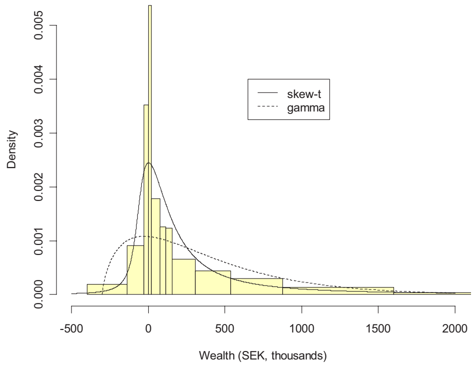


Looking at Figure 4, it should be stressed that this is the year when the gamma distribution performs best, relatively speaking. With few exceptions, it performs less well as we move back in time. However, as can be seen from the graph above, the difference is not very large.

Fitting the distributions to data on wealth, the gamma distribution performs rather poorly when compared to the skew-t distribution. Graphing an arbitrary year, this can easily be seen; see Figure 5 below using data for 1988 as an example.

Here, as with income, the gamma distribution performs more poorly as we move back in time. Also speaking against the gamma distribution is the fact that net wealth can take on negative values, which is why we need to adjust the original data so that γ, the location parameter, equals zero.

Figure 5: Fitting a gamma $(1.68, 397, -300)$ and a $ST(-63, 195, 2.86, 0.99)$ distribution to data on wealth in 1988 (Source: Jansson (2000)).



However, although the skew-t seems to offer a better description of the data, it is not without flaws in the current context. As can be seen from eq.(2) and eq.(3), the two first moments tend to grow exponentially as ν becomes smaller. The pedagogical problem for the skew-t distribution as seen, for instance, in Figure 5, is that if $\nu < 1$, it is not possible to calculate any moments at all. If ν were to be just above unity, then even very small alterations of it would greatly affect the mean. This effect is due to the fatness of the skewed tail; in our case the right, since we have a positive skewness.[11] These potential pitfalls and others, however, will be discussed further in Section 2.3 below, as we consider the results from the actual estimations from fitting the distributions to the data.

To conclude, both distributions will be used, for income as well as wealth. As argued earlier, I believe that comparisons of the two distributions and their performances will be considerably valuable. Moreover, the results will not be dependent upon the choice of a single distribution.

[11] As an illustrative example of ν's effect on the mean, we can look at the parameter values for 1997; see p.103 in the appendix. Keeping the other parameter values constant, an estimated mean of 11,104 for $\nu = 1.01$ and a mean of 512 for $\nu = 1.25$ was obtained. If the two different distributions were to be graphed, it would require a very large figure indeed to visually notice the difference between the two distributions.

2.2 The dispersion of income

Data on the distribution of income has been obtained from Statistics Sweden's Statistical Yearbook for each year between 1965 and 2003 (Statistics Sweden (various years)), data which stems from the compilation of individuals' tax reports for 1963-2000. As can be seen in Figure 2, p.62, the data is grouped. These percentiles vary both in magnitude and number between years. For a fuller presentation, see the appendix, p.101.

2.2.1 Fitting the gamma distribution to income data

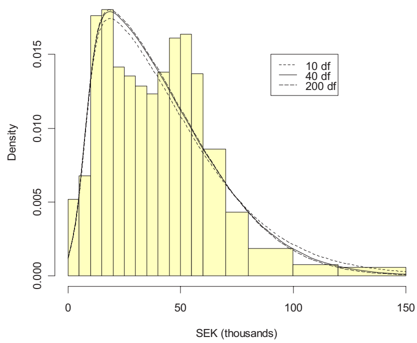
Given the precision of the data set, fitting the gamma distribution to the data on income is straightforward. Also, since the data is on income and thereby has a lower bound of 0, the procedure is reduced to estimating a two-parameter distribution. Thus, the scale invariant measure of income dispersion using the gamma distribution, is calculated as $ID_t^{\gamma} = \frac{Var_t^{\gamma}[.]}{(E_t^{\gamma}[.])^2}$.[12]

2.2.2 Fitting the skew-t distribution to income data

In contrast to the previous section, fitting the skew-t distribution to income data requires the use of restrictions on parameters, albeit small. Given a data set, the estimation of the parameters in a skew-t distribution is rather sensitive to how detailed the information is, especially when regarding the skewed tail, which is the right tail in our case. Before 1978, the information is rather less detailed and even more so before 1974. In consequence, the last cell in the skewed tail in the years before 1978 is significantly larger, around 1% of the total rather than well below 0.1% after 1978. The difference might seem insignificant at first glance. In order to adjust to this cell being larger, however, the degrees of freedom parameter, ν, grows very large resulting in a slightly thinner tail. The problem is that the description of the underlying data improves very little, if at all, as can been seen in Figure 6 below. In contrast, the consequences for calculation of the moments and thus the measure of dispersion, are rather large.

[12] For the complete table of results from the estimations, see the appendix, p.107.

Figure 6: Fitted $ST(7.3, 42, 8.8, \nu)$ for $\nu = 10, 40, 200$ to income data for 1977 (Source: Statistics Sweden (various years)).



As a result, the description of the data varies in an undesired fashion between years. The problem is handled through the placing of a restriction on the fourth parameter, the validity of which can be tested. Given the earlier unrestricted estimations, a guesstimate of $\nu = 10$ as the restriction for the years 1963-1977 is used. Reassuringly, the log likelihood ratio test returns test statistics for all years in the neighborhood of 0.01, well below any conventional significance level. Actually, even applying a highly unconventional 50% significance level, we would still be unable to reject the null hypothesis of the restriction being correct. A relevant question is why was only the restriction $\nu = 10$ applied for the years in question, 1963-1977; why not the whole sample? The answer is to be found in eq.(3), that describes the second moment of the distribution. There we see that as ν increases $\frac{\nu}{\nu-2} \rightarrow 1$, meaning the relative importance of ν diminishes as it grows. Conversely, the relative importance of ν for the second moment grows as it approaches 2 which is why the consequences of the restriction $\nu = 10$ would increase with time as the unrestricted estimates of ν gets smaller.[13]

Thus, we can now summarize the results. As expected, especially since the data is in current prices, the values of both the location and scale parameters,

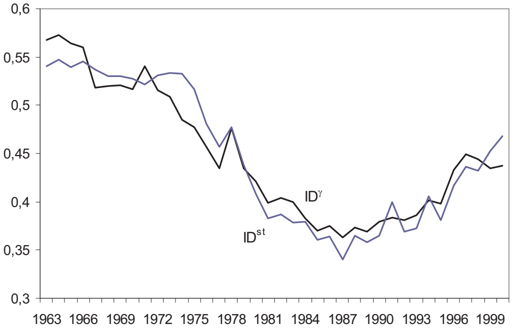
[13] For a better appreciation of this argument, see the results from fitting the skew-t distribution to income data for all years, 1963-2000, p.106 in the Appendix.

ξ and ω, grow with time. As for the value of the two other parameters, α and ν, we see that both shape and the degrees of freedom decline over time, although a few exceptions between adjacent years can be found. However, for our purposes the first and second moment are of more interest and especially the scale invariant measure of income dispersion, $ID_t^{st} = \frac{Var_t^{st}[.]}{(E_t^{st}[.])^2}$. Next, we look at the results from fitting the two distributions to data on income.

2.2.3 Results from fitting the gamma and skew-t distributions to income data

For income as well, we graph the two measures of dispersion, see Figure 7 below, as a complement to the tables with the complete list of estimation results in the appendix, p.106 and p.107.

Figure 7: The two measures of income dispersion, ID_t^{st} and ID_t^{γ}, for 1963 to 2000.



The two measures are, as expected, almost unanimous in their verdict on the development of income dispersion for the period in question. Furthermore, the overall picture is broadly in line with what has been found in earlier studies of income distribution; see for instance Gustavsson and Palmer (2002), although they place the trough a few years earlier.[14]

[14] A recent simulation study by Armelius (2004), suggest that tax aviodance can be at least partly responsible for the increased dispersion in the wake of the large tax reform in 1990-1991.

2.3 The dispersion of wealth

The estimations in this section are based on information from two sources. The first source is a collection of register data (seldom undertaken) by Statistics Sweden, which has been summarized in Jansson (2000), and contains information on the whole distribution of wealth. The second source of information is Statistics Sweden (various years), which in turn draws on households' tax reports. Here, there is detailed information on the right tail of the distribution; i.e. for all households falling above the threshold value above which one is obligated to pay capital tax. For a more thorough presentation of the data, see the appendix p.101. The presentation of the results from fitting the two selected distributions to the data follows next.

2.3.1 Fitting the gamma distribution to wealth data

The gamma distribution is a less flexible distribution than the skew-t, as we only estimate two parameters. Besides this potential advantage, the gamma distribution can achieve what the skew-t could not given the data at hand; that is, calculating the second moment. Although, as argued earlier, this is not paramount given the aim of finding a useful measure of dispersion, it has to be considered an advantage. However, the gamma distribution also has drawbacks, at least potentially. The problem regards the need for an assumption on γ. Since the data concerns net wealth, we have to normalize the distribution so that the sample starts at 0, i.e. $\gamma = 0$. This is really just a matter of shifting the distribution to the right, meaning we need to assume a lower bound for the year of interest. If we assume this lower bound to be -100, then we simply increase the value for the breaks of the groups in the data by 100. After estimation of the two-parameter distribution, we reintroduce the value of γ, in this case -100, to be able to calculate the moments. Thus, the estimations in this section form a two-step approach. First, for the practical considerations regarding the assessment of γ, we turn to the information in Jansson (2000). Second, we use the adjusted information for fitting the gamma distribution to tax report data.

When deciding on the lower bound of the distribution, the information in Jansson (2000) is of great help. There we find the median value for the lowest 10% ($m_{10\%}$) and the median value for the next 10% ($m_{20\%}$) of the sample for each of the seven years the data has been collected. The difference between these two values is then considered to represent the development in the left tail of the distribution. Using the seven observations on $m_{10\%} - m_{20\%}$, a series describing $m_{10\%} - m_{20\%}$ for the whole sample, 1963-2000, is fitted. As for the fitted values, a linear interpolation between the actual observations is made. Before 1978 and after 1997 we have no information, which is why the value for 1978 for all years before 1978 and the value for 1997 for all years after 1997 are chosen. Next, the distance between $m_{10\%}$ and $m_{20\%}$ was assumed to be a good approximation of the distance between $m_{10\%}$ and the left bound of the distribution; i.e. γ.[15]

The results from estimations and calculations, as well as the assigned values of γ, are found in the appendix, p.105.[16] As for the calculations of the measure of wealth dispersion, WD_t^{γ}, problems are encountered regarding the scale invariant fraction $\frac{Var[.]}{(E[.])^2}$. Unfortunately, for certain values of γ, the first moment comes very close to zero for some years, which is why the test for robustness would have been impossible. Instead, the standard deviation was deflated using the implicit total consumption price deflator, $Defl_t$, and taking the natural logarithm. Thus, the measure of wealth dispersion using the gamma distribution is $WD_t^{\gamma} = \ln\left(\frac{Var_t^{\gamma}[.]}{(Defl_t)^2}\right)$. Next, the fitting of the skew-t distribution.

2.3.2 Fitting the skew-t distribution to wealth data

In order to use the skew-t distribution to describe how the dispersion of wealth has changed over time, I draw on information from two sources. Besides the yearly tax report summaries (Statistics Sweden (various years)), an important

[15] Admittedly, this is more than a fair share of assumptions, even for an economist. However, the results show that assigning other values for γ based on other assumptions, only affect the estimations marginally. More importantly, all major conclusions and results later in the paper are unaffected by such alterations.

[16] WD_t^{γ} is affected by the detailed information being concentrated in the right tail, as can be seen in the table in the appendix. The low values of α mean that the distribution is very close to being L-shaped. However, as argued earlier, this does not disqualify it as a useful measure of dispersion.

source is Jansson (2000); the reason both are used is twofold. First, as mentioned in the previous sections, using both allows us to see how a skew-t distribution fit the wealth data during the period of interest. Second, it helps us solve the problem of information in the tax reports being insufficient for a 4-parameter estimation. Using the results from the estimations based on data from Jansson (2000), it is possible to decide on sensible parameter restrictions, thus circumventing the problem by reducing the number of parameters to be estimated.[17]

If we do not impose such restrictions, the combination of the flexibility of the distribution and the scarcity of the data results in highly unrealistic estimates; all parameters take on extreme values making the distribution look like a nail standing on its head. If this is true, why choose such a flexible distribution at all? The reason is that in the first phase it describes the data better, and using the information that characterizes the distribution, we can better describe the less-detailed data in the second phase. Thus, through placing restrictions on the skewness (α) and degrees of freedom (ν) parameters, we achieve three things. First, we are able to capture the features that set the skew-t distribution apart from other distributions. Second, the restrictions are based on knowledge about the distribution thereby contributing positively to the second-stage estimations. Third, we can devote the remaining estimation effort to obtaining parameter estimates for location (ξ) and scale (ω), parameters that are material to calculating the measure of dispersion.

Although the parameter values from the estimations based on the data in Jansson (2000) suggest that such restrictions do not affect the estimations significantly, we want to make sure that the restrictions cannot be rejected which is why a likelihood ratio test is applied.

Let $\hat{\theta}_U$ be the maximum likelihood estimate of θ obtained without regard to the constraints, and let $\hat{\theta}_R$ be the constrained maximum likelihood estimator. If $\hat{L}_U$ and $\hat{L}_R$ are the likelihood functions evaluated at these two estimates, then

[17] See Table 20 p.103 in the appendix for the results from fitting a skew-t distribution to the data in Jansson (2000).

the log likelihood ratio is

$$\lambda = \hat{L}_R - \hat{L}_U.$$

The test statistic -2λ is then compared to the appropriate critical value from a χ^2_p table. Looking at the results from the seven estimations in Table 5 below, we see that the restriction of setting $\alpha = 5$ and $\nu = 1.01$ is valid for all years.[18]

Table 5: Testing restrictions in estimated wealth distribution

	$\hat{L}_R$	$\hat{L}_U$	-2λ	c.v.[1]
1978	-251.1	-248.4	5.42	5.99
1983	-242.1	-241.3	1.62	5.99
1985	-272.0	-271.3	1.36	5.99
1988	-259.5	-258.3	2.42	5.99
1990	-260.7	-259.2	2.88	5.99
1992	-259.9	-257.7	4.28	5.99
1997	-260.0	-258.2	3.61	5.99

(5)

[1] c.v. is the critical value for a χ^2_2 distribution at the 5% level

A choice of $\nu = 1.01$ might seem odd at first glance. The explanation is to be found in the skew-*t* moments' properties. Based on the results from the estimations on the data in Jansson (2000), the choice of setting the restriction to $\nu = 1$ seems reasonable. However, as is clear from equation (2), we have to have $\nu > 1$ in order to calculate the first moment which is the reason behind letting $\nu = 1.01$. As for the second moment, which requires $\nu > 2$, it does not exist given the restriction on ν. However, the aim when fitting a distribution to the data is not explicitly to calculate moments, but rather to find a useful measure of dispersion. Although the second moment does not exist, the scale parameter ω is still a measure of dispersion. Thus, $(\omega_t[.])^2$ instead of $Var_t[.]$ is used for the calculations of the measure of wealth distribution, leaving the measure looking like $WD_t^{st} = \ln\left(\frac{(\omega_t[.])^2}{(Defl_t)^2}\right)$.[19] For a complete list of all estimates, see

[18] The validity of the restrictions is rather sensitive to the values chosen for α and ν. Various combinations of the two in the neighbourhood of $(5, 1.01)$ were tried but the combinations $(3, 1.01)$, $(5, 1.1)$ and $(7, 1.01)$ were all rejected. $(5, 0.9)$ is a possibility but it makes little sense in not choosing the alternative where we can calculate the first moment, i.e. $(5, 1.01)$.

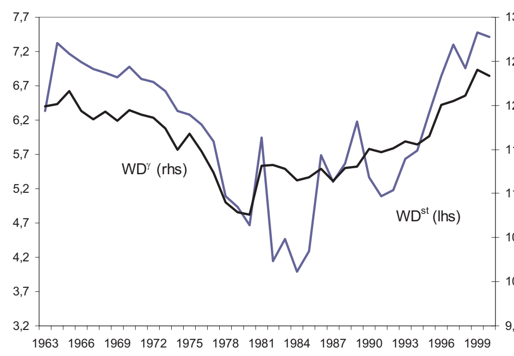
[19] For the skew-*t* distribution, it is possible to use the squared expected value in the de-

the appendix p.104.

2.3.3 Results from fitting the gamma and skew-t distributions to wealth data

As a complement to the tables in the appendix, and to get an intuitive idea of how the two measures look, Figure 8 below depicts the two measures of wealth dispersion.

Figure 8: The two measures of wealth dispersion, WD_t^{st} and WD_t^{γ}, for 1963 to 2000.

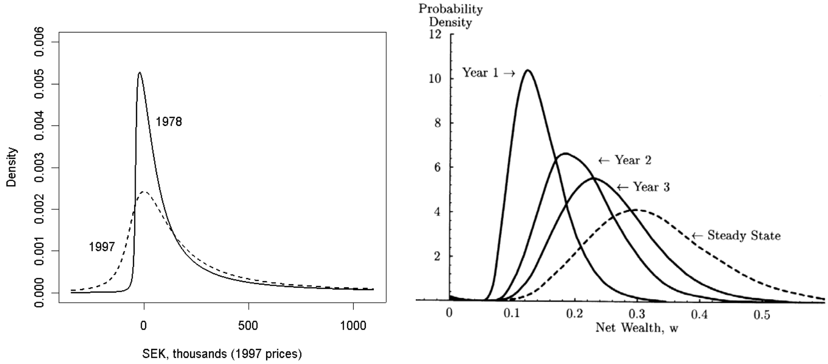


Here we see that the two measures depict a decreasing dispersion until the early- to mid-1980's, after which the dispersion increases. Given the deregulation of credit markets during the period, this is what theory would predict (especially the permanent income hypothesis); lifting liquidity constraints increases wealth dispersion. However, against the backdrop of the buffer-stock hypothesis predicting rapid convergence, i.e. within 4 time periods to a steady-state distribution, the ongoing increase in dispersion for almost two decades seems to disagree with that prediction. Furthermore, the actual changes in

nominator , as we did for ID_t^j. However, since we want to to compare the results from using WD_t^{st} and WD_t^{γ}, a measure that can be used for both distributions have been chosen. Here, we want to highlight that results from using $WD_t^{st} = \frac{(\omega_t[.])^2}{(E_t^{st}[.])^2}$ are almost identical to the ones reported in the paper; more importantly, all conclusions remain intact.

the distribution of wealth have properties that are different from the changes predicted by the buffer-stock hypothesis, as can be seen in Figure 9 below.

Figure 9: Wealth distributions: empirical in left panel is $ST(-40, 105, 11.8, 0.78)$ for 1978 and $ST(-62, 187, 2.47, 0.91)$ for 1997; theoretical in right panel is from the buffer-stock hypothesis.



Sources: Left panel, data from Jansson (2000). Right panel, figure from Carroll (1997), reprinted with permission, © 1997 by the President and Fellows of Harvard College and the Massachusetts Institute of Technology.

Over time, we see an increased dispersion as well as a more symmetric, i.e. less skewed, distribution in both cases. The difference is instead to be found in the location parameter. It clearly grows in the theoretical case (right panel), shifting the distribution to the right, while for the actual distributions (left panel) it is virtually unchanged.

Compared with earlier empirical work, this seems to be in line with earlier findings, although I have not been able to find a comparable measure for the specific period in question. However, we can conclude that the observation in Bager-Sjögren and Klevmarken (1996) that inequality (measured as the coefficient of variation for net wealth) rose from 1.14 in 1983/84, to 2.06 in 85/86, to 2.25 in 92/93, is largely consistent with the estimates, especially WD_t^{st}; see Table 21 in the appendix, p.104.[20]

[20] Bager-Sjögren and Klevmarken (1996) reports other estimates of the coefficient of varia-

The results are also reassuring regarding internal consistency, not between distributions but between different data sets. Using the results from fitting a skew-t distribution to the data in Jansson (2000), see p.103 in the appendix, we can create a similar measure of dispersion. Table 6 below contains the results and a comparison with the WD_t^{st} series in Figure 8.[21]

Table 6: Calculation of wealth dispersion, WD_t^{st}, based on two different data sources

Period	Statistics Sweden (various years)	Jansson (2000)
1978-1983	-	+
1983-1985	-	-
1985-1988	+	+
1988-1990	-	-
1990-1992	-	-
1992-1997	+	+

(6)

+ indicates increase in WD_t^{st}
- indicates decrease in WD_t^{st}

As is evident, the measures of wealth dispersion based on two different data sets yield the same verdict in all cases but one. Given the use of the information in Jansson (2000) when calculating WD_t^{st} as well as the possibility of calculating an internally consistent measure of dispersion, WD_t^{st} is favored as the measure of wealth dispersion.[22]

tion for net wealth using different data and other definitions of wealth. The main conclusion regarding the here reported measure however, remains largely unchanged.

[21] Since the measures in levels differ somewhat in size and since it is the first difference that will enter the consumption functions later, the table only depicts whether the measures from estiamtions based on the two different data sources, yields the same verdict in terms of decreasing or increasing dispersion between observations.

[22] Two circumstances that might amplify the estimated changes in dispersion over time are acknowledged. First, since the estimations are based on information in the very right tail of the distribution, this is likely to amplify the changes in the variance of the distribution. It is plausible that the tails of the distribution might change somewhat between years, without affecting the characteristics for the whole distribution. Second, and only applying to WD_t^{st}, since there is no detailed information on the left tail of the distribution, it leads to an inflexible left bound in estimations. This is likely not the case as deregulation of credit markets, for instance, have led to a limited increase in dispersion in the left tail as well. Taking both these observations into account, however, I do not believe that they invalidate the measures of dispersion and consequently not the results from augmentations of the consumption functions.

3 The consumption functions

The consumption functions are chosen against the backdrop of the hypothesis that we want to test; namely are the distributions of income and wealth omitted variables in certainty-equivalent and perfect-certainty consumption functions?[23] In addition, the consumption functions should preferably be well known, thoroughly tested, previously augmented and estimated using Swedish data. Thus, the sights were set on the rule-of-thumb model (aka the λ-model) and the error correction model (ECM), which both fulfill all the requirements.[24]

Adding to the flavor, the two complement one another from several perspectives. First, applying a theoretical perspective, the λ-model incorporates two different types of consumer behavior while the ECM is derived from a general budget restriction.[25] The two approaches are sometimes labeled the Euler approach and the solved-out approach, respectively.[26] Second, from an empirical perspective, the models can be said to represent the two strands of research on aggregate consumption that long dominated the field after 1978. The λ-model stems from the paper of Hall (1978), in which he combines the permanent income/life-cycle hypothesis with rational expectations, thereby concluding that consumption should follow a random walk. In contrast, the ECM has its roots in the paper of Davidson et al (1978), by and large a product of rigorous econometric evaluation. Their specification of the model can be viewed as a result of extensive and heterogeneous research and publication within the field. A decade later, their econometric method was given its statistical foundation through the papers of Engle and Granger (1987) and Johansen (1988). Third, the respective specifications differ as the ECM is highly parameterized while the λ-model contains few parameters. Also, in the former specification a disaggregated wealth

[23] See the appendix p.102 for a presentation of the data used in this section.

[24] The two publications that this paper draws upon regarding theory and method are Carroll et al. (1994) and Lettau and Ludvigson (2004) for the λ-model and the ECM, respectively; see sections 3.1 and 3.2 for a more thorough presentation.

[25] However, the ECM can be combined with models of consumer preferences, see Lettau and Ludvigson (2004).

[26] However, the solved-out approach is based on the Euler-equation; more precisely, it substitutes the budget constraint into the Euler equation - hence the term 'solved out'.

variable enters explicitly. To conclude, the results from augmentation are not likely to be dependent upon model specification. Thus, general conclusions regarding the empirical importance of the effect on consumption from changes in the dispersion of income and wealth should be possible.

As for the augmentation, WD_t^j was used as the measure of dispersion in period $t+1$, this since wealth is a stock variable and is measured at the end of the period. The same reasoning applies for the aggregate wealth measures introduced in the error correction model. Next, the two consumption functions are presented, estimated and augmented.

3.1 The rule-of-thumb model

The presentation here follows Carroll et al. (1994), which in turn is a slightly modified version of the influential work of Campbell and Mankiw (1989, 1990 and 1991).[27]

Assume that one part of the population, λ, receives a constant proportion of total income, defined as labor income (y_t^{li}), and consumes it in the current period $\Delta c_t^{ROT} = \Delta y_t^{li}$ while the rest of the population follows the proposition from Hall (1978), thus the consumption of life-cyclers is a white-noise process $\Delta c_t^{LC} = \varepsilon_t$. This means that aggregate consumption is given by

$$\Delta c_t = \lambda \Delta y_t^{li} + \varepsilon_t. \tag{7}$$

The time-averaging problem[28] is then solved by directly estimating the $MA(1)$ error term, why equation (7) is modified as follows:

$$\Delta c_t = \lambda \Delta y_t^{li} + \upsilon_t - \theta \upsilon_{t-1}. \tag{8}$$

[27] Agell and Berg (1996) have previously estimated this type of model on Swedish annual data.

[28] "Many authors have noted that if consumption decisions are made continuously but the data are measured as time aggregates, the observed series on spending will follow an $IMA(1,1)$ even if consumer behavior conforms exactly to the life-cycle model and the consumption good is completely nondurable."(Carroll et al. (1994), p.1402)

As it is now possible to use instruments dated at $t-1$ and not $t-2$, the loss of information for the instrumental variables is minimized while we still satisfy the theoretical considerations.

3.1.1 Estimating the rule-of-thumb model

As mentioned earlier, instruments are necessary when estimating eq.(8).[29] Previous studies provide some guidance but to prevent limiting ourselves to a particular set of instruments, the same estimations using two different sets have been conducted.[30] The first set of instruments, IS1, consists of the first difference of labor income (Δy_t^{li}), consumption expenditure on durable goods (Δcd_t) and the discount rate (Δr_t), all measured at time $t-1$. The second set of instruments, IS2, is set one augmented with the first difference of gross fixed capital formation (Δq_t), also measured at time $t-1$. The results from the estimations are found in Table 9 below.

Table 9: Results from estimation of eq.(8)[1].

Instr. set	First stage $\bar{R}^2$	λ	θ	$\bar{R}^2$	Overidentification test	BG	ARCH
IS1	0.312	0.576	0.165	0.271	*0.125*	*0.094*	*0.392*
		0.001	*0.379*				
IS2	0.341	0.585	0.441	0.284	*0.122*	*0.171*	*0.603*
		0.001	*0.017*				

(9)

Italics denote p-values
IS1: $\Delta y_{t-1}^{li}, \Delta r_{t-1}, \Delta cd_{t-1}$ IS2: $\Delta y_{t-1}^{li}, \Delta r_{t-1}, \Delta cd_{t-1}, \Delta q_{t-1}$

The second column, first stage $\bar{R}^2$, reports the results from evaluation of the instruments.[31] Turning to the estimations of eq.(8), the results differ somewhat from previous estimations using Swedish data, which is why a stability test is

[29] All estimations have been carried out using RATS 5.0 and EViews 3.1.
[30] Since the aim of this paper is to evaluate an augmentation, we would not like our results being possibly dependent on a single set of instruments.
[31] The results are not dependent on instrumental variable estimation; similar results can be reproduced using OLS estimation.

also undertaken.[32] Given the positive, i.e. stable, outcome of the test we can only conclude that the results differ from previous estimations and that the value of λ is higher than previously estimated; 0.576 (for IS1) and 0.585 (for IS2) is clearly higher than the previous average estimate of around 0.3 and also outside the earlier range of estimates of -0.05 to 0.52. Next, the model is augmented with measures of income and wealth dispersion.

3.1.2 Augmenting the rule-of-thumb model

Augmenting the λ-model is not an original idea; it is rather the opposite.[33] Besides developing a more efficient way of estimating the model, as mentioned earlier, Carroll et al. (1994) augments it using an index of consumer sentiment; similar examples for Sweden are found in Agell and Berg (1996) and Berg and Bergström (1996).

The logic behind including a measure of income dispersion is straightforward. For the measure of wealth dispersion, the logic is that one of the two types of consumers is affected by changes in wealth, although a wealth variable does not enter the model specification explicitly. For the fraction of life-cycle consumers, $1 - \lambda$, wealth is an important determinant of consumption since consumers' consumption in the current period is determined by their lifetime wealth. For the fraction of rule-of-thumb consumers, λ, changes in wealth dispersion do not affect them. As they consume all their disposable income each period (making saving impossible), they cannot accumulate any wealth. This leaves the fully augmented version of eq.(8) looking like:

$$\Delta c_t = \lambda \Delta y_t^{li} + \mu_1^j \Delta ID_t^j + \mu_2^j \Delta WD_t^j + v_t - \theta v_{t-1}, \qquad (10)$$

where $j = \gamma$ or st, i.e. denoting whether the gamma distribution or skew-t distribution has been used for calculation of the measure of dispersion.

[32] The results from Chow's breakpoint test seem very robust for both set of instruments given that all p-values stayed well clear of 0.1, despite trying different breakpoints between 1978 and 1985. For an overview of previous results, see for instance Hansson (2001).

[33] All estimations have been done using RATS 5.0 and EViews 3.1.

Looking at the results from the estimations of the partially augmented λ-model in Table 11 below, we see some interesting results.[34]

Table 11: Results from estimation of partially augmented λ-model

Model	λ	μ_1^j	μ_2^j	θ	$\bar{R}^2$
Eq.(8)	0.585			0.441	0.284
	0.001			*0.017*	
$+\Delta ID_t^{st}$	0.518	0.113		0.333	0.356
	0.002	*0.237*		*0.079*	
$+\Delta ID_t^{\gamma}$	0.636	0.147		0.509	0.262
	0.000	*0.124*		*0.007*	
$+\Delta WD_t^{st}$	0.556		0.005	0.378	0.359
	0.001		*0.082*	*0.050*	
$+\Delta WD_t^{\gamma}$	0.569		0.019	0.475	0.354
	0.000		*0.080*	*0.010*	

(11)

Eq.(8): $\Delta c_t = \lambda \Delta y_t^{li} + \upsilon_t - \theta \upsilon_{t-1}$
Italics denote p-values

For making comparison more easy, the results from the estimation of the original model have been included; eq.(8). Turning to the new parameters, we see that μ_1^j, the income dispersion parameter, is more than a little shy of conventional significance levels for the skew-t distribution, while for the gamma distribution it lies in the neighborhood of the 10% significance level. Neither of them, though, yield a satisfying result. Turning to the two measures of wealth dispersion, the picture changes as both μ_2^{γ} and μ_2^{st} are significant at the 10% level. For the models augmented by WD_t^j, we also see that the fraction of life-cycle consumers has increased somewhat, i.e. λ is slightly lower than in the original model. Importantly, given the turbulent macroeconomic years of the late 1980's and early 1990's, all augmented models pass the Chow breakpoint test.[35] Next, we

[34] For the estimations in Table 11, only the second instrument set have been used. The reason is that using the first set only changes the results marginally. The results are available upon request. I have also, for expository convinence, excluded the results from the three tests used in table 11, as all four augmented models clearly fail to reject the null hypothesis in all tests.

[35] All years between 1979 and 1986 were tried as breakpoints; in none of the cases a p-value below 0.1 was reported.

turn to the fully augmented λ-model.

Table 12: Results from estimation of fully augmented λ-model

Model	λ	μ_1^j	μ_2^j	θ	$\bar{R}^2$
Eq.(8)	0.585			0.441	0.284
	0.001			*0.017*	
$+\Delta ID_t^{st}$,$+\Delta WD_t^{st}$	0.471	0.137	0.005	0.421	0.449
	0.002	*0.115*	*0.054*	*0.025*	
$+\Delta ID_t^{st}$,$+\Delta WD_t^{\gamma}$	0.507	0.119	0.016	0.426	0.411
	0.001	*0.184*	*0.135*	*0.025*	
$+\Delta ID_t^{\gamma}$,$+\Delta WD_t^{st}$	0.602	0.184	0.006	0.556	0.388
	0.000	*0.036*	*0.029*	*0.003*	
$+\Delta ID_t^{\gamma}$,$+\Delta WD_t^{\gamma}$	0.585	0.150	0.019	0.508	0.390
	0.000	*0.084*	*0.067*	*0.012*	

(12)

Eq.(8): $\Delta c_t = \lambda \Delta y_t^{li} + \upsilon_t - \theta \upsilon_{t-1}$
Italics denote p-values

All the augmented models have significantly improved the fit of the model, as can be seen in the $\bar{R}^2$ column in Table 12 above. Looking at the original parameters, we see that they have changed somewhat, although without a distinct pattern. As for the measures of dispersion, the picture is by and large what we expected, given the results from the partial augmentation; for all four models, μ_2^j is more significant than μ_1^j. The size, sign and significance of the μ's are roughly the same although μ_1, the parameter for ΔID_t, now takes on lower values in all cases. Thus, there is a correlation between the two measures of dispersion that positively affects the significance of ΔID_t. Studying the measures one by one, we see that in five out of eight cases the dispersion parameter is significant and when it is not, it only misses being significant by a couple of percentage points; all p-values are below 0.2. Furthermore, when using the Wald test to try the restriction of $\mu_1^j = \mu_2^j = 0$, we are able to reject it in all cases.[36] As for the

[36] For three of the four models we have a p-value below 0.05, and for the augmentation with ΔID_t^{st} and ΔWD_t^{γ} it is 0.08.

parameter stability in the models, the Chow breakpoint test is adopted, using different breakpoints. All models pass without any problems. Next, we look at the results from estimation and augmentation of the error correction model.

3.2 The error correction model

Regarding the ECM, the theoretical and methodological framework follows Lettau and Ludvigson (2004), while the specification of the model follows earlier studies on Swedish data; see for instance Johnsson and Kaplan (1999) and Lyhagen (2001). The variables entering the ECM are thus consumer expenditure on non-durable goods and services (c_t), household disposable income (y_t^d), household financial wealth (w_t^f) and household net non-financial wealth (w_t^{nnf}), while the sample's time span is 1970 to 2000.[37] Unfortunately, the variables on wealth are not available before 1970 which is why the sample for the ECM estimation is shorter than the sample for the λ-model.

When compared to the λ-model, it is perhaps less clear how the ECM specification follows from theory. However, Lettau and Ludvigson (2004) shows (on the basis of work by Campbell and Mankiw (1989)), that the ECM specification can be derived from a general budget constraint relation. The intuition behind the result is in the accounting-like relationship among the variables. Given a stationary savings rate in the long run, the variables entering the budget constraint cannot drift too far apart. Rather, they share the same long-run trend; i.e. they are cointegrated.

The ECM is the single equation version of the vector error correction model (VECM), which in turn, making use of Granger's representation theorem, is a reparameterized version of a vector autoregressive model (VAR). The VECM

[37] Given the measure of wealth dispersion, it would perhaps seem logical to use a model with only one net wealth variable, as in Lettau and Ludvigson (2004). However, in contrast to their findings, but in line with Berg and Bergström (1995), I find that the model has a very poor fit. Thus, the current disaggregated specification is chosen.

can in our case be described by

$$\Delta \mathbf{x}_t = \mathbf{\Phi D}_t + \mathbf{\Pi x}_{t-1} + \sum_{i=2}^{k} \boldsymbol{\gamma}_i \Delta \mathbf{x}_{t-(i-1)} + \boldsymbol{\varepsilon}_t \tag{13}$$

where $\mathbf{D}_t$ is a vector of determinants, including a VAT dummy, Δ denotes first difference, k equals the number of lags in the system, $\mathbf{x}_t \equiv (c_t, y_t^d, w_t^f, w_t^{nnf})'$, $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$ and $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}_i$ are (4×4) matrices.[38] The intuition behind the model is that variables in $\mathbf{x}_t$ share the same trend, i.e. they are cointegrated, which in turn requires that they are integrated of the same order; in our case $I(1)$. Often the first differences are referred to as the short-run part of the model while the levels are referred to as the long-run part of the model. In the long-run part, the value of α determines at which speed the error, or disequilibrium, is corrected while $\boldsymbol{\beta}$ determines the relationship between the variables.

3.2.1 Estimating the error correction model

When trying to form a stable VAR specification of the model, it is of obvious interest to try reducing the number of lags so that the system is kept as small as possible.[39] However, this should not be done without considering the properties of the residuals since they should not deviate "too much from Gaussian white noise" (Johansen (1995), p. 20). This consideration, combined with evaluation of information criteria, provides the guidelines for empirical modeling. In our case, a system with two lags, i.e. $k = 2$, yields a p-value in the vector test for autocorrelation of 0.42 along with no signs of heteroscedasticity and approximate normality.[40]

Next, we test for the number of cointegrating relationships in $\mathbf{x}_t$. To do this, one out of the five models proposed by Johansen (1988) has to be chosen. The choice between the models concerns the possible inclusion of a constant and/or

[38] The use of VAT dummy, D9192, follows Johnsson and Kaplan (1999) who include it to control for the large tax reform and the abondonment of the fixed exchange rate in the early 1990's. D9192 takes the value 0 in 1970-1990, 0.33 in 1991, 0.66 in 1992 and 1 thereafter.

[39] All estimations have been carried out using PcGive 10.0 and RATS 5.0.

[40] Using a dummy for 1986, it is possible to reduce the system to $k = 1$ - a specification for which all major conclusions are intact.

trend in the short-run as well as the long-run part of the model. Conveniently, the current specification offers some help in our choice. First, we want to allow for a constant in the short-run part of the model as well as in the cointegrating space, i.e. the long-run part of the model, since all variables expressed in levels display a clear time trend. However, I am somewhat reluctant to include a time trend in the cointegrating space since this would imply that there is a time trend in the cointegrating residual, i.e. a time trend in saving. Consequently, we end up with what is called model 3.

The trace test for evaluating the number of cointegrating vectors yields clear results. With a p-value of 0.000 we reject the null of $r(\mathbf{\Pi}) = 0$ ($H_1 : r(\mathbf{\Pi}) \geqslant 1$) while we in the next step of the test fail to reject the null of $r(\mathbf{\Pi}) = 1$ ($H_1 : r(\mathbf{\Pi}) \geqslant 2$). Now, we know that we can model the consumption function as a single equation but have yet to determine the number of error correction mechanisms. To obtain "super consistent" estimates of the four β's, dynamic least squares estimation was used (Stock and Watson (1993)). Normalizing for β_0, the parameter associated with c_t, the parameter values are $\beta_1 = 0.529, \beta_2 = 0.137, \beta_3 = -0.003$.[41] This means, as expected, that income dominates the cointegrating relationship while financial wealth is by far the more important variable of the two wealth variables. Using the estimates of $\boldsymbol{\beta}$, we can now test for weak exogeneity and thus answer the question of how many correction mechanisms there are and thus which is the statistically endogenous variable. Since most economic theory views c_t as being the endogenous variable in the cointegrating relationship, the restriction that $\boldsymbol{\alpha} = (\alpha_{c_t}, 0, 0, 0)$ is first tried. The LR test yields a test statistic of 4.1, meaning that the restriction cannot be rejected at the 5% or 10% levels. This means that c_t is endogenous and the value of the adjustment coefficient, $\alpha_{c_t} = -0.37$, indicates that a disequilibrium is corrected after approximately three years. Finally, this means that it is possible to model the consumption function as a single equation with one error correction parameter and estimate it using ordinary least squares (OLS)

[41] Following Lettau and Ludvigson (2004), an explanation for the β's not summing up to 1 is that the used measure of consumption is a subset of the theoretical definition of consumption.

(Engle and Granger (1987), Stock (1987)).[42] Thus the model to be estimated by OLS looks like:

$$\boldsymbol{\gamma}\boldsymbol{\Delta}\mathbf{x}_t' = \Phi D_t + \alpha\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1} + \varepsilon_t, \tag{14}$$

where $\boldsymbol{\gamma} \equiv (1, -\gamma_1, -\gamma_2, -\gamma_3)$, $\mathbf{x}_t \equiv (c_t, y_t^d, w_t^f, w_t^{nnf})$ and $\hat{\boldsymbol{\beta}} \equiv$ (1, -0.529, -0.137, 0.003) is the vector of previously estimated coefficients. The estimates from this version of eq.(14) is found in Table 15 below.

Table 15: Results from estimation of eq.(14)[1].

γ_1	γ_2	γ_3	α	$\bar{R}^2$
0.256	0.048	0.006	-0.280	0.283
0.030	*0.251*	*0.644*	*0.036*	

(15)

Italics denotes p-values

[1] Eq.(14): $\gamma\Delta x_t' = \Phi D_t + \alpha\hat{\beta}' x_{t-1} + \varepsilon_t$

Here we can see that the parameters in the short-run part, the γ's, follow the same pattern as in the long-run part. Income changes dominate in the short-run and looking at the wealth parameters, we see that changes in financial wealth are more important than changes in net non-financial wealth, although both are not significant. Next, the ECM is augmented.

3.2.2 Augmenting the error correction model

As with the λ-model, the ECM had previously been augmented; see for instance Lyhagen (2001) for Sweden and Davis and Palumbo (2001) for the US. However, unlike the λ-model, here we have a choice regarding the augmentation; should it be in the short-run and/or the long-run part of the model in eq.(14)? As it turns out, given the limited sample, this is as much a practical as a theoretical concern. As for the short-run part of the model, including additional regressors in a finite sample would be justified if efficiency gains could be made, as I believe they can. What we should refrain from, is drawing conclusions

[42] OLS estimation of an autoregressive function yields estimates that are biased towards zero. However, this, if anything, works to my disadvantage since I am interested in testing the augmented ECM for parameter significance.

regarding the parameters in the cointegrating relationship when using a model that is augmented in the short-run part. For such conclusions to be possible, the additional explanatory variables have to be orthogonal to the cointegrating error, i.e. to $\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1}$ (see, for instance, Johansen (1992)).[43] As for the long-run part, the picture is quite different. Here, respecification of the original VAR is needed along with a decision on whether the new variables should be treated as exogenous or endogenous. From a theoretical point of view I tend to support labeling ID_t and WD_t exogenous, given that neither of the two measures of dispersion could be claimed to be dependent on the original variables.[44] Also, given the limited sample, trying to incorporate ID_t and WD_t as endogenous variables leads to VAR specifications with tremendous autocorrelation in the residuals. Despite additional lags and various dummies, the tests clearly reject the hypothesis of well-behaved residuals in almost all aspects. Thus, we can conclude that they deviate too much from Gaussian white noise. Also, as the number of endogenous variables grows, adding an extra lag or two means that the system expands rapidly making it impossible to achieve an acceptable VAR, as we rapidly approach zero degrees of freedom. For the case of adding ID_t and WD_t to the short-run part of the model, no such concerns are necessary.

When augmenting the cointegrating relationship using ID_t^j and/or WD_t^j, it results in multiple cointegrating vectors and multiple error correction mechanisms in each vector, leaving the model impossible to interpret.[45] Thus, we refrain from augmenting the model in the long-run part. This means that we end up with the following specification for the fully augmented ECM:

$$\boldsymbol{\gamma}\Delta\mathbf{x}_t' = \Phi D_t + \alpha\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1} + \boldsymbol{\mu}\Delta\mathbf{ID}_t + \boldsymbol{\varepsilon}_t$$

[43] This however, is not a problem as the assumption of orthogonality not can be rejected in any of the models.

[44] Here, a valid objection could be that labelling any variable exogenous in a VAR goes against the underlying idea of the model. However, our starting point is that economic theory suggests that an ECM specification likely is a good depictor of aggregate consumption behavior and says nothing about the underlying VAR.

[45] Experimenting with the specification of the equation, using model 3 and 4, a lag of 1 or 2, and dummies, *one* model specification with $r(\mathbf{\Pi}) = 1$ and $\boldsymbol{\alpha} = (\alpha_{c_t}, 0, 0, 0)$ can be found. However, given the instability, i.e. the results being very sensitive to model specification, I clearly want to state that this specification *not* is robust to changes.

where $\mathbf{ID}_t = (ID_t^j, WD_t^j)'$ and $\boldsymbol{\mu}$ is the associated parameter vector.

As before, augmentation using the measures of dispersion on their own as well as together, will be tried. Looking at Table 16 below, we see the results from the partial augmentation of the ECM.

Table 16: Results from estimation of partially augmented ECM

	eq.(14)	$+\Delta ID_t^{st}$	$+\Delta ID_t^{\gamma}$	$+\Delta WD_t^{st}$	$+\Delta WD_t^{\gamma}$
γ_1	0.256	0.218	0.250	0.299	0.288
	0.030	*0.068*	*0.038*	*0.008*	*0.014*
γ_2	0.048	0.043	0.049	0.048	0.060
	0.251	*0.299*	*0.254*	*0.213*	*0.149*
γ_3	0.006	0.002	0.007	0.005	0.007
	0.644	*0.856*	*0.599*	*0.704*	*0.613*
α	-0.280	-0.293	-0.262	-0.295	-0.327
	0.036	*0.028*	*0.060*	*0.018*	*0.015*
μ_1^j		0.139	-0.069		
		0.234	*0.626*		
μ_2^j				0.007	0.021
				0.032	*0.101*
$\overline{R}^2$	0.283	0.298	0.258	0.395	0.327

(16)

Eq.(14): $\gamma\Delta x_t' = \Phi D_t + \alpha\hat{\beta}' x_{t-1} + \varepsilon_t$
Italics denote p-values

When compared to the augmentation of the λ-model, we see both similarities and dissimilarities. As for the μ's, the sign is the same as before while the size changes for μ_2^{γ}. Looking at significance, both parameters for income dispersion are still clearly insignificant. The parameters associated with wealth dispersion, μ_2^j, are almost the same as in the λ-model although the p-value of μ_2^{γ} now *just* breaches the 10%-level at 0.101. The dissimilarities concern the original parameters as we discern a pattern here. γ_1, γ_2 and α are all larger and more significant in the two models augmented by ΔWD_t. The Chow breakpoint test is not possible to conduct, given the short sample in combination with the highly parameterized ECM specification. Next, the results from the fully augmented version:

Table 17: Results from estimation of fully augmented ECM

	eq.(14)	$+\Delta ID_t^{st}$ $+\Delta WD_t^{st}$	$+\Delta ID_t^{st}$ $+\Delta WD_t^{\gamma}$	$+\Delta ID_t^{\gamma}$ $+\Delta WD_t^{st}$	$+\Delta ID_t^{\gamma}$ $+\Delta WD_t^{\gamma}$
γ_1	0.256	0.259	0.253	0.291	0.281
	0.030	*0.021*	*0.035*	*0.011*	*0.018*
γ_2	0.048	0.043	0.055	0.049	0.061
	0.251	*0.258*	*0.186*	*0.206*	*0.144*
γ_3	0.006	0.000	0.003	0.006	0.008
	0.644	*0.969*	*0.804*	*0.608*	*0.542*
α	-0.280	-0.310	-0.335	-0.266	-0.304
	0.036	*0.012*	*0.013*	*0.038*	*0.028*
μ_1^j		0.155	0.121	-0.119	-0.099
		0.146	*0.285*	*0.366*	*0.471*
μ_2^j		0.008	0.019	0.008	0.022
		0.023	*0.125*	*0.026*	*0.091*
$\overline{R}^2$	0.283	0.428	0.344	0.391	0.324

(17)

Eq.(14): $\gamma\Delta x_t' = \Phi D_t + \alpha\hat{\beta}' x_{t-1} + \varepsilon_t$

Italics denote p-values

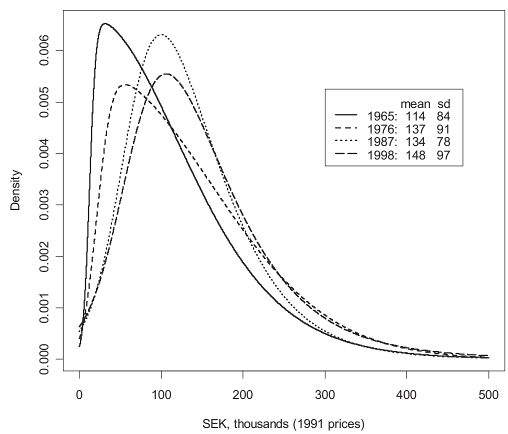
In Table 17 we see that the results differ from the λ-model somewhat, as the pattern from the partial augmentation remains largely intact. Here, all μ_1's are insignificant while the reverse is true for three out of four μ_2's.

3.3 Results

Although the results are not unanimous, the picture that emerges is consistent and robust. In a Swedish aggregate consumption model that is augmented with measures of income and wealth dispersion, changes in consumption depend significantly and positively on changes in wealth dispersion and to a much lesser degree on changes in income dispersion. How can this be?

For income dispersion the results are rather clear; income dispersion is not a significant determinant of consumption. An intuitive explanation can be found by studying examples of the distributional development over time, as described by the skew-t distribution. Looking at Figure 10 below, we see some interesting characteristics. First, a closer look at the parameters that distinguish this

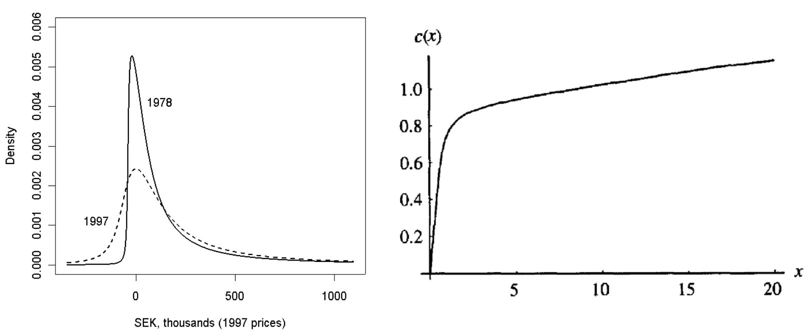
Figure 10: Fitting a skew-t distribution to income data for 1965, 1976, 1987 and 1998; in constant 1991 prices (Source: Statistics Sweden (various years)).



distributional form is warranted; namely the skewness parameter α and the degrees of freedom parameter ν. Overall, the change between 1965 and 1998 can be described by a significant drop in both α and ν; that is, the distribution is now less skewed but has got fatter tails instead. Put differently, we can liken the change to going from a distributional form in 1965 that is closer to skew-normal, to a distributional form in 1998 that is closer to a normal t-distribution. Second, and more important for explaining the results, two interesting features can be identified. First, more people have received higher incomes and fewer people have recieved lower incomes over the years, meaning that, if anything, this fact should have pushed more people into groups with lower MPC's. Second, the changes are relatively small over the years, apart perhaps from the changes in the very left of the left tail. Presumably, regardless of what measure of dispersion constructed, the effect on consumption would be negative, if discernible at all. Combining this observation with the results that the parameters associated with income dispersion turned out to be non-significant, we can conclude that the simple measures of income dispersion are not able to capture any significant empirical effects on aggregate consumption.[46]

The picture is quite different for wealth, as the dispersion of wealth has a significant effect on consumption. The reason for this effect from increased dispersion, for example, is most likely twofold; Figure 11 below illustrates the argument.

Figure 11: Left panel: estimated wealth distribution for 1978, $ST(-40, 105, 11.8, 0.78)$, and 1997, $ST(-62, 187, 2.47, 0.91)$. Right panel: stylized consumption function from the buffer-stock saving hypothesis ($c = C/wL$ and $x = X/wL$ where C is consumption, X is current resources and wL is permanent labor income).



Source: Left panel, data from Jansson (2000). Right panel, figure from Carroll (2000), reprinted with permission, © 2000 by the American Economic Association.

In the left panel we see that the increased dispersion has meant quite a large increase of the probability mass in a narrow band of the left tail and a modest probability mass increase in a very wide band in the right tail.[47] However, the

[46] Given the relatively poor performance in the partially augmented models, the observation that income dispersion comes out significant in two of the fully augmented λ-models, is attached little weight.

[47] The increase in the left tail is not an estimation anomaly. In the original data (Jansson

strict concavity of the consumption function, as follows from the "buffer-stock" saving hypothesis, would seemingly imply that the two effects to some extent would be offsetting. The stylized consumption function from Carroll (2000) in the right panel of Figure 11 tells the story.[48] For small values of x, the BSH consumption function has a very large MPC while for large values of x it has a considerably smaller MPC and also is approximately linear. Thus, the increased probability mass in the left tail in the left panel of Figure 11, most likely means that more households now have a much higher MPC. Furthermore, it is plausible that the wide-spread increase in probability mass in the right tail would not have caused MPC's to change due to the approximate linearity of the consumption function for higher values of net wealth. Taken together, the arguments constitute a viable explanation for why the observed increase/decrease in net wealth dispersion has had a significantly expansionary/contractionary effect on consumption growth. Last, we observe that the estimated effect is not only statistically significant. On average, increased wealth dispersion has added approximately $.05 - .07$ percentage points to yearly consumption growth between 1980 and 2000, a figure that is economically significant as well as plausible.[49]

Against the backdrop of the changes in the distribution of wealth over time and their importance for the results, a measure of disperison based on the third moment would be interesting. However, for the skew-t distribution the third moment cannot be calculated since $\nu < 3$ for all years and using the skewness parameter is also ruled out given the restriction of $\alpha = 5$ for all years.

As for the difference between the distributions of income and wealth, where only the latter is significant, the results in this paper find support in research using household level data. Bjellerup (2005), in a study using Swedish house-

(2000)), measured in constant 1997 prices, 5% of the distribution is below SEK -32,800 in 1978 while 15% of the distribution is below SEK -38,800 in 1997. Thus, the change as depicted in Figure 11 seem reasonable.

[48] c is defined as the ratio of consumption C to permanent labor income wL and x as the ratio of current resources (nonhuman wealth plus current income X) to permanent labor income, for a microeconomic consumer for whom interest rates, wages and labor supply are fixed at their steady-state levels.

[49] The average growth of consumption of nondurable goods and services between 1980 and 2000 was approximately 1.1%.

hold level data for 1999-2001, finds that consumption is approximately linear in income, while the marginal propensity to consume is strictly decreasing in net wealth. Although far from exhaustive, the results there and the results here seem to support one another.

The results here can also be viewed as being a part of the discussion on aggregation as well as on representative-agent models. Krusell and Smith (1998) show that using a calibrated version of the stochastic growth model with partially uninsurable idiosyncratic risk and movements in aggregate productivity, "the behavior of the macroeconomic aggregates can be almost perfectly described using only the mean of the wealth distribution" (Abstract). However, as Carroll (2000) argues, this is because their model using a high-mean, low-variance wealth distribution, "generates an attractive 'approximate aggregation' result: behavior of the economy is very similar in essentially all respects to behavior in the representative-agent model." (p.113) Hence, it is *not* enough to "track only the evolution of the 'aggregate budget' in order to analyze the dynamic behavior of the macroeconomic aggregates" (Krusell and Smith (1998), p.889); this given the high-skewness, high-variance features of the actual estimated empirical distributions.

3.3.1 Generality of the results

As previously argued, I believe the results are not dependent on model or method, an opinion based on several observations. First, the question of the quality of the underlying data is answered positively, in my opinion, given the similarities between the results presented in this paper and those of previous research. Second, two different distributions have been utilized when calculating the measures of dispersion. Although the results differ somewhat, I contend that the differences between distributions are logical given the benefits and drawbacks of the different methods of estimation and the characteristics of the distributions. Third, as argued in section 3, the attributes of the λ-model and the ECM are highly complementary which is why the results most likely are valid for other specifications of the consumption function. Fourth, in three out of the four partially augmented models we clearly reject the hypothesis that μ_2^j is equal to zero, while the p-value in the fourth case *just* misses at 0.101. When using the preferred measure of wealth dispersion, WD_t^{st}, we observe that μ_2^{st} is larger (0.007 versus 0.005) and has a lower p-value (0.032 versus 0.082) in the ECM than in the λ-model. This is harmonoius with it being the model containing explicit wealth variables and it is also here that we see a more improved fit of the model ($\Delta\bar{R}^2$ is 0.112 versus 0.075). Furthermore, it seems logical that the λ-model, in which not all consumers are assumed to consume out of wealth, is the one where the estimates of μ^2 are lower. Staying with the λ-model, we also observe that λ is smaller in the models augmented by ΔWD_t^j, implying that the fraction of life-cycle consumers is larger when taking changes in wealth dispersion into account. Finally, we know from the Chow breakpoint test in the λ-model that we cannot reject parameter stability for the whole sample. Given the profound effects of the deregulation of the credit markets in the 1980's and the severe recession of the early 1990's, this is a surprisingly positive result.

4 Conclusions and comments

In this paper I find that the wealth distribution is an omitted variable in aggregate consumption functions based on representative-agent models. When the dispersion of wealth is included, the associated parameter becomes significant, independent of model specification as well as the choice of distribution. The result implies a rejection of the proposition in Hall (1978) that consumption should follow a random walk, and thus support the prediction of the buffer-stock saving hypothesis (BSH). Furthermore, the development of the estimated wealth distribution for 1963-2000 seems to support both theories in their prediction that easing liquidity constraints will cause increased wealth dispersion. However, the BSH's prediction that the wealth distribution will converge rapidly to a steady state finds no clear support; rather the evidence after credit market deregulation in the early 1980's seems to support the PIH's prediction of a continuously growing dispersion.

The explanation for the results can be found in the observation that increased wealth dispersion has meant a marked increase in the group of households with negative net wealth, a group that according to the BSH has the highest marginal propensity to consume. The finding is also supported by research at the micro level, given that the results in Bjellerup (2005) suggest that the marginal propensity to consume out of net wealth, but not income, is strictly decreasing. Furthermore, we observe that the estimated effect is not only statistically significant. On average, increased wealth dispersion has added approximately $.05 - .07$ percentage points to yearly consumption growth between 1980 and 2000; a figure that is economically significant as well as plausible.

These findings have at least one important implication for economic policy. Lettau and Ludvigson (2004), reinforcing the recommendation in Bernanke and Gertler (2001), claim that policy makers interested in aggregate demand should disregard most changes in asset values, since they "are transitory and unrelated to consumer spending" (p.294). Contrary to their conclusions, the results here indicate that not only have the level and mean of net wealth to be considered, changes in the distribution must also be taken into account.

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Appendix

Data

The wealth and income data sets

The data on income has been obtained from Statistics Sweden's yearbook for each year in the sample and, given a varying administrative lag, the data for 1963-2000 has been obtained from the yearbooks for 1965-2003; series IF 20. The measure of income used is total earned and capital income for persons aged 20 or older. The information has been by group, i.e. the information available is on how many persons there are in each income group; the group reporting zero income has been deleted. The number of groups have changed over the years, although never fewer than 7 and never more than 22. An example is found below in Table 18.

Table 18: Income data, defined as total earned and capital income for persons aged 20 or older, for 1969
(Source: Statistics Sweden (various years))

Group limit[1]	Observations
0.01-2.49	171688
2.5-4.99	282532
5-9.99	898489
10-19.99	1276645
20-29.99	1120143
30-49.99	514781
50-	146020

[1] SEK, thousands

(18)

The data on wealth has been obtained from two sources. The first source is Jansson (2000) from which register data for 1978, 1983, 1985, 1988, 1990, 1992 and 1997 has been obtained. The information used is comprised of the median values for 13 percentiles for each year, thereby providing a rather detailed picture of the distribution as can be seen in Figure 5, p.67 for instance. The second source is Statistics Sweden's yearbook for each year in the sample. Given a varying administrative lag, the data for 1963-2000 has been obtained from the yearbooks for 1965-2003; series HE 20. The measure of wealth is assessed wealth.[50] The information have been of the same type as for income, i.e. by group. However, the information used has started at the current threshold value above which the household is obligated to pay capital tax. I was advised by Statistics Sweden that below this threshold value, the quality of the data is very poor. The number of groups has varied here as well between years, from 3 to 9. An example of the data from Statistics Sweden (various years) is found in Table 19 below.

Table 19: Wealth data, defined as net total household wealth, for 1998 (Source: Statistics Sweden (various years))

Group limit[1]	Observations
901-1000	50059
1001-1400	138038
1401-2000	90159
2001-3000	50788
3001-4000	16764
4000-	21555

[1] SEK, thousands

(19)

The macroeconomic time series

The data used in this study has been obtained from the National Institute of Economic Research (Konjunkturinstitutet), Statistics Sweden (Statistiska Centralbyrån) and the International Monetary Fund (IMF). The data set comprises yearly observations of consumer expenditure on non-durable goods and services (c_t), consumer expenditure on durable goods (cd_t), household disposable income (y_t), household labor income (y_t^{li}), gross fixed capital formation (q_t), the discount rate (r_t) from 1963 to 2000 and household financial (w_t^f) and net non-financial wealth (w_t^{nnf}) from 1970 to 2000. All series are in 1991 prices and

[50] Since the data on wealth stems from tax reports, not all wealth is accounted for due to taxation rules. Art, gold and stocks listed on smaller exchanges, for instance, are exempt. Furthermore, the method of calculation of the taxable value differ between asset classes. For instance, for real estate it is approximately 75% of the market value while it for stocks and bonds is the full market value.

have been deflated by the implicit total consumption price deflator.

Results from fitting distributions to income and wealth data

The fitting of a distribution to wealth was carried out in two steps for the skew-t distribution, and in one step for the gamma distribution. For income, both distributions were fitted in a single step.

As described earlier in the paper, fitting the skew-t to wealth data in the yearly compilation of tax reports, required fixing two of the parameters. To aid my decision on an appropriate approach, the register data of Jansson (2000) was used and a skew-t distribution was fitted for each year; the results from these estimations are found below in Table 20. Also, the information in Jansson (2000) helped me form a picture of the distribution of wealth, as described in section 2, which in turn was the basis for choosing the gamma and skew-t distributions.

Table 20: Parameter estimates from fitting a skew-t distribution to the Jansson (2000) data on wealth.

Year	ξ	ω	α	ν
1978	-40	105	11.8	0.78
1983	-80	207	10.3	1.34
1985	-41	151	3.31	0.99
1988	-63	195	2.86	0.99
1990	-58	185	2.67	0.89
1992	-56	183	2.39	0.98
1997	-62	187	2.47	0.91

(20)

Table 21: Parameter estimates, mean and WD_t^{st} from fitting a skew-t distribution to data on wealth for 1963 to 2000; the restrictions $\alpha = 5$ and $\nu = 1.01$ apply for all years (Statistics Sweden (various years)).

	Parameters			
Year	ξ	ω	$E[.]$	WD_t^{st}
2000	416	49.4	3539	7.41
1999	413	50.9	3627	7.48
1998	459	38.6	2897	6.95
1997	344	45.5	3218	7.30
1996	383	35.4	2616	6.84
1995	331	26.7	2020	6.31
1994	442	19.6	1683	5.75
1993	531	18.1	1673	5.63
1992	559	13.6	1419	5.18
1991	568	12.7	1373	5.09
1990	530	13.2	1366	5.36
1989	249	18.1	1393	6.18
1988	278	12.4	1063	5.56
1987	284	10.3	932	5.30
1986	273	11.8	1021	5.69
1985	325	5.57	677	4.28
1984	330	4.50	614	3.99
1983	312	5.29	646	4.47
1982	325	4.06	581	4.14
1981	152	9.05	723	5.94
1980	143	4.27	413	4.67
1979	133	4.32	406	4.93
1978	128	4.34	403	5.09
1977	117	5.80	483	5.89
1976	115	5.91	488	6.13
1975	112	5.71	472	6.27
1974	84.4	5.30	419	6.33
1973	72.8	5.57	425	6.63
1972	70.5	5.53	420	6.76
1971	62.3	5.31	398	6.80
1970	66.0	5.36	404	6.97
1969	52.8	4.72	351	6.82
1968	51.9	4.72	350	6.89
1967	47.1	4.76	348	6.94
1966	47.6	4.75	348	7.04
1965	45.8	4.76	346	7.17
1964	20.6	4.88	329	7.33
1963	58.9	2.88	241	6.34

$$WD_t^{st} = \ln\left(\frac{(\omega_t)^2}{(Defl_t)^2}\right) \tag{21}$$

Table 22: Parameter estimates, mean and WD_t^γ from fitting a gamma distribution to data on wealth for 1963 to 2000 (Statistics Sweden (various years)) ($WD_t^\gamma = \ln\left(\frac{Var_t^\gamma[.]}{(Defl_t)^2}\right)$)

	Parameters					
Year	α	β	γ	$E[.]$	$\sqrt{Var[.]}$	WD^γ
2000	0.1821	1431	−282	−21.2	611	12.44
1999	0.1810	1475	−279	−12.3	627	12.51
1998	0.1992	1197	−276	−37.8	534	12.21
1997	0.1932	1169	−274	−47.7	514	12.15
1996	0.1279	1378	−268	−91.9	493	12.11
1995	0.1388	1090	−265	−114	406	11.75
1994	0.1585	946	−258	−108	377	11.66
1993	0.3419	640	−253	−33.8	374	11.69
1992	0.3531	572	−239	−37.4	340	11.61
1991	0.3361	561	−225	−36.8	325	11.57
1990	0.2712	577	−196	−39.9	300	11.61
1989	0.5373	337	−169	12.2	247	11.41
1988	0.5062	322	−149	14.1	229	11.39
1987	0.5129	279	−125	18.2	200	11.24
1986	0.4109	318	−104	26.4	204	11.38
1985	0.3012	337	−85.4	16.1	185	11.29
1984	0.2609	332	−71.9	14.8	170	11.25
1983	0.1944	381	−59.5	14.6	168	11.38
1982	0.1911	353	−48.0	19.5	154	11.42
1981	0.4547	207	−38.3	55.8	140	11.41
1980	0.1831	221	−29.5	10.8	94.4	10.86
1979	0.1518	218	−22.2	10.9	84.9	10.88
1978	0.1273	233	−16.8	12.9	83.2	11,00
1977	0.1452	232	−15.0	18.7	88.5	11.34
1976	0.1240	256	−13.5	18.2	90.1	11.58
1975	0.1094	271	−12.2	17.4	89.6	11.78
1974	0.0692	281	−11.0	8.42	73.8	11.60
1973	0.0975	242	−10.0	13.6	75.6	11.84
1972	0.0901	248	−9.3	13.1	74.5	11.96
1971	0.0802	251	−8.7	11.4	71.2	11.99
1970	0.0832	235	−8.1	11.5	67.8	12.05
1969	0.0611	244	−7.7	7.26	60.4	11.92
1968	0.0570	259	−7.4	7.33	61.8	12.03
1967	0.0564	244	−7.3	6.50	58.0	11.94
1966	0.0580	240	−6.9	7.00	57.8	12.04
1965	0.0517	267	−6.5	7.30	60.7	12.26
1964	0.0391	271	−6.2	4.43	53.6	12.12
1963	0.0385	260	−6.0	4.05	51.0	12.09

(22)

Table 23: Parameter estimates, moments and ID_t^{st} from fitting a skew-t distribution to data on income for 1963 to 2000 (Statistics Sweden (various years))

	Parameters						
Year	ξ	ω	α	$\nu*$	$E[.]$	$\sqrt{Var[.]}$	ID_t^{st}
2000	79.7	131.4	2.30	4.29	198.0	135.4	0.467
1999	79.0	124.2	2.18	4.38	189.3	127.4	0.453
1998	69.4	123.8	2.37	5.11	177.2	116.5	0.432
1997	63.6	121.9	2.52	5.28	170.0	112.2	0.436
1996	63.8	115.6	2.44	5.42	163.8	105.8	0.417
1995	68.4	104.3	2.13	5.53	156.3	96.5	0.381
1994	66.1	99.7	2.16	4.86	152.5	97.1	0.406
1993	64.9	97.5	2.14	5.61	146.9	89.7	0.373
1992	63.1	94.4	2.16	5.62	142.6	86.6	0.369
1991	55.8	95.9	2.64	5.15	140.4	88.7	0.399
1990	50.0	94.9	2.83	6.86	130.6	78.8	0.364
1989	46.8	82.4	2.70	6.49	116.9	70.0	0.358
1988	40.6	75.6	2.83	6.60	105.1	63.5	0.365
1987	40.3	68.8	2.52	7.45	97.3	56.8	0.341
1986	31.8	67.0	3.15	7.72	88.5	53.4	0.364
1985	29.4	62.3	3.21	7.90	82.0	49.2	0.36
1984	23.6	60.7	3.86	8.68	75.1	46.2	0.379
1983	21.8	55.9	3.80	8.8	69.1	42.5	0.378
1982	18.7	53.5	4.31	9.31	64.1	39.9	0.388
1981	18.0	49.1	4.19	8.96	59.7	36.9	0.382
1980	14.0	47.2	4.97	9.77	54.1	34.6	0.409
1979	10.2	44.1	6.37	10.4	47.7	31.6	0.438
1978	7.59	42.5	7.98	10.1	44.0	30.4	0.477
1977	8.01	39.1	7.61	10.0	41.5	28.1	0.458
1976	6.37	36.7	8.64	10.0	37.9	26.3	0.482
1975	4.39	34.0	12.5	10.0	33.7	24.2	0.516
1974	3.3	30.2	20.3	10.0	29.4	21.4	0.53
1973	2.8	26.5	64.4	10.0	25.7	18.8	0.535
1972	2.69	24.8	88.5	10.0	24.1	17.6	0.532
1971	2.64	22.6	69.7	10.0	22.2	16.0	0.52
1970	2.53	21.2	12.1	10.0	20.8	15.1	0.527
1969	2.32	19.9	12.3	10.0	19.5	14.2	0.532
1968	2.21	19.0	12.6	10.0	18.6	13.5	0.528
1967	2.00	18.5	14.3	10.0	18.0	13.2	0.54
1966	1.61	17.0	97.8	10.0	16.3	12.0	0.541
1965	1.57	15.6	82.3	10.0	15.1	11.1	0.543
1964	1.37	14.6	71.4	10.0	14.0	10.3	0.542
1963	1.34	13.5	88.9	10.0	13.0	9.6	0.544

(23)

* = for 1963 through 1977 the restriction $\nu = 10$ applies

$ID_t^{st} = \frac{Var_t^{st}[.]}{\left(E_t^{st}[.]\right)^2}$

Table 24: Parameter estimates, moments and ID_t^γ from fitting a gamma distribution to data on income for 1963 to 2000 (Statistics Sweden (various years))

	Parameters				
Year	α	β	$E[.]$	$\sqrt{Var[.]}$	ID_t^γ
2000	2.29	87.6	200.4	132.5	0.437
1999	2.30	83.3	191.5	126.3	0.435
1998	2.25	79.5	178.9	119.3	0.444
1997	2.23	77.1	171.8	115.1	0.449
1996	2.31	71.6	165.4	108.8	0.433
1995	2.51	62.9	157.9	99.6	0.398
1994	2.49	62.0	154.4	97.9	0.402
1993	2.59	57.4	148.5	92.3	0.386
1992	2.62	54.9	144.1	89.0	0.381
1991	2.61	54.4	141.8	87.8	0.384
1990	2.63	50.1	131.9	81.3	0.380
1989	2.71	43.6	118.1	71.7	0.369
1988	2.68	39.7	106.4	65.0	0.373
1987	2.76	35.7	98.3	59.2	0.363
1986	2.67	33.4	89.1	54.5	0.375
1985	2.70	30.6	82.6	50.3	0.370
1984	2.61	29.0	75.7	46.9	0.383
1983	2.50	27.8	69.6	44.0	0.400
1982	2.48	26.1	64.7	41.1	0.404
1981	2.51	24.0	60.2	38.0	0.399
1980	2.38	23.0	54.6	35.4	0.421
1979	2.30	21.1	48.4	31.9	0.435
1978	2.10	21.2	44.6	30.8	0.476
1977	2.30	18.2	41.9	27.6	0.435
1976	2.19	17.5	38.3	25.9	0.457
1975	2.09	16.3	34.2	23.7	0.478
1974	2.06	14.5	30.0	20.9	0.485
1973	1.97	13.2	25.9	18.5	0.508
1972	1.94	12.5	24.3	17.5	0.516
1971	1.85	11.9	22.1	16.2	0.540
1970	1.94	10.8	20.9	15.0	0.517
1969	1.92	10.2	19.6	14.1	0.521
1968	1.92	9.7	18.7	13.5	0.520
1967	1.93	9.4	18.1	13.	0.518
1966	1.79	9.1	16.3	12.2	0.560
1965	1.77	8.5	15.0	11.3	0.564
1964	1.75	8.0	14.0	10.6	0.573
1963	1.76	7.4	13.0	9.8	0.567

$ID_t^\gamma = \frac{Var_t^\gamma[.]}{\left(E_t^\gamma[.]\right)^2}$

(24)

Essay 4

Does Consumption Function Concavity Vary with Asset Type?*

Mårten Bjellerup†

May 9, 2005

Abstract

The optimal intertemporal consumption problem has been largely addressed by using linear or approximately linear consumption functions, based on the Euler equation. However, by introducing labor income uncertainty, as in the buffer-stock saving version of the permanent income hypothesis, the consumption function becomes strictly concave for a wide class of models. Using grouped Swedish household level data for 1999-2001, I find evidence suggesting that the marginal propensity to consume out of current resources, i.e. current income plus net wealth, is strictly decreasing in net wealth and in current resources, while approximately constant in income. What could appear to be a contradiction is explained by the joint distribution of net wealth and income, where income is not strictly increasing in net wealth. The results thus suggest that consumption function concavity varies with asset type.

*I am indebted to Lena Bjerke, Kjell Jansson and several others at Statistics Sweden for their help as well as to Jonas Månsson for his generous help with SPSS. Bengt Assarsson's feedback at the final seminar is very much appreciated and last but not least, I am very grateful for Håkan Locking's continuous support and encouragement.

†School of Management and Economics, Växjö University, SE-351 95, Växjö, Sweden; *e-mail*: marten.bjellerup@ehv.vxu.se.

1 Introduction

Ever since the appearance of the prominent paper of Hall (1978), the optimal intertemporal consumption problem has been largely addressed by using linear or approximately linear consumption functions, based on the Euler equation.[1] The standard perfect certainty and certainty equivalent versions of the consumption decision imply a marginal propensity to consume that is unrelated to the level of household wealth. However, introducing labor income uncertainty makes the consumption function strictly concave, as noted by, e.g., Zeldes (1989) and proved by Carroll and Kimball (1996), meaning that the implication of Hall (1978) that consumption should follow a random walk at the household level, no longer holds true. Furthermore, the strict concavity implies a marginal propensity to consume that declines with the level of current resources, defined as current income and non-human wealth.

Carroll and Kimball (1996) see this step forward, in effect, as a step backward against the backdrop of Keynes (1936), arguing explicitly for a concave consumption function. Developing a version of the permanent income hypothesis with a model in which there are patient and impatient consumers yielding buffer-stock saving behavior (BSH hereafter), Carroll (2001) accounts for the above-mentioned findings and argues that this version "...matches Milton Friedman's (1957) original intuitive description of the permanent income hypothesis much better than the subsequent perfect foresight and certainty equivalence models did." (Abstract).

Several recent papers contain microeconomic empirical evidence on the concavity of the consumption function; e.g. McCarthy (1995), Ekman (1996), Lusardi (1996), Souleles (1999) and Parker (1999).[2] None of the papers I am aware of, however, consider the type of specific questions on the nature of the consumption function's non-linearity, addressed in this paper. The earlier liter-

[1] As Carroll and Kimball (1996) notes: "Of the 25 household-level studies summarized in the recent survey by Browning and Lusardi (1996), only two allow for a nonlinear consumption function..." (p.982, note 5)

[2] Four of these find evidence in favor of concavity while the fifth (Ekman (1996)), using a less direct test, finds no evidence.

ature following the influential publication of Friedman (1957), also found some evidence of concavity.[3] However, the buffer-stock hypothesis only proposes that consumption is concave in current resources, not that the propensity to consume could vary differently with net wealth and income, the two components of current resources.

At the aggregate level, concavity means that the wealth distribution should be included as an explanatory variable in aggregate consumption Euler equations. The results in Bjellerup (2005), using Swedish data for 1963-2000, support this prediction by the BSH. The explanation put forward rests on the observation that the probability mass in the left tail of the net wealth distribution has changed markedly over time. The assumption that this group of consumers has the highest marginal propensity to consume (as in the BSH) would explain the results. This paper will address the more general issue of the concavity of the consumption function using Swedish household data for 1999-2001, thereby also testing the proposition in Bjellerup (2005).

To preview, the results support the view that the marginal propensity to consume out of current resources, MPC_{CR}, is strictly decreasing in net wealth and in current resources, while approximately constant in income. Thus, the concave consumption function suggested by Carroll (1997) is supported, while the type of representative-agent model as proposed in Hall (1978) is rejected. The result that households with negative net wealth consume more than slightly more wealthy households, with respect to net wealth, is probably due to a higher average disposable income for the first group. Taken together, the results seem to support the finding in Bjellerup (2005); the increase in the dispersion of net wealth due to the increase of households with negative net wealth has had a positive effect on growth between 1980 and 2000, while changes in the distribution of income have had no significant effect.

The paper is organized as follows. Section 2 briefly describes the theoretical background while the following section presents the data and the creation of the

[3] See, e.g., Souleles (1999) for a brief account or Mayer (1972) for a summary of these early findings.

data set. Section 4 starts with the specification of the chosen linear consumption function and then proceeds with estimation and a test of the hypothesis of linearity. Section 5 comments and concludes.

2 Theoretical background

The basic representative-agent, perfect certainty and certainty equivalent consumption function based on the Euler equation, as referred to by Carroll (1997), can be illustrated, for instance, using a derivation of the two-period consumption function under point expectations, thus following most consumption literature; see e.g. Romer (1996) and Muellbauer and Lattimore (1995).

A representative individual is assumed to have intertemporally additive preferences, resulting in an instantaneous utility (aka felicity) function

$$U = u(c_t) + \frac{1}{1+\delta} u(c_{t+1}), \tag{1}$$

where $u' > 0$, $u'' < 0$ and δ is the individual's subjective discount rate. A special case of equation (1) is its homothetic form

$$U = c_t^{-\rho} + \frac{1}{1+\delta} c_{t+1}^{-\rho}, \tag{2}$$

where the elasticity of substitution $\sigma = \frac{1}{1+\rho}$ measures consumption's responsiveness to the opportunity cost, that is to the inverse of the expected real rate of return. Assume further that the representative individual neither receives nor desires to leave any inheritance, and that it is possible to save or borrow at an exogenous interest rate, r. Hence, we have the budget constraint

$$A_t + y_t + \frac{1}{1+r_t^e} y_{t+1} = c_t + \frac{1}{1+r_t^e} c_{t+1} \equiv W \tag{3}$$

where A_t are assets, y_t is income, r_t^e is the expected real interest rate and W is life-cycle wealth. Maximizing eq.(2) subject to eq.(3) using the Lagrangian method yields the first order conditions

$$\frac{\delta L}{\delta C_t} = 0 = -\rho c_t^{-(\rho+1)} + \lambda$$
$$\frac{\delta L}{\delta C_{t+1}} = 0 = -\rho c_{t+1}^{-(\rho+1)} + \frac{1}{1+r_t^e}\lambda$$

which, after rearranging, yields the Euler equation

$$c_t^{-(\rho+1)} = \frac{(1+r_t^e)}{(1+\delta)} c_{t+1}^{-(\rho+1)}. \tag{4}$$

Combining the Euler eq.(4) with the identity for W, eq.(3), we obtain the solved-out form of the consumption function:

$$c_t = \frac{1}{\kappa_t}\left(A_t + y_t + \frac{1}{1+r_t^e} y_{t+1}\right), \tag{5}$$

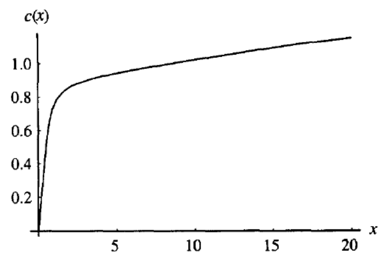
where κ_t is the inverse of the marginal propensity to consume out of life-cycle wealth.

Omitted up to this point, because of the two-period framework, is perhaps the most classic implication of the model; given a normal income pattern, the wealth trajectory will exhibit a hump-shape. Thus, A_t and c_t in eq.(5) will be age dependent. One other feature of the derivation above (in this context very important), is that the consumption function is linear.

The finding in Zeldes (1989) using numerical methods, that introducing income uncertainty into the problem made the consumption function concave, is given a more analytical explanation in Carroll and Kimball (1996). In fact, they prove that the consumption function becomes strictly concave for a wide class of problems. Besides offering a plausible theoretical explanation for these results, Carroll (1997) argues that a buffer-stock version of the permanent income hypothesis is not only a better model, but also that it "represents a close approximation" (Carroll (1997), p.4) of the original ideas of Friedman (1957).

Briefly, Carroll (1997) argues as follows: Define c as the ratio of consumption C to permanent labor income wL and x as the ratio of current resources (nonhuman wealth plus current income X) to permanent labor income, for a microeconomic consumer for whom interest rates, wages and labor supply are fixed at their steady-state levels.[4] Then, $c(x)$ is globally concave but relatively smooth and almost linear at large values of x, as can be seen in Figure 1 below.[5]

Figure 1: Stylized illustration of the concavity of the consumption function, according to the buffer-stock saving hypothesis (Source: Carroll (2000), reprinted with permission, © 2000 by the American Economic Association.).



For our purposes, the observation that $c(x)$ is highly non-linear for lower values of x is important. What is to be tested is the linearity of the consumption function, or equivalently, whether the marginal propensity to consume out of current resources, MPC_{CR}, is constant or not. In other terms, can we find evidence in support of the claim in Carroll (1997) that the perfect foresight and certainty equivalent models are incorrect when they assume linear consumption functions?

The contribution of this paper is twofold. First to be addressed is the more precise question of in what variable(s) - net wealth, income and/or current resources - consumption is concave. It is possible that MPC_{CR} is, e.g., strictly decreasing in one variable, but not in another; the outcome depends on the joint distribution of net wealth and income. Second, the BSH has previously only been tested using US data, why a test using Swedish data is interesting given the different characteristics of the social security systems in the two countries. It

[4] For a more thorough exposition, see Carroll (1997).

[5] Note that the x-axis lower bound not should be interpreted literally; x can take negative values depending on whether or not liquidity constraints are present (Carroll (2001)).

is conceivable for instance, that an almost all-encompassing Swedish-type social security system would induce less buffer stock saving behavior.

The database used in this paper contains detailed information on income and wealth, but no measure of consumption which thus must be imputed. A presentation of the data with the construction of the data set follows next.

3 Data

The data is obtained from Statistics Sweden and their cross-sectional data base HEK, which consists of a representative sample, drawn each year, of approximately 35,000 individuals. The sample contains information on income with a wide range of background variables; the information originates from surveys and register data. This paper utilizes data from 1999-2001 against the backdrop of a special wealth supplement that only exists for these three consecutive years. The wealth supplement contains detailed register data covering a wide range of assets, including low-level disaggregation of financial assets and debt as well as numerous categories of real estate holdings. Next follows a description of how the data set is constructed.

3.1 Construction of the data set

After aggregation of all individuals into households, 14,465 in 1999, 12584 in 2000 and 12710 in 2001, the monetary variables for 1999 and 2001 are deflated using the implicit total consumption deflator with 2000 as base year.[6] Next comes the question of how to impute consumption. Basically, there are two ways to obtain micro data on consumption. The first is to ask the households in the sample to keep records of consumption. The second is to calculate imputed consumption using measures of income and wealth, see e.g. Hurd (1989). Since the current database does not contain a variable based on records of consumption, the latter approach is chosen, with the proxy for consumption calculated

[6] A more detailed desricption of the construction of the data set is found in the appendix, p.135.

as

$$C \equiv \text{labor income + transfers + capital income - net savings.} \tag{6}$$

In terms of the available data, we can reformulate eq.(6) as

$$C_{t+1} = Y_{t+1} - (NW_{t+1} - (1+r)NW_t), \tag{7}$$

where C_{t+1} is consumption in period $t+1$, Y_{t+1} is the household's disposable income in period $t+1$, NW_{t+1} is net wealth in period $t+1$ and $(1+r)NW_t$ is the value at time $t+1$ of the stock of net wealth held at time t. Thus, $(NW_{t+1} - (1+r)NW_t)$ represents net savings for the period $t+1$. Given the large ex-post return differences between asset classes, an advantage of the HEK data base is the level of disaggregation of wealth and debt enabling us to get a better estimate of $(1+r)NW_t$. As an example, it is possible to take into account the geographic variability of changes in house prices as well as whether the holdings are in the form of mutual funds based on stocks, fixed income instruments or a mix between the two. Finally, since we need NW_{t-1} to calculate C_t, we lose one observation, meaning that the final data set covers 2000 and 2001.

3.2 Independent samples in HEK

In HEK, a new sample is drawn each year, but observations on wealth for adjacent years are needed to calculate the imputed measure of consumption. Given the large number of observations, the solution is found using household grouping; i.e. we create groups of households, according to certain characteristics, that have similar wealth properties over time. One consideration when forming household groups, is that as the number of observations increases within the groups, the number of household groups decreases. Thus, there is a trade-off between the size of the groups and the number of the groups. Another consideration is, of course, what characteristics to use when forming the groups.

Given the aim of similar wealth properties, the two main characteristics used are disposable income and age. A benefit from using these characteristics is that they take into consideration the life-cycle aspect of household behavior. Also, to create an alternative grouping characteristic, the two will be used together with a dummy variable for place of residence.[7]

The first household-grouping, G1, contains 20 classes in disposable income and 15 classes in age, totaling 300 household-groups. The second household-grouping, G2, contains 10 classes in disposable income, 10 classes in age and 3 classes in place of residence, also totaling 300 household-groups. From this point, increasing the number of classes, thereby increasing the number of household-groups, yields very few households in the groups with the fewest observations.[8]

The new observations, i.e. the household-groupings, will be characterized using the median value of all continuous variables.[9] This, of course, affects the calculation of imputed consumption, as described in section 3.1. Now, we instead have

$$\overline{C}_{gi,t+1} \equiv \overline{DI}_{gi,t+1} - \left(\overline{NW}_{gi,t+1} - (1+r)\overline{NW}_{gi,t}\right)$$

where the bars denote median and the subscript gi denotes "group individual". Henceforth, the bars will be omitted for notational convenience.

4 Empirical analysis

Against the backdrop of cross-sectional data and the theoretically derived optimal consumption rule under perfect foresight, eq.(5), the econometric specifi-

[7] Previewing the reults, the choice of grouping does not affect any of the major conclusions in the paper. As for the variable 'place of residence', it takes the value 1 if the household is located in large city, 2 if the household is located in southern Sweden but not in a large city, and 3 if the houshold is located in the north of Sweden.

[8] With 300 household-groups, the smallest group consists of approximately 10 observations. Increasing the number of households causes the number of households in the smallest group to drop to approximately 3.

[9] Besides follwing earlier litterature, the choice is based on the effect of skewness and kurtosis of the distributions, especially for net wealth. Using the mean, the characteristics of the original data set are less well described by the household-grouping observations.

cation should be linear and it should be possible to test the MPC for current resources, i.e. current income and wealth. Furthermore, we want to estimate the level of consumption, as suggested by the BSH and Figure 1. A specification that takes these considerations into account is

$$C_k = \alpha + \beta_1 Y_k + \beta_2 NW_k + \varepsilon_k, \tag{8}$$

where k denotes group individual gi at time t.[10] The problem, shared with all other papers, of not being able to observe future income is here solved indirectly through the household-grouping made earlier, thereby taking the life-cycle perspective into account in combination with the assumption that future income is a function of current income.[11]

4.1 Estimation

Given the construction of the data, it is necessary to take into account the heteroscedasticity introduced by the creation of the household-groups.[12] However, as could be suspected, there are other origins of heteroscedasticity. Using a Goldfeld-Quandt test and ordering the observations by the size of Y, we clearly see that there are still problems with heteroscedasticity, despite the group size correction.[13] In order for inference to be possible, a consistent estimate of the covariance matrix allowing for heteroscedasticity will be used, as in White (1980).[14] Also, to address endogeneity as well as measurement problems, GMM estimation will be employed. The set of instruments used in all estimations consists of paid taxes, the number of household members and a dummy for the intensity of work.[15] There is abundant literature on outliers in micro eco-

[10] However, net wealth is measured as end of period stock, which in line with most previous work is assumed to affect the consumption decision in the next period.

[11] Given the use of household-grouping, it is plausible that the logarithm of income, y, obeys $y = g + y_{t-1}$, where g is the growth rate of income.

[12] See the appendix p.134 for a description of this problem.

[13] The test statistics from the GQ test are between 5 and 30, corresponding to p-values of virtually zero. The option of taking the logarithm of the series does not exist in this context as net wealth can take negative values.

[14] In order to avoid correcting the correction, only the White (1980) matrix will be utilized.

[15] The main conclusions in the paper are not sensitive to the selection of instruments; different measures of debt as well as the sex of the household head, have been utilized in various combinations with the previously mentioned instruments, without any significant changes to

nomic data, see e.g. Flavin (1991) and Hoynes and McFadden (1994), a problem which is handled here through deletion; an outlier is defined with respect to the assumed data-generating process, meaning that all observations with a standardized residual greater than 3 standard deviations have been deleted.[16] The results from GMM estimation of the consumption function (eq.(8)) can be seen in Table 9 below.[17]

Table 9: Results from GMM estimation of eq.(8)

α	β_1	β_2	$\bar{R}^2$	J
−23197	1.17	−0.108	0.398	0.662
[12871]	[0.063]	[0.029]		

(9)

Standard errors are in brackets
J: p-value from test for over-identifying restrictions
Eq.(8): $C_k = \alpha + \beta_1 Y_k + \beta_2 NW_k + \varepsilon_k$

The point estimates do not seem unreasonable. Comparing them to previous results we find that they are somewhat larger than what Ekman (1996) found using Swedish data for 1982-1991.[18] Next, we turn to the augmented model, now including dummies, to see whether the constant, income and net wealth parameters vary by level of income, net wealth and/or current resources.

4.2 Augmentation

The aim of this section is twofold. It is of general interest to try to construct an empirical reciprocal of the BSH consumption function in Figure 1, p.116, as well as to try to determine how the concavity relates to the aggregate (current resources) and the disaggregate (disposable income and net wealth). At a more specific level, I am interested in how results from the household level approach relate to the results in Bjellerup (2005), where aggregate estimation suggested that the distribution of net wealth and its development over time are

the results.

[16] This choice has a very limited impact on the results and do not change the major conclusions in the paper.

[17] Given that the results and main conclusions in the paper are unaffected by the choice of grouping, only the results for the G1 grouping will be reported.

[18] However, Ekman (1996) used a disaggregated wealth variable, why the results are not directly comaprable.

of importance for consumption.

The BSH suggests that the MPC differs between groups of people and that it changes with the level of current resources (CR), i.e. current income plus net wealth. However, the results in Bjellerup (2005) possibly indicate that consumption is concave in net wealth, but not in income. Thus, it is of interest to set up a test where the intercept and the MPCs, i.e. α, β_1 and β_2 in eq.(8), are allowed to vary (in intervals) with the level of the variable of interest, i.e. Y, NW and CR respectively. Given a total of 600 observations, a choice of 6 non-overlapping intervals ($i = 1, ..., 6$) yields 100 observations per interval and using interval dummies, we get the sought after parameters: α_i, $\beta_{1,i}$ and $\beta_{2,i}$. For each variable of interest (x), the observations (k) are ordered increasingly; why, e.g., $x = Y$ and $i = 1$ corresponds to the 100 observations with lowest income and $x = NW$ and $i = 6$ to the 100 observations with the highest net wealth.

The baseline model we want to estimate for different x's, is consequently

$$C_k = \sum_{i=1}^{6} \alpha_i + \sum_{i=1}^{6} \beta_{1,i} Y_k + \sum_{i=1}^{6} \beta_{2,i} NW_k + \varepsilon_k. \tag{10}$$

where the last interval ($i = 6$) is used as the reference interval in estimations since it is not possible to use 6 dummies for 6 intervals, as the last dummy will be a linear combination of the previous dummies. Thus, when checking for interval varying parameters, the joint hypothesis to be tested for interval i is $\alpha_i = \alpha_6$, $\beta_{1,i} = \beta_{1,6}$ and $\beta_{2,i} = \beta_{2,6}$.

Apart from the baseline model, a specification where consumption only depends on current resources will also be estimated. The BSH-model we want to estimate for different x's, is consequently

$$C_k = \sum_{i=1}^{6} \alpha_{BSH,i} + \sum_{i=1}^{6} \beta_{BSH,i} CR_k + \varepsilon_k. \tag{11}$$

Although this specification greatly reduces the fit of the model, it yields two things. First, it is a test of how dependent the results are on the chosen functional form. Second, it gives us the results when the BSH dictates functional form.

To be able to compare the results with the BSH's stylized consumption function, projected consumption for each interval, $C^*_{x_i}$, is calculated as

$$C^*_{x_i} = \alpha_i + \beta_{1,i}\bar{Y}_{k,x_i} + \beta_{2,i}\overline{NW}_{k,x_i}, \tag{12}$$

for the baseline model and as

$$C^*_{x_i} = \alpha_{BSH,i} + \beta_{BSH,i}\overline{CR}, \tag{13}$$

for the BSH model, where $\bar{Y}_{k,x_i}$, $\overline{NW}_{k,x_i}$ and $\overline{CR}_{k,x_i}$ are the means of disposable income, net wealth and current resources in interval x_i, respectively. Also, the marginal propensity to consume out of current resources, MPC$_{CR}$, is calculated between each interval; $\text{MPC}^{x_i}_{CR} = \frac{\left(C^*_{x_{i+1}} - C^*_{x_i}\right)}{\left(CR_{x_{i+1}} - CR_{x_i}\right)}$. Augmentation and estimation to uncover potential concavity follows next.

4.2.1 MPC$_{CR}$ and its relation to disposable income

First, we turn to the results from estimation of eq.(10) and eq.(11) with $x = Y$, i.e. the observations are ordered according to the level of disposable income.[19] The results are found in Figure 2 and Table 14 below.

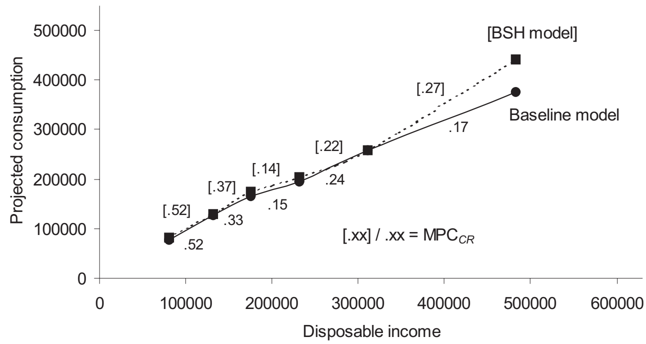
Table 14: Test statistics and critical values from testing parameters for equality across intervals in the Baseline (eq.(10)) and BSH (eq.(11)) models for $x = Y$.

Estimated equation	Interval (i) 1	2	3	4	5	c.v.	
Baseline, eq.(10)	10.7*	5.54	6.64	3.27	14.7*	7.81	(14)
BSH, eq.(11)	68.4*	82.0*	88.6*	80.6*	51.9*	5.99	

* indicates rejection at 5% significance level

[19] See the appendix pp.137-138 for a fuller presentation of the results from estimation of the augmented models.

Figure 2: Results from estimation of the Baseline (eq.(10)) and BSH (eq.(11)) models for $x = Y$.



Not immediately evident in the graph but in the table, is the fact that only two of the five sets of interval dummies are significant in estimation of the Baseline model. Furthermore, approximate linearity is suggested by the rather stable MPC's as well as the almost linear development of projected consumption. Taken together, when disposable income is conditioned upon the support for nonlinearity in general and concavity in particular is rather weak.

4.2.2 MPC$_{CR}$ and its relation to net wealth

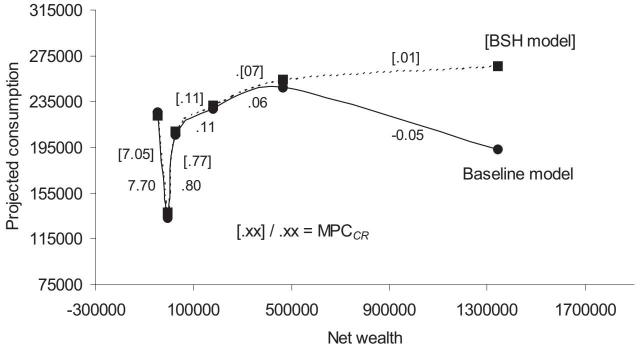
Figure 3 and Table 15 below presents the results of estimation of eq.(10) and eq.(11) with $x = NW$, i.e. when the sample has been ordered according to the value of net wealth.

Table 15: Test statistics and critical values from testing parameters for equality across intervals in the Baseline (eq.(10)) and BSH (eq.(11)) models for $x = NW$.

Estimated	Interval (i)						
equation	1	2	3	4	5	c.v.	
Baseline, eq.(10)	37.1*	40.1*	34.6*	32.0*	23.2*	7.81	(15)
BSH, eq.(11)	50.4*	85.6*	130*	70.0*	35.8*	5.99	

* indicates rejection at 5% significance level

Figure 3: Results from estimation of the Baseline (eq.(10)) and BSH (eq.(11)) models for $x = NW$.



In contrast to the previous section when Y was conditioned upon, here, the dummies are jointly significant for all intervals. Furthermore, clear support for concavity is found as MPC$_{CR}$ is strictly decreasing in net wealth. As for the level of projected consumption, we see a distinct negative spike for a value of net wealth of approximately 0, after which consumption rises quite rapidly to its previous level. When compared to the previous graph, Figure 2, the difference is striking. The third and last augmentation addresses the question of what happens when the aggregate measure, current resources, is considered.

4.2.3 MPC$_{CR}$ and its relation to current resources

As for the BSH and its view that the MPC changes with current resources, Figure 4 with the plot of projected consumption for each interval and model and Table 16 provide valuable information.

Figure 4: Results from estimation of the Baseline (eq.(10)) and BSH (eq.(11)) models for $x = CR$.

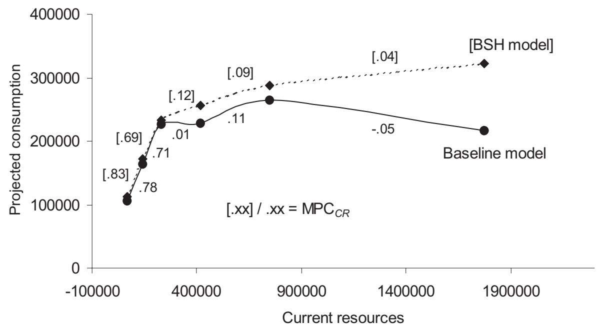


Table 16: Test statistics and critical values from testing parameters for equality across intervals in the Baseline (eq.(10)) and BSH (eq.(11)) models for $x = CR$.

Estimated equation	Interval (i) 1	2	3	4	5	c.v.	
Baseline, eq.(10)	56.1*	22.9*	26.3*	29.5*	33.5*	7.81	(16)
BSH, eq.(11)	16.3*	18.9*	12.2*	8.71*	8.76*	5.99	

* indicates rejection at 5% significance level

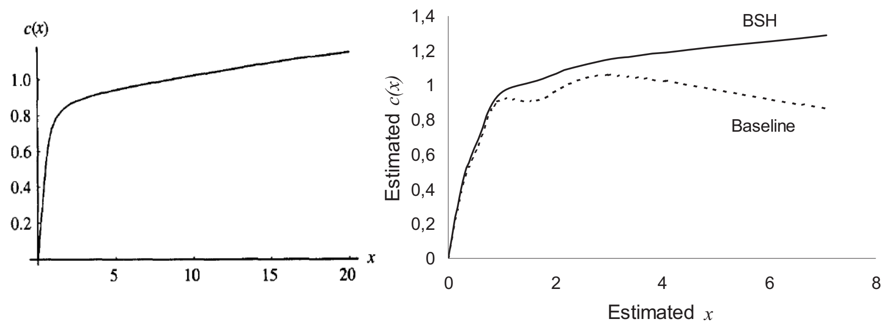
Once more, the dummies are jointly significant for all intervals in both models thus implying non-linearity. Although the pattern is not as clear as in the previous graph, the development of the MPC$_{CR}$'s suggests concavity. The first two are approximately .7 while the last three are rather close to 0. As for the level of projected consumption, it rises rapidly at first after which it levels off.

4.3 Results

The BSH states that consumption is concave in current resources, i.e. income and net wealth. However, given the possibility to disaggregate current resources, it is possible to ask the question whether the marginal propensity to consume out of current resources, MPC_{CR}, is strictly decreasing in net wealth, income and/or current resources. The answer, according to the estimations in this paper, is that MPC_{CR} is strictly decreasing in net wealth, almost strictly decreasing in current resources while it is approximately constant in disposable income. Taken together, the results seem to offer support for the buffer-stock saving version of the life-cycle hypothesis (Carroll (1997)), which consequently means that the rational expectations version of the life-cycle hypothesis as presented in Hall (1978), is rejected.

As for the question of an empirical reciprocal to the stylized theoretical BSH consumption function, we first have to approximate permanent labor income (wL), remembering that $x = \frac{CR}{wL}$ and $c = \frac{C}{wL}$. Using the average disposable income in 2001 as a proxy for wL, we can express the empirical consumption function in Figure 4 in correspondence with the BSH's theoretical version. It seems reasonable to use both the Baseline and the BSH models with slightly more emphasis on the latter, given the aim. Extrapolating the estimated empirical consumption functions to the point $(0, 0)$, we get the result as seen in Figure 5 below.

Figure 5: Consumption functions. Left panel: stylized theoretical according to BSH (Source: Carroll (2000), reprinted with permission, © 2000 by the American Economic Association.). Right panel: empirically estimated, based on the Baseline (eq.(10)) and BSH (eq.(11)) models.

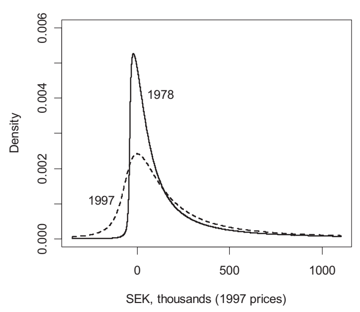


Although not a spitting image, the empirical estimation in the right panel and especially the BSH model, bears a close resemblance to the theoretical version in the left panel. We also see that the kink, admittedly influenced by the proxy for permanent labor income, seem to be in a similar position in the two panels, close to the point $(.9, .9)$.

The quite large changes in MPC_{CR} for low values of NW and CR respectively, as seen in Figure 3 and Figure 4, should not be considered an anomaly *per se*. Rather, such jumps should generally be expected for functions having a kink point. In particular, Carroll and Kimball (2001) states that this "will occur (for example) in the transition between levels of wealth where a constraint is not binding and where it is binding" (p.22). Given the low levels of net wealth and current resources for which the jumps in MPC_{CR} are present in this paper, an explanation involving liquidity constraints does not seem implausible.

Furthermore, the results in Bjellerup (2005), that the distribution of wealth, but not income should be included in linear aggregate consumption functions, seem to be supported. As for wealth, Bjellerup (2005) suggests that the distributional developments seen in Figure 6 below would mean, *ceteris paribus*, a

Figure 6: Estimated net wealth distributions for 1978 and 1997 (source: Bjellerup (2005)).



boost to aggregate consumption.[20] As can be seen, there is considerably less probability mass in the SEK $0 - 200,000$ range in 1997; most of this is found

[20] Given the very low rate of inflation during the last years of the 1990's, the fact that this paper uses 2000 as base year while the data in Figure 6 uses 1997, is attached little weight.

below SEK 0, and a smaller part above SEK $200,000$. Thus, households with negative net wealth have to consume more than those households with slightly positive net wealth for the proposition to be true.[21] The results depicted in Figure 3, p.125 suggest that this might be the case. The second and third data points in Figure 3, those with lower projected consumption, correspond to values of net wealth of $-5,000$ and $25,000$ respectively, while the adjacent first and fourth data points, correspond to values of net wealth of $-45,000$ and $175,000$ respectively, implying higher projected consumption. Given the approximate linearity of the consumption function with respect to income and the limited support for a changing MPC_{CR}, it is not surprising that Bjellerup (2005) does not find any effect from changing income dispersion at the aggregate level.

In previous research, the approach in Parker (1999) and Gourinchas and Parker (2001), is closest to the approach chosen here. Their finding for the US, that it is the combination of low income and little liquid wealth that yields high MPC_{CR}'s, contrasts somewhat with the findings in this paper, given that the importance of the level of income finds little support.

So, why is it that for consumption behavior, the distribution of net wealth dominates the distribution of disposable income in the Swedish case? One possible explanation for the importance of net wealth consist of two related observations. First, the joint distribution of net wealth and disposable income reveals that average income for the group of households with negative net wealth, is approximately 10% higher than for the group of households with little or moderately positive net wealth ($0 < NW < 75,000$).[22] Second, the group of households with negative net wealth has a considerably higher average debt incurred for higher education than all other groups; slightly more than twice that of the average for the whole sample. Relating to the result in Bjellerup (2005) where the dispersion of wealth was found to have increased since the early 1980's, the

[21] It should be noted that, of course, the results in this paper, based on data for 1999-2001, are limited when being used for explaining results in time series estimation based on data for 1963-2000.

[22] Reassuringly, this a property that not is due to the process of creating household-groups, the conclusion holds for the original data set as well.

increase could possibly arise from more than deregulation of credit markets. The boom witnessed in higher education participation in combination with more generous rules regarding eligibility for borrowing for higher education, has meant a marked increase over time for debt incurred for higher education.[23] Thus, the relaxation of liquidity constraints that most likely has been important for the increased dispersion of wealth, has possibly two sources. As for the lack of importance of the distribution of income, in contrast to the US findings in Parker (1999), this could perhaps be explained by the wage structure and progressive tax system in Sweden resulting in a more compressed distribution for disposable income.

Summing up, the main results in this paper support the buffer-stock saving version of the life-cycle hypothesis and its main prediction - the consumption function is concave, not linear. The MPC_{CR} is strictly decreasing in net wealth, almost strictly decreasing in current resources, and the projected consumption function (Figure 5, p.127) bears a close resemblance to the theoretical stylized consumption function of Carroll (2000).

5 Conclusions and comments

The buffer-stock saving version of the permanent income hypothesis (Carroll (1997)) specifies that consumption is strictly concave in current resources, i.e. net wealth plus current income. The results in this paper, using Swedish grouped household level data for 1999-2001, support the view that the marginal propensity to consume out of current resources is strictly decreasing in net wealth and in current resources, while it is almost constant in disposable income. Put differently, consumption function concavity is dependent on asset type. The result that households with negative net wealth consume more than slightly richer households, with respect to net wealth, is probably due to the average disposable income of the first group being higher. Taken together, the results

[23] After a reform in 1977, all students enrolled in post-upper secondary school educations became eligible for state sponsered borrowing.

seem to support the finding in Bjellerup (2005); the increase in the dispersion of net wealth, due to the increase of households with negative net wealth, has had a positive effect on growth between 1980 and 2000 while changes in the distribution of income has had no significant effect.

Taken together, the results strongly support a rejection of the type of representative agent model suggested by Hall (1978). Turning from theory to policy, the view of Greenspan (1998) that "[w]e need also to examine trends in the distribution of wealth, which, more fundamentally than earnings or income, represents a measure of the ability of households to consume", thus finds support.

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Appendix

Estimation from grouped data and heteroscedasticity

The main concern when using grouped data is that heteroscedasticity becomes a problem when creating the groups. Following Kmenta (1971), we consider a simple linear regression. Suppose the n sample observations are divided into G groups. Then, let n_1 be the number of observations in the first group, n_2 be the number of observations in the second group, and so on. Thus, we must have

$$\sum_{g=1}^{G} n_g = n.$$

Let us denote ith observation in the gth group by the double subscript ig, so that the regression equation can be written as

$$Y_{ig} = \alpha + \beta X_{ig} + \varepsilon_{ig} \qquad (i = 1, 2, ..., n_g; g = 1, 2, ..., G). \tag{17}$$

Suppose now that instead of being given a complete enumeration of all observations in each group, we are only given their numbers and the mean values (or totals) of X and Y, presented as follows:

Group	Number of observations	Mean of X	Mean of Y
1	n_1	$\overline{X}_1$	$\overline{Y}_1$
2	n_2	$\overline{X}_2$	$\overline{Y}_2$
⋮	⋮	⋮	⋮
G	n_g	$\overline{X}_G$	$\overline{Y}_G$

where

$$\overline{X}_g = \frac{1}{n_g}\sum_{i=1}^{n_g} X_{ig} \quad \text{and} \quad \overline{Y}_g = \frac{1}{n_g}\sum_{i=1}^{n_g} Y_{ig} \quad (g = 1, 2, ..., G).$$

Let us now take the regression equation (eq.(17)) and condense it by averaging over all observations within each group. In this way we obtain

$$\overline{Y}_g = \alpha + \beta\overline{X}_g + \overline{\varepsilon}_g \quad (g = 1, 2, ..., G). \tag{18}$$

That is, we are replacing the original n observations with a smaller number of G group means. First, we note that

$$E(\overline{\varepsilon}_g) = E\frac{1}{n_g}(\varepsilon_{1g} + \varepsilon_{2g} + ... + \varepsilon_{n_g g}) = 0,$$

which means that ordinary least squares estimators of α and β based on group means are unbiased. Second, and more importantly,

$$\mathrm{Var}(\overline{\varepsilon}_g) = \frac{1}{n_g^2}(\sigma^2 + \sigma^2 + ... + \sigma^2) = \frac{n_g\sigma^2}{n_g^2} = \frac{\sigma^2}{n_g},$$

which means, that unless the number of observations is the same in each group, the disturbance in eq.(18) is heteroscedastic. However, since we know the size of each group, it is possible to construct a series of weights so as to remedy the problem of heteroscedasticity. It is important to note that this only concerns the heteroscedasticity introduced by the creation of group means. It is of course possible that the regression despite this correction exhibits nonspherical disturbances, whether heteroscedasticity or something else.

Handling of the data base

First, all individuals were aggregated into households. Next, households that had changed compositions during the year (BHELAR = 0) were deleted, as well as small business owners (TFOAB $\neq$ 0), and thereby farmers, and households with missing values.

Second, all monetary variables were deflated using the implicit total consumption price deflator. Also, in order to be able to calculate $(1 + r)W_t$ in eq.(7), total return on a large class of assets was needed. Table 19 below contain a brief summary of the ex-post return.[24]

[24] For 4 classes of real estate holdings, geographical disaggregation is possible; tenant owned

Table 19: Total ex-post return for different asset classes

	2000	2001
Cash	.005	.005
Stocks, A-list	-.067	-.189
Stocks, O-list	-.374	-.063
Stocks, other listed	-.478	-.539
Fixed income instruments	.070	.058
Mutual fund, stocks (aktiefond)	-.104	-.149
Mutual fund, fixed income (räntefond)	.024	-.096
Mutual fund, mixed (blandfond)	-.077	-.171
Other real estate	.127	.036
Residential house (hyreshus)	.184	.116
Industrial real estate (industri)	.159	.055
Debt, according to financial statements	.066	.064
Debt, incurred for higher education	.032	.031
Other debt	.054	.050

(19)

In addition, for a number of asset classes it was not possible to find a measure of performance that directly corresponded to that class of assets. As an example, for capital insurances the total ex-post return for mutual funds (mixed) has been used.

Next, group-individuals were created, as described in section 3.2, after which the construction of the data set was completed through the calculation of imputed consumption, as described in section 3.1.

apartment (4 geographical districts), house (5), leisure house (5) and farms (5). For practical reasons, the complete list of returns have been omitted.

Creation of aggregate measures

Table 20: Composition of variables

Short form		Plain English (Statistics Sweden abbreviation)
Y	=	Disposable income (CDISPH)
$Debt$	=	Debt from tax authorities register data (FKURTA)
	+	Other debt (FSKURST)
	+	Issued options (FKUSKOP)
	+	Debt, incurred for higher education (FSKUST)
W^F	=	Stocks (FAKTIBMV+FOTCMV+FAALMV)
	+	Bonds (FKUPREM+FKUVPR)
	+	Mutual funds (FKUPREM+FKUVPR)
	+	Options (FOPMV)
	+	Capital insurances (FFORS)
	+	Other securities (FOVRMV)
W^{NF}	=	Tenant-owned apartment (FBORMH)
	+	House (FFSTEGM + FTOMTEM)
	+	Leisure house (FFSTEGO + FFSTFRM + FTOMTFM)
	+	Other real estate (FFSTSF)
	+	Other assets (FOVRTMV)

(20)

Results from estimation of augmented models

Table 21: Results from estimation of Baseline and BSH models for $x = Y$

	Model				
	Baseline			BSH	
Interval (i)	α	β_1	β_2	α_{BSH}	β_{BSH}
1	-17375	1.36	-0.63	121901	-0.40
2	-74675	1.63	-0.21	154154	-0.12
3	-833348	1.58	-0.20	226285	-0.17
4	-123962	1.70	-0.25	317802	-0.22
5	479022	-0.33	-0.25	481795	-0.28
6	-3035145	2.10	-0.34	465062	-0.02
$\bar{R}^2$	0.64			0.40	

Baseline model: $C_k = \sum_{i=1}^{6} \alpha_i + \sum_{i=1}^{6} \beta_{1,i} Y_k + \sum_{i=1}^{6} \beta_{2,i} NW_k + \varepsilon_k$

BSH model: $C_k = \sum_{i=1}^{6} \alpha_{BSH,i} + \sum_{i=1}^{6} \beta_{BSH,i} CR_k + \varepsilon_k$

(21)

Table 22: Results from estimation of Baseline and BSH models for $x = NW$

	Model				
	Baseline			BSH	
Interval (i)	α	β_1	β_2	α_{BSH}	β_{BSH}
1	-44746	1.28	-0.88	78747	1.09
2	-17046	1.13	-1.62	-3861	1.18
3	-7285	1.18	-0.20	12284	0.93
4	-6321	1.23	-0.31	-159610	0.95
5	-111919	1.30	0.02	-213899	0.64
6	65784	1.94	-0.51	252897	0.01
$\bar{R}^2$	0.66			0.21	

Baseline model: $C_k = \sum_{i=1}^{6} \alpha_i + \sum_{i=1}^{6} \beta_{1,i} Y_k + \sum_{i=1}^{6} \beta_{2,i} NW_k + \varepsilon_k$

BSH model: $C_k = \sum_{i=1}^{6} \alpha_{BSH,i} + \sum_{i=1}^{6} \beta_{BSH,i} CR_k + \varepsilon_k$

(22)

Table 23: Results from estimation of Baseline and BSH models for $x = CR$

	Model				
	Baseline			BSH	
Interval (i)	α	β_1	β_2	α_{BSH}	β_{BSH}
1	-5203	1.06	-0.78	103736	0.13
2	1149	1.06	-0.47	-3216	1.25
3	-114934	1.67	0.01	99769	0.58
4	-36617	1.30	-0.28	157754	0.24
5	-63273	1.23	-0.07	115486	0.23
6	40199	1.84	-0.47	566350	-0.14
$\bar{R}^2$	0.68			0.11	

Baseline model: $C_k = \sum_{i=1}^{6} \alpha_i + \sum_{i=1}^{6} \beta_{1,i} Y_k + \sum_{i=1}^{6} \beta_{2,i} NW_k + \varepsilon_k$

BSH model: $C_k = \sum_{i=1}^{6} \alpha_{BSH,i} + \sum_{i=1}^{6} \beta_{BSH,i} CR_k + \varepsilon_k$

(23)

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