Performance Evaluation of Simple Space-Time Block Coding on MIMO Communication System
Abstract

This thesis discuss on new technique called space time block coding (especially Alamouti’s code) which is used to increase capacity and reliability of data transmission over time varying multi-path fading channel. The over all work of the thesis included in the following four chapters.

In chapter-1 we are going to cover some theoretical part which is useful to understand thesis work and in chapter-2 we will discuss the comparison between simple space time block code (Alamouti’s code) and MRRC (Maximum Ratio Receiver Combining) which is receiver diversity and then in chapter-3 we will see the channel capacity & probability error performance for 2x2 Alamouti code over Rayleigh and Rice fading channel .Finally the conclusion and further work included in chapter-4.
Acknowledgements

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CHAPTER 1: WIRELESS COMMUNICATION

Introduction
The history of wireless communication starts from 100 years ago. This technology has removed so many problems associated by cables or cords. Today, it is possible for portable device to transfer data without being physically connected to computer or any other device. Wireless technology makes our life easy and comfortable. And also the demand on bandwidth and spectral availability are endless. However, the designers have got difficult task of limited availability of radio spectrum, fading, multi-path, interference, to meet the demand for high data rate [1].

The 2G and 3G standards are not good enough to satisfy the demand of high capacity. Therefore, the new standard 4G which is the successor of 2G and 3G are in the way to provide fully broadband internet for mobile and stationary users and it uses the new technology called MIMO (Multiple Input Output). Table 1.1 shows different standard’s operating frequency and supported data rates.

<table>
<thead>
<tr>
<th>No</th>
<th>Standards</th>
<th>Generation</th>
<th>Operated frequency</th>
<th>Supported data rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GSM</td>
<td>2G</td>
<td>1.8GHz</td>
<td>22.8 Kbit/s</td>
</tr>
<tr>
<td></td>
<td>Global system for Mb. Communication</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>UMTS</td>
<td>3G</td>
<td>2GHz</td>
<td>38.4kbit/s or 2Mbit/s for stationary</td>
</tr>
<tr>
<td></td>
<td>Universal Mobile T.S.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>LTE</td>
<td>4G</td>
<td>1.4MHz-20MHz</td>
<td>100Mb/s or 1Gbit/s</td>
</tr>
<tr>
<td></td>
<td>Long term Evolution</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table 1.1: Comparisons of data rates between different standard*
1.1. Wireless communication systems
The communication process consists of five basic elements, namely, source information, transmitter, channel, receiver and user information as shown in Fig 1.1 below.

Fig 1.1: Five basic elements of communication systems.

1. The source information: generates a signal that carries the message. The message may be various types, either analog form such as audio signal or digital form like bit stream from computer. The message signal from source is usually in the form of baseband signal.

2. Transmitter: it operates in some way on message signal and produces a signal suitable for transmission to receiving point over the specific channel. The transmitted signal always much higher the maximum frequency component of the message signals.

3. Channel: It is a media used to transmit the signal from transmitting to receiving point. During transmission noise and different kind of interfering signals added to transmitting signal.

4. Receiver: It operates on receive signal and trying to reproduce original signal from it since the received signal is corrupted version of transmitted signal.

5. User of information: It is a person or thing for which the message is intended.

1.2. Propagation characteristics in wireless channel
Wireless channel experience a lot of limitation on the performance of wireless system. It is extremely random and difficult to analyse since the signal transmitted over wireless channel arrives at the destination in a number of different path or multi-path. The multi-path arises from scattering, reflection, diffraction or refraction of radiated energy off that objects lie in the environment. The actually received signal is often weaker than transmitted signal due to propagation loss and fading.
1.2.1. Average propagation loss
The effect of multi-path propagation and time variations it observed directly on received signal power level over wireless channel. With path loss model we can characterized the relation between average received power signal ($P_r$) and transmitted power signal [2].

$$P_r \propto \frac{PG_tG_r}{d^n}$$  \hspace{1cm} (1.1)

$$P_t = K \frac{PG_tG_r}{d^2} \hspace{1cm} \text{(For free space n=2)}$$  \hspace{1cm} (1.2)

where $K$ is constant (for fixed value of n), $P_t$ is transmitted power signal, $d$ is the separation distance between receiver and transmitter, $G_t$ & $G_r$ the power gain of transmitted and received antenna respectively, and $n$ is the path loss exponent. The value $n$ is between 2 and 6. For free space and with direct line of sight $n$ is 2, whereas for an indoor transmission with so many obstacles it could be reach up to 6. The constant $k$ for $n=2$ is given by.
where $\lambda$ the wave length. The gain of antenna is given in terms of effective aperture ($A_e$) by

$$G = \frac{4\pi A_e}{\lambda^2}$$

Equation 1.2 shows that Power falls off proportional to distance squared.

1.2.2. Small –Scale Fading
Fading is rapid change in amplitude, phases or multi-path delays of transmitted radio signal over short period of time or travel distance. It is caused by interference of one or more signal which arrives at receiver with slightly different times.

The following factors influence small-scale fading:

1. Multi-path Propagation
It is a condition where the transmitted radio signal is reflected or diffracted or scattered by physical structure and produce multiple signal path between transmitter and receiver station. This effect produces fading and distortion on received signal.

2. Speed of the Mobile
The relative motion between the base station and the mobile is result in random frequency modulation due to different Doppler shifts on each of the multi-path components.

3. Speed of the Surrounding Objects
If the surrounding objects in motion they induce Doppler shift on multi-path components.

4. The Transmission Bandwidth of the Signal
If transmitted signal bandwidth is greater than the multi-path channel bandwidth, a distorted signal received. The channel bandwidth quantified by the coherence bandwidth.
1.2.3. Types of small - Scale Fading

We classify small-scale fading in two broad parts depending on the relation between the signal parameters (such as bandwidth and symbol period) and the channel parameters (such as rms delay spread and Doppler spread) [3].

a) Based on multi-path time Delay spread

- Flat fading
  When the transmitted signal bandwidth much lesser than the coherence bandwidth, the wireless channel is called flat fading. If the channel has a constant gain and linear phase response over a bandwidth of the transmitted signal, then the received signal will undergo flat fading. In flat fading, the multi-path structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multi-path.

- Frequency selective fading
  When the channel bandwidth is smaller than transmitted signal then the channel creates frequency selective fading on received signal. Frequency selective fading occurs when multi-path delay greater than the symbol period of the transmitted symbol.

b) Based on Doppler spread

- Fast fading
  It is a kind of fading that the multi-path waves components such as amplitude, phases, time delay change faster than the rate of change of the transmitted signal.

- Slow fading
  Slow fading results from a blocking effect due to buildings and natural features and is also known as long-term fading, or shadowing. The statistical distribution of the mean is influenced by the antenna heights, operating frequency and specific type of environment.
1.3. Wireless Channel
Basically wired and wireless channel have fundamental differences. For example, wireless channel time-variant because of changes in environment and the relative motion of transmitter and receiver. And also signal received from transmitter through multi-path.

The signal propagated from transmitter may reflect, refract or diffract. This shows us the signals transmitted over wireless channel are received via multiple secondary paths in addition to direct of line of sight. Due to multi-path the original signal changes in phase and in strength. To understand clearly wireless channel we should characterized properly the multi-path and time variations and then suitable channel model should be developed.

1.3.1. Channel Model
To model wireless channel we should first understand the effect of the channel on transmitted signal [4]. We can generally represent the transmitted signal as

\[ x(t) = \text{Re} \left[ x_l(t) e^{j2\pi f_c t} \right] \]  \hspace{1cm} (1.5)

where \( x_l(t) \) the baseband signal.

Let assume there are multi-path propagation over channel and associated to each path there is exist propagation delay and attenuation factor. The bandpass signal in receiver side can be written in the form of

\[ y(t) = \sum_n \beta_n(t) x(t - \tau_n(t)), \]  \hspace{1cm} (1.6)

where \( \beta_n(t) \) and \( \tau_n(t) \) are the attenuation factor and propagation delay for nth path received signal. By substituting \( x(t) \) from equation (1.5) in to (1.6)

\[ y(t) = \text{Re} \left\{ \left[ \sum_n \beta_n(t) e^{-j2\pi f_c \tau_n} x_l(t - \tau_n(t)) \right] e^{j2\pi f_c t} \right\} . \]  \hspace{1cm} (1.7)

From the above equation we can write the equivalent low pass received signal as

\[ r(t) = \sum_n \beta_n(t) e^{-j2\pi f_c \tau_n} x_l(t - \tau_n(t)) \]  \hspace{1cm} (1.8)
Also the equivalent low pass channel described by time-variant impulse response as

\[ c(\tau; t) = \sum_n \beta_n(t) e^{-j2\pi f_c \tau_n t} \delta(t - \tau_n(t)). \]  

(1.9)

When the impulse response \( c(\tau; t) \) is modeled as zero mean complex–valued Gaussian process the envelope \(|c(\tau; t)|\) at any instant \( t \) is Rayleigh distributed. In this case the channel said to be Rayleigh fading channel.

### 1.3.2. Types of channel

**Rayleigh Fading Channel**

There are many probability distributions choice can be considered in attempting to model the statistical characteristics of the fading channel. However, the Rayleigh distribution is commonly used to model the statistics of signals transmitted through radio channels such as cellular radio and the envelope of the channel response given by the following equation [4].

\[
P_R(r) = \frac{2r}{\Omega} e^{-r^2/\Omega}, \quad r \geq 0
\]

(1.10)

Where \( \Omega = E(R^2), \quad R = \sqrt{X^2 + Y^2}, \quad X \sim N(0, \sigma^2), \quad Y \sim N(0, \sigma^2) \)

**Ricean Fading Channel**

If there are fixed scatters or signal reflectors in the medium, in addition to randomly moving scattered then the envelope of \(|c(\tau; t)|\) no longer modeled as having zero mean. In this case probability distribution function that has been used to model the envelope of fading signal is Rice distribution and the channel is called Ricean fading channel. The Rice distribution for statistical model is represented by the PDF.

\[
P_R(r) = \frac{r}{\sigma^2} e^{-(r^2 + s^2)/2\sigma^2} I_0\left(\frac{rs}{\sigma^2}\right), \quad r \geq 0
\]

(1.11)
where \( R = \sqrt{(X^2 + Y^2)} \), \( s^2 = m_1^2 + m_2^2 \), \( X \sim N(s \cos(\theta), \sigma^2) \), \( Y \sim N(s \sin(\theta), \sigma^2) \)

**Nakagami-m Distribution**

We have Nakagami-m distribution as alternative statistical channel modeling for the envelope the channel response and the distribution is given by the PDF.

\[
P_R(r) = \frac{2}{\Gamma(m)} \left( \frac{m}{\Omega} \right)^m r^{2m-1} e^{-m r^2/\Omega},
\]

where \( \Gamma(.) \) the Gamma function, \( \Omega = E(R^2) \), \( m \) (fading figure) the inverse normalized variance of \( r \) which must satisfies \( m \geq 1/2 \). By setting \( m=1 \) the equation (1.12) reduced to a Rayleigh PDF.

**1.4. Introduction to Antenna**

The performance of wireless communications is obviously depending on the advance of antenna system refers to smart or intelligent antenna. Recently multiple antenna technologies are emerging to achieve the goal of 4G (fourth generation) system such as high rate reliability and long range communication. Therefore antenna has a vital role in successfully design of wireless communication.

**1.4.1. Antenna**

Antenna is a device used to efficiently to radiate electromagnetic wave and receive transmitted wave. We have different kind of Antenna for different applications. To choose suitable antenna for the right application we should be careful consider the following important antenna charters tics shown in table 1.2 [5].

**A. Antenna Radiation Patterns**

It is 3-D plot of radiation of electromagnetic wave from antenna. The two types of radiation patterns are Broadside and Endfire.

**B. Power Gain**

Power gain is the ratio of power input in to antenna to power output from the antenna. It is measure that takes in to account the efficiency and the directivity of antenna.
C. **Directivity**  
It is a method to determine the direction of propagation of wave form antenna.

D. **Polarization**  
It is the orientation of electromagnetic wave far from the source. To get maximum performance from the antenna the polarization of transmitting antenna must match to receiving antenna.

<table>
<thead>
<tr>
<th>Antenna type</th>
<th>Radiation pattern</th>
<th>Power gain</th>
<th>Directivity</th>
<th>Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole</td>
<td>Broad side</td>
<td>Low</td>
<td>Low</td>
<td>Linear</td>
</tr>
<tr>
<td>Flat panel antenna</td>
<td>Broad side</td>
<td>Low/medium</td>
<td>Low</td>
<td>Linear</td>
</tr>
<tr>
<td>Parabolic dish</td>
<td>Broad side</td>
<td>High</td>
<td>High</td>
<td>Linear /Circular</td>
</tr>
<tr>
<td></td>
<td>End fire</td>
<td>Medium / High</td>
<td>Medium/High</td>
<td>Linear</td>
</tr>
<tr>
<td>Microstrip antenna</td>
<td>End fire</td>
<td>Medium</td>
<td>Medium</td>
<td>linear</td>
</tr>
</tbody>
</table>

*Table 1.2: Antenna comparison depending on characteristics of Antenna.*

1.5. **Diversity**  
In wireless communication we encounter multi-path fading which is fluctuate in signal strength through out the channel. This problem gives raise to high data Bit error rates. To solve this problem we can use diversity technique. It is achieved by providing a copy of transmitted signal over frequency, time and space. We have the three kind of diversity technique in wireless communication [1].

1.5.1. **Time Diversity**  
Transmitting the replica of signal at the same moment of time and it arrives at receiver different moment in time if they travel different physical paths of different lengths. To get effective diversity the channel should provide sufficient variation on time. we have to sure that the interleaved symbol independent of the previous symbol at time of transmission to get good result.

1.5.2. **Frequency Diversity**  
The technique is similarly to time diversity. However, here provides diversity of the replica of the original signal in frequency domain. This method is to be practicable the coherent bandwidth of the channel should be less than the signal bandwidth. It assures signal at different frequencies will suffer independent level of attenuation or fading.
1.5.3. Space Diversity
It is commonly known as antenna diversity. The replicas of the original signal from transmitter are provided across different antenna of the receiver. Here also to insure independent fades across different antennas, the spacing of antennas should be greater than the coherent distance.

We can further divide the space diversity into receiver and transmitter diversity which is depend on whether the diversity is applied to transmitter or to receiver.

Receiver Diversity
We use multiple antennas at receiver to improve the signal quality. It is costly and difficult to implement in hand held mobile devices like cell phone. A good example for this is MRRC (maximum Ratio combining) diversity.

Transmitter Diversity
This method uses multiple antennas at receiver to improve the signal quality. Today it is more popular than receiver diversity since it so easier to implement at the base station. This scheme needs complete channel knowledge at the transmitter. However the new simple transmit diversity technique like Alamouti’s scheme they do not need channel information.

1.6. Modulation
We have different kind of modulation scheme. It is chosen according to channel characteristics in order to optimize channels’ performance.

Amplitude Modulation
Amplitude modulation is the change in amplitude of the carrier signal in accordance with information bearing signal.

Frequency Modulation
Frequency modulation is the change in frequency of carrier signal in accordance with information bearing signal. But the amplitude of carrier remains constant.

Phase Modulation
It is similarly to frequency modulation but instead of frequency the phase of the carrier signal change in accordance with information bearing signal.

Binary Phase Shift Keying
The two signals with different phase (it is normally 0, π ) represent the binary data.
\[ s_1(t) = A \cos(2\pi f_c t) \quad 0 \leq t \leq T \quad 0 \]
\[ s_2(t) = -A \cos(2\pi f_c t) \quad 0 \leq t \leq T \quad 1 \]

**High order Phase Shift Keying (M-PSK)**

It uses M point on the constellation diagrams. It is possible to use any number of constellations to construct PSK. The highest order with very low probability error deployed so far is 8-PSK.

**1.7. Information Theory**

Information theory is mainly the branch of mathematics and Electrical engineering. The main goal information theory to determine the ultimate data compression (the entropy \( H \)) and ultimate transmission rate of communication (the channel capacity) which are fundamental issue in communication theory [6].

**Entropy**

Entropy is *the number of bits on average required to describe the random variable*. It is also a measure of the average uncertainty in random variable. We can define entropy mathematical for random variable \( x \) with probability mass function \( p(x) \) as follows

\[
H(x) = - \sum p(x) \log_2 p(x) \text{ Bits.} \tag{1.17}
\]

And the conditional entropy represented by \( H(X \mid Y) \) is the entropy of random variable conditional on the knowledge of another random variable.

**Mutual Information**

*Mutual information is the reduction in uncertainty due to another random variable.* It is measure the relation or dependence between two random variables.

\[
I(X;Y) = H(X) - H(X \mid Y) = \sum_{x,y} p(x, y) \log \frac{p(x, y)}{p(x)p(y)}. \tag{1.18}
\]

\( I(X;Y) = 0 \) when \( X \) and \( Y \) are independent.
1.7.1. Communication Channel

*Communication channel is a system in which the output depends probabilistically on its input.* The capacity of communication channel with input $X$ and output $Y$ defined by

$$ C = \max_{p(x)} I(X;Y). \quad (1.19) $$

The above equation one can interpret that capacity is the maximum rate at which we can send information over the channel and recovery the information at the output with almost zero probability error.

1.7.2. Gaussian Channel

$$ Y_i = X_i + n_i \quad n_i \sim N(0,N_0), \quad (1.20) $$

where $Y_i$: The output at time $i$.
$X_i$: The input at time $i$.
$n_i$: The noise independent to signal $X_i$ at time $i$.

In this model we can generally say the capacity of the channel will be infinite either the noise variance is zero or the input is unconstrained. The most common limitation on the input is energy or power constraint. For example, for any code word $x_1, x_2, x_3 \ldots x_m$ transmitted over the channel the average power constraint

$$ \frac{1}{m} \sum_{i=1}^{m} X_i^2 \leq P. \quad (1.21) $$

We can define the information capacity of Gaussian channel with power constraint $P$ is

$$ C = \max_{f(x):EX^2 \leq P} I(X;Y). \quad (1.22) $$

To calculate the information capacity let expand $I(X;Y)$

$$ I(X;Y) = h(Y) - h(Y \mid X) $$
$$ = h(Y) - h(X + n \mid X) $$
$$ = h(Y) - h(n \mid X) $$
$$ = h(Y) - h(n), \quad (1.23) $$
where \( n \) is independent of \( X \), and \( h(n) \) is:

\[
h(n) = \frac{1}{2} \log 2\pi e N_0
\]

\[
\begin{align*}
\{ & \text{for normal } \alpha \sim N(0, \sigma^2) \\
& h(\alpha) = -\int \alpha \ln \alpha \ d\alpha = \frac{1}{2} \log 2\pi e \sigma^2 \\
\}
\end{align*}
\]

\[
E(Y^2) = E(X + n)^2 = EX^2 + 2EXEn + En^2
\]
\[
= P + N_0 \quad \text{since } X \text{ and } n \text{ indpendant } & \mathbb{E}(n) = 0 \tag{1.24}
\]

Then entropy of \( Y \) is

\[
h(Y) = \frac{1}{2} \log 2\pi e E(Y^2)
\]
\[
= \frac{1}{2} \log 2\pi e(P + N_0)
\]

Substituting \( h(Y) \) and \( h(n) \) in \( I(X;Y) \)

\[
I(X;Y) = h(Y) - h(n)
\]
\[
\leq \frac{1}{2} \log 2\pi e(P + N_0) - \frac{1}{2} \log 2\pi eN_0
\]
\[
= \frac{1}{2} \log(1 + \frac{P}{N_0}).
\]

Therefore, the information capacity of the Gaussian channel is:

\[
C = \max_{EX^2 \leq P} I(X;Y) = \frac{1}{2} \log(1 + \frac{P}{N_0}). \tag{1.25}
\]

We get maximum capacity when \( X \sim N(0, P) \).
1.7.3. Band Limited Channels

Band limited channel with white noise is very common model over radio network or a telephone line. The output of such channel describes as convolution.

\[ Y(t) = ((X(t) + n(t)) \ast h(t)), \quad (1.26) \]

where \( X(t) \) is the input signal, \( n(t) \) the wave form of white Gaussian noise, and \( h(t) \) the impulse response of the band pass filter to filter out all frequency greater than \( B \).

Let assume we have channel bandwidth \( B \) and sampling frequency for both input and output samples taken \( \frac{1}{2B} \) second apart. Each of the input samples is corrupted by noise to produce corresponding output sample. Each noise sample is i.i.d since noise is white and Gaussian.

By assuming our function has most of its energy in Bandwidth \( B \) and in finite time interval say \((0, T)\). In this case the energy per sample is \( PT/2Bt = p/2B \) the noise variance per sample \( \frac{n_o}{2B} \ast \frac{T}{2BT} \) (where \( \frac{n_0}{2} \) power spectral density in watt/Hz). Earlier we derived the capacity for Gaussian channel

\[ C = \frac{1}{2} \log(1 + \frac{P}{N_0}) \text{ Bits per transmission.} \]

And the we can determine the capacity per sample is

\[ C = \frac{1}{2} \log(1 + \frac{P}{n_o/2}) = \frac{1}{2} \log(1 + \frac{P}{n_o B}) \text{ bits per sample.} \]

For \( 2B \) samples for each second the capacity will be

\[ C = B \log(1 + \frac{P}{n_o B}). \quad (1.27) \]

The above formula gives the capacity of a band limited Gaussian channel with noise spectral density \( \frac{n_o}{2} \) watts/HZ and power \( p \) watts and very famous formula in formation theory.
In the next section we will discuss MIMO and its application. Further more we will see the concept behind space time coding in MIMO.

2.1. MIMO
MIMO stand for multiple inputs and multiple outputs. It is a system that uses several antennas at the transmitter and receiver links. Because of the invention of new technology like, portable terminal (laptop), mobile terminal (cell phones) and consumer devices demand on bandwidth constantly increasing and frequency spectrum is too crowded. Moreover, we faced challenge with both an operator and a subscriber side. The operator needs high coverage and high capacity whereas subscriber needs high quality and high speed. During design Engineers face uphill challenge to increase capacity, reliability, and speed and spectrum efficiency in order to meet both the subscriber and the operator interest.

Before introduction of MIMO there was a popular technology called smart antenna is used to improving wireless communication in adverse propagation conditions such as fading, multi-path and interference. The main idea in smart antennas is that of beamforming by which one increase the average signal noise ratio (SNR) through focusing into desire direction. This can be achieved by optimally combining the element with weight selected as function of each element response. And also using spatially diversity, which is use antenna array, the probability of losing the signals altogether are decrease exponential with the number of decorrelated antenna elements.
But the use of MIMO link goes beyond that of smart antennas. It transmit different signals from each transmits element so that the receiving antennas received the superposition of all transmitted signals. Multiple antenna both transmitter and receiver creates a matrix of possible transmission modes. The main advantage is depending on the possibility of transmitting over several spatial modes of the matrix with in the same time-frequency slot without extra power waste.

2.2. Comparison Between SISO, SIMO, MISO and MIMO Systems

2.2.1. Single Input Single Output (SISO)
It is a traditional model in wireless system which uses one antenna at transmitter and one antenna at receiver. Its over all performance largely dependant on channel behavior and environment. It is used in radio and TV broadcast and our personal wireless technologies such us wi-fi and Bluetooth. To improve the channel performance we can use either single input multiple output (SIMO) or multiple input single output (MISO) [7].

![Figure 2.1: Single Input Single output (SISO) 1x1.](image)

2.2.2. Single Input Multiple Outputs (SIMO)
It uses one antenna at transmitter and multiple antennas at receiver. It is logical to use SIMO for uplink scenarios. The receiver can either choose the best antenna to receive the stronger signal (selection diversity) or combine signals from all antennas in order to increase SNR (Maximal Ratio Receiver Combining or MRRC).
2.2.3. **Multiple Input Single Output (MISO)**
We use several antenna at transmitter side whereas single antenna at receiver side. It is more usually to use MISO for downlink scenarios.
2.2.4. **Multiple Input and Multiple Outputs (MIMO)**

We use multiple antenna both transmitter and receiver side. It is very hot topic today in wireless technology such as PAN, LAN, MAN and WAN to increase data rate multiple times to satisfy the bandwidth demand of broadband users.

![2x2 MIMO Diagram](image)

Figure 2.4: Multiple Input Multiple Output (MIMO) 2x2.

2.3. **The Factors Limits Wireless Communication**

The Shannon capacity result is the reason for today’s wireless MIMO communication research and application. The analysis of information theory based channel capacity used to determine the maximum transfer data rate between transmitter and receiver and it gives information how the channel model or the antenna configuration may affect the transmission rate. In addition to this it helps system designer benchmark transmitter and receiver algorithm performance [8] [9].

2.3.1. **Shannon’s Capacity for SISO**

By assuming our single channel corrupted by an additive white Gaussian noise (AWGN) the capacity can be expressed as

\[ C = \log_2 (1 + \xi) \text{ Bit} / \text{Sec} / \text{Hz}, \]  

(2.1)

where \( \xi \) is signal Noise ratio (SNR).

Practically the wireless channels are time-varying and affected by random fading. Therefore the capacity written with some modification:
\[ C = \log_2 (1 + \xi |h|^2) \text{ Bit / Sec / Hz}, \quad (2.2) \]

where $|h|^2$ is normalized channel power transfer characteristic.

### 2.3.2. Shannon’s Capacity for MISO and SIMO

The capacity for conventional antenna system (MISO or SIMO) depends on number of elements used and it can be generalized

\[ C = \log_2 (1 + \xi h h^*) \text{ Bit / Sec / Hz}, \quad (2.3) \]

where $h = h_0, h_1, h_2, \ldots, h_{M-1}$ is the channel amplitudes from transmitter to M elements of receiver. $^*$ denotes the transpose conjugate.

### 2.3.3. Shannon’s Capacity for MIMO Links

For N and M number of antenna at transmitter and receiver respectively the capacity is expressed without transmit channel information in the following way.

\[ C = \log_2 \left[ \det \left( I + \frac{\xi}{N} H H^* \right) \right] \text{ Bit / Sec / Hz}, \quad (2.4) \]

where $\xi$ is the average SNR and $I$ is identity matrix and $H$ is transfer matrix.

The capacity of MIMO channel increase when the number of elements grows. We can say generally the capacity increase linearly with smallest number of antennas $\min (N, M)$ outside and no longer inside the log function. For large number of antenna $M=N$ the capacity can be expressed

\[ C \approx M \log_2 (1 + \xi) \text{ Bit / Sec / Hz}. \quad (2.5) \]

The equation (2.5) clearly show that capacity increase linearly with the $M$.
2.3.4. Transmission of Data in MIMO System
Today MIMO use either to maximize data rate or diversity or both. Incase of maximizing the data rate we improve average capacity by performing spatial multiplexing. We can get good result in diversity by using multiple antennas. But it is not increase data rate.

2.4. Channel Modeling in MIMO
In wireless communication especially in MIMO system the channel modeling is the key area and needs great effort. The main objective of MIMO technology is to increase capacity which is depending on decorrelation properties between antennas and the full rankness of the channel matrix.

Why channel modeling?
- To fully understand channel behavior and then we extract formula which represents the channel.
- To determine the impact of the propagation parameters on the capacity of the channel.

Good channeling modeling clearly put the following points:
- It is exactly put the capacity of outdoor and indoor MIMO channel.
- Identifies the important parameters governing capacity.
- Put very simplify conditions to get full rank.

Generally, we can represent wireless channel between N transmit antenna and M receive antenna by channel matrix H

\[
H(\tau, t) = \begin{bmatrix}
    h_{i,1}(\tau, t) & h_{i,2}(\tau, t) & \ldots & h_{i,N}(\tau, t) \\
    h_{2,1}(\tau, t) & h_{2,2}(\tau, t) & \ldots & h_{2,N}(\tau, t) \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{M,1}(\tau, t) & h_{M,2}(\tau, t) & \ldots & h_{M,N}(\tau, t)
\end{bmatrix}, \tag{2.6}
\]

where \( h_{ij}(\tau, t) \) the impulse response between \( i \) transmit antenna and \( j \) receive antenna.

Let say the signal \( x_j(t) \) is transmitted from \( j \)th transmit antenna, then the signal \( y_j(t) \) received at \( i \)th received antenna is expressed by:
\[ y_j(t) = \sum_{j=1}^{M} h_{i,j}(\tau, t) \ast x_j(t) + n_i(t), \] (2.7)

where \( n_i(t) \) white Gaussian noise.

### 2.5. Space Time Block Coding

Today wireless system supposed to have better quality, high bit rate, large coverage, be more power and bandwidth efficient over time varying multi-path fading channel. But it is difficult to achieve this in time varying multi-path fading environment.

There are different technique is proposed to mitigate multi-path fading in a wireless channel.

- **Transmitter Power Control**
  It is assumed that the transmitter has some knowledge about the channel and it increases its power by the same level to reduce fading effect over channel. This method is not practical since radiation power limitation and cost of power amplifier, and wastage in bandwidth due to feed back channel information.

- **Time and Frequency Diversity**
  Both time and frequency diversity are ineffective due to large delay in slow varying channel and small delay spread in the channel respectively.

- **Antenna Diversity**
  It is very effective technique to reduce multi-path fading. We are two types of antenna diversity technique. Receiver diversity (e.g. MRRC) and transmitter diversity (e.g. space time code) which is depend on whether the diversity is applied to transmitter or to receiver.

Recently the transmit diversity scheme become more popular to solve multi-path fading problem which use multiple antenna at transmitter to improve reliable data transmission. This scheme called combination of spatial (antenna) and temporal processing known as space time block coding (STBC). In STBC Alamouti’s scheme is the most famous and popular one since the only STBC which can achieve both full diversity and full code rate for complex constellation.
2.5.1. Maximal Ratio Receive Combining (MRRC)

It is a method used several antennas at receiver side to receive the same signal through different propagation path and perform combining or selection and switching to improve the received signal quality. This approach practically applied only to base station to improve their reception quality because of the cost, size and power of remote units [10].

![Diagram of MRRC with two transmit antennas]

Fig 2.5: MRRC with two transmit antenna.

The channel is modeled by complex multiplicative distortion. For two receive antenna channel between the transmitter antenna and the receive antenna one is \( h_1 = \beta_1 e^{j\alpha} \) and the channel between the transmitter and receive antenna two is \( h_2 = \beta_2 e^{j\alpha} \). Noise would be add at two receiver so that the two received base band signal would be

\[
\begin{align*}
    y_1 &= h_1 x_1 + n_1 \\
    y_2 &= h_2 x_1 + n_2
\end{align*}
\]  

(2.8)

where \( x_1 \) the signal sent from transmitter, \( n_1 \) and \( n_2 \) represent complex noise which is assumed to be Gaussian distributed with zero mean.
The two-branch MRRC receiver will combine the receive signals as follows:

\[
\tilde{x}_1 = h_1^* y_1 + h_2^* y_2 \\
= h_1^* (h_1 x_1 + n_1) + h_2^* (h_2 x_1 + n_2) \\
\tilde{x}_1 = (\beta_1^2 + \beta_2^2) x_1 + h_1^* n_1 + h_2^* n_2 ,
\]  

(2.9)

Where: \( \beta_1^2 = h_1^* h_1, \beta_2^2 = h_2^* h_2 \), \( \tilde{x}_1 \) the maximum likelihood estimate of transmitted signal of \( x_i \)

After all, the maximum likelihood decision rule is used at receiver to choose which symbol was actually transmitted i.e. Choose signal \( x_i = x_i \) if and only if

\[
(\beta_1^2 + \beta_2^2 - 1) |x_i|^2 + d^2(\tilde{x}_i, x_i) \leq (\beta_1^2 + \beta_2^2 - 1) |x_k|^2 + d^2(\tilde{x}_i, x_k) \\
\forall i \neq k
\]

(2.10)

Where, \( d^2(\tilde{x}_1, x_i) \) is squared Euclidean distance between signals \( \tilde{x}_i \) and \( x_i \) given by:

\[
d^2(\tilde{x}_1, x_i) = (\tilde{x}_1 - x_i)(\tilde{x}_1^* - x_i^*) .
\]

### 2.5.2. Alamouti Scheme

Alamouti scheme is a simple transmit diversity scheme suitable for two transmit antennas. Before coding the base band signal would be modulated at each antenna using an M-PSK modulated scheme. Then the modulated signal is encoded by space time block code technique. We assume here the transmitter does not have channel knowledge but the receiver full knowledge about the channel. We will first see, two transmit antenna and one receive scheme then two transmit and two receive scheme.

#### Two transmitter and one receiver scheme

Two symbols are considered at a time, say \( x_1 \) and \( x_2 \) they are transmitted in two consecutive time slots. In first time slot, \( x_1 \) is transmitted from antenna one and \( x_2 \) is transmitted from antenna two. In the second time slot \(-x_2^*\) is transmitted from antenna one while and \( x_1^* \) is transmitted from antenna two as shown in Fig 2.2 below. Also the equivalent space time block coding matrix shown in table 2.1
Fig 2.6: Alamouti scheme with two transmit and one receive antenna.

\[ C_{alamouti} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \]

<table>
<thead>
<tr>
<th>Time</th>
<th>antenna 1</th>
<th>antenna 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>( x_1 )</td>
<td>( x_2 )</td>
</tr>
<tr>
<td>( t + T )</td>
<td>( -x_2^* )</td>
<td>( x_1^* )</td>
</tr>
</tbody>
</table>

Table 2.1: shows space –Time coding.
The fading coefficient from antenna one and two are denoted by $h_1(t)$ and $h_2(t)$ respectively at time $t$. By assuming these coefficients are constant across two consecutive symbols.

$$h_1(t) = h_1(t + T) = h_1 = \beta_1 e^{j\alpha_1}$$

$$h_2(t) = h_2(t + T) = h_2 = \beta_2 e^{j\alpha_2}.$$  

(2.11)

The received signal at time $t$ and $t + T$ in matrix form expressed as

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix},$$

(2.12)

where $y_1$ and $y_2$ the received signal at time $t$ and $t + T$ and $n_1$ and $n_2$ are independent zero mean additive white Gaussian noise.

The combiner combines the received signal as

$$\begin{align*}
\bar{x}_1 &= h_1^* y_1 + h_2^* y_2^* \\
\bar{x}_2 &= h_2^* y_1 - h_1^* y_2^*.
\end{align*}$$

(2.13)

By substitution some terms in equation we get

$$\begin{align*}
\bar{x}_1 &= (\beta_1^2 + \beta_2^2) x_1 + h_1^* n_1 + h_2 n_2^* \\
\bar{x}_2 &= (\beta_1^2 + \beta_2^2) x_2 - h_1 n_1^* + h_2^* n_2.
\end{align*}$$

(2.14)

Finally similar to MRRC, the maximum likelihood decision rule is used to at receiver to choose which symbol was actually transmitted for each of the signals $x_1$ and $x_2$.

We can conclude that from equation (2.14) 2x1 Alamouti scheme gives the same diversity order with that of 1x2 MRRC.
Two transmitters and two receiver scheme

Let us consider the channel model for two consecutive channel uses at time $t$ and $t+T$ \cite{10,11}.

$$
y(t) = Hc(t) + n(t)$$
$$y(t + T) = Hc(t + T) + n(t + T),$$

(2.15)

where $H$ is 2x2 channel matrix, $c(t)$ and $c(t+T)$ Alamouti code words, $n(t)$ and $n(t+T)$ are zero mean uncorrelated additive noise.

$$H = \begin{pmatrix} h_1 & h_2 \\ h_3 & h_4 \end{pmatrix}, \quad c(t) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad c(t+T) = \begin{pmatrix} -x_2^* \\ x_1^* \end{pmatrix}, \quad n(t) = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}, \quad n(t+T) = \begin{pmatrix} n_3 \\ n_4 \end{pmatrix}. \quad (2.16)$$
By inserting the terms in equation (2.16) into (2.15) we get

\[
y_1(t) = h_1 x_1 + h_2 x_2 + n_1 \\
y_2(t) = h_3 x_1 + h_4 x_2 + n_3 \\
y_1(t + T) = -h_1 x_2^* + h_2 x_1^* + n_2 \\
y_2(t + T) = -h_3 x_2^* + h_4 x_1^* + n_4.
\]  

(2.17)

The two signals built by combiner sent to maximum likelihood decoder for decision criteria are:

\[
\tilde{x}_1 = h_1^* y_1(t) + h_2^* y_1^*(t + T) + h_3^* y_2(t) + h_4^* y_2^*(t + T) \\
\tilde{x}_2 = h_2^* y_1(t) - h_1^* y_1^*(t + T) + h_4^* y_2(t) + h_3^* y_2^*(t + T).
\]  

(2.18)

By inserting the signal model in equation (2.17) in to the signal built by the combiner equation (2.18) will be

\[
\tilde{x}_1 = (h_1^2 + h_2^2 + h_3^2 + h_4^2) x_1 + h_1^* n_1 + h_2^* n_2^* + h_3^* n_3 + h_4^* n_4^* \\
\tilde{x}_2 = (h_1^2 + h_2^2 + h_3^2 + h_4^2) x_2 - h_1^* n_2^* + h_2^* n_1^* - h_3^* n_4^* + h_4^* n_3^*.
\]  

(2.19)

Finally the maximum likelihood decision rule is used at receiver to choose which symbol was actually transmitted for each of the signals \( x_1 \) and \( x_2 \).

Alamouti’s 2x2 schemes give the same diversity order with 1x4 MRRC. Generally we can conclude that Alamouti scheme with two transmit and M receive antennas is equivalent to MRRe with one transmit antenna and 2M receive antenna.

2.5.3. Comparison Between Alamouti and MRRC
As mention before we can get equivalent result from Alamouti and MRRC scheme. But during implementation the two scheme quite different from each other. In the Fig 2.4 below we can see clearly the error performance between transmit (Alamouti) and receive diversity (MRRe) scheme using Quadarture phase shift keying modulation.
Fig 2.8: Bit error performance comparison between Alamouti scheme and MRRC.

1.0: Power Requirements
Receive diversity requires less power when compare to transmit diversity since transmission carried out by single antenna whereas in transmit diversity two antenna simultaneously require to transmit the data. If system is radiation power is limited, it would degrade the performance of scheme. If we assume that total power transmitted from two transmit antenna and one receive antenna equal that of receive diversity, the power transmitted from two transmit antenna should be half of the total power. This leads to 3 dB penalty in the error performance. It is possible to get identical performance with receive diversity by doing each transmit antenna must radiate the same energy as the single transmitted antenna received diversity. But, assuming equal radiated power has advantage since transmit scheme requires two half power amplifiers compared to one full power amplifier for MRRC which may be less expensive for system implementation [10].

As shown Fig 2.4 to achieve bit error rate (BER) of $10^{-4}$ the MRRC with one transmit antenna and two receive antenna we need about a SNR of 16 dB and for the Alamouti scheme two transmit antenna and one receive antenna we need SNR about 18 dB. This shows us the Alamouti scheme 2dB (practically 3dB) worse than MRRC due to system radiation limitation.
2.0: Delay Effect and Interference
Decoding delay is two symbol period for MRRC whereas for the N transmit diversity (Alamouti) is N symbol period. Alamouti scheme is affected by interference due to simultaneous transmission of signal from two antennas.

3.0: Soft Failure
The key advantages of receiver diversity are soft failure. If any of receive chains fail the signal still be received with other receive chains even though the performance adversely affected. This advantage is preserved in the transmit diversity scheme because if any of the transmit chains fail, the others will still be transmitting and hence the signal will still be received but with some performance degradation.

4.0: Sensitivity for Channel Estimation Error
The Alamouti scheme requires twice number of pilot symbols for channel estimation when pilot insertion and extraction used compare to MRRC.

2.5.4. Orthogonal Space Time Block Coding
More recently there are many works emerged on orthogonal space time coding to extend the work of Alamouti more than two transmit antennas. By different author proposed more than two straightforward coding for 3 and 4 transmit antenna. However, none of them achieve to construct full rate orthogonal code. Therefore, this led to variety of code design strategies to prolong Alamouti’s where one sacrifices the data rate to preserve a simple decoding structure or the orthogonality of the code to retain a full data rate. We will some example to clarify the above statement.

Space time block code for 3 transmit antenna

\[
C_{3,1/2} = \begin{pmatrix}
x_1 & x_2 & x_3 \\
-x_2 & x_1 & x_4 \\
-x_3 & x_4 & x_1 \\
-x_4 & x_3 & x_2 \\
x_1^* & x_2^* & x_3^* \\
-x_2^* & x_1^* & x_4^* \\
-x_3^* & x_4^* & x_1^* \\
-x_4^* & x_3^* & x_2^*
\end{pmatrix}
\]  \hspace{1cm} (2.20)
The rate of the above code is $\frac{1}{2}$. The code for more than two transmit antenna to achieve the orthogonality the only way is sacrifice its rate.

Note: Code Rate is defined by how many symbols per time slot it transmits on average over the course of one block.

**Quasi-orthogonal STBC**

\[
C = \begin{pmatrix}
    x_1 & x_2 & x_3 & 0 \\
    -x_2^* & x_1^* & 0 & x_3 \\
    -x_3^* & 0 & x_1^* & -x_2 \\
    0 & -x_3^* & x_2^* & x_1 \\
\end{pmatrix}.
\]  

(2.21)

The rate for this code is full. However orthogonality hold only for some columns in code matrix. This makes decoding slightly more complex than for orthogonal STBCs.
CHAPTER 3: SIMULATION AND RESULTS

In chapter one we tried to cover some theoretical part which is useful to understand this thesis work and in chapter two we discussed simple space time block code (Alamouti code) with comparing with MRRC which is receiver diversity.

In this chapter we will try to demonstrate simulated result of ideas discussed in previous two chapters using MATLAB.

3.1. Flow Chart
It is very important to represent the simulation using flow charts since it is clarify the ones understanding of the whole process easily and it gives a chance to think where the process can be improved. Flow chart is shown in Fig 3.1 below.

3.2. Channel Capacity
As we defined in previous chapter channel capacity is the maximum rate at which we can send data over the channel and recovery the data at the output with vanishing probability error. Channel capacity depend on signal noise ratio (SNR), in Number of antenna element (in MIMO), in bandwidth (for band limited channel). In the next section we will see the simulation result for Shannon’s channel capacity for Rayleigh fading channel.

3.2.1. Simulated Result
In Fig 3.2 below we can see that when we increase the SNR, the average channel capacity increases too. But practically we can increase the SNR ratio only to certain limit because of system radiation power limitation and cost and size of the equipment to implement the system.
In MIMO if the number of antenna element grows the capacity will increase. It increases linearly in proportion to large number of antenna elements. In Fig 3.3 clearly show that the system with two transmit and two receive antenna (TxR 2x2) better channel capacity than system with MISO (TxR 2x1), SIMO (TxR 1x2) and SISO(TxR 1x1).

Fig 3.1: Flow chart for the whole process of simulation.
Fig 3.2: Shows that the relation between average channel capacity Vs. SNR for Rayleigh fading channel.

Fig 3.3: Shannon’s channel capacity for different number of antenna element over Rayleigh fading channel.
3.3. Probability Error and Channel Capacity

Let say we have N transmitted bits then the probability error is the average number of bits received with error from N transmitted bits. Normally probability error small number like $10^{-5}$. It is not difficult to achieve very small error probability in wireline channel without wasting large amount transmit energy since wireline channel is less susceptible with noise and interference than wireless channel. However, in wireless channel due to multi-path fading it is difficult to get very small probability error but still it is possible to get it using either transmit or receive diversity technique.

3.3.2. Simulated Result

In theoretical we know that for indoor scenario the behavior of the channel better characterized by Ricean fading than Rayleigh fading. For outdoor and for long distance communication Rayleigh fading gives good performance.

Alamouti’s 2x2 channel capacity over Rayleigh and Ricean fading shown in Fig 3.6 and 3.8 respectively. The alamouti’s channel capacity very close to Shannon capacity channel that means Alamouti code is maybe good code. By comparing the two figures we can conclude that Ricean fading channel gives the same channel capacity as Rayleigh fading channel but with lesser SNR.

![Probability Error vs SNR](image)

**Fig 3.4:** shows that the relation between probability error and SNR for two transmits and two receive Antenna for Alamouti code with Qpsk over Rayleigh fading channel.
Fig 3.5: Shows probability error vs SNR over Rayleigh fading channel for PSK, QPSK, 3-ary and 4-ary in which the number of bit per symbol is 1, 2, 3 and 4 respectively.

Fig 3.6: Compares the Shannon’s Rayleigh fading channel with Alamouti’s channel capacity (The four * in fig represent the value of SNR at which the probability error almost zero for 1, 2, 3 and 4 bits per symbol respectively).
Fig 3.7: shows probability error vs. SNR over Rice channel for PSK, QPSK, 3-ary and 4-ary in which the number of bit per symbol is 1, 2, 3 and 4 respectively.

Fig 3.8: Compares the Shannon’s Rice channel with Alamouti’s channel capacity (The four * in fig represent the value of SNR at which the probability error almost zero for 1, 2, 3 and 4 bits per symbol respectively).

3.4. Transmit and Receive Diversity
In transmit diversity using different kind of modulation and coding technique we can achieve with almost zero probability error whereas in receiver diversity we can achieve it by increasing SNR which is improved by increasing the number of antenna element receiver side . I t is shown below in Fig 3.9 the relation between number of antenna element and SNR.
3.4.1. Simulated Result

As can be seen in Fig 3.10 using Alamouti coding technique we get better error probability performance than SISO. However MRRC with the same diversity order with transmit diversity gives the same error performance if there is no power limitation. In our case, in the graph below MRRC 2x1 seems better than Alamouti 2x1 this because we used transmit energy for Alamouti 2x1 for each antenna element half of the total power. That is why in around 3dB SNR penalty.

![SNR improvement with Maximal Ratio Combining](image1)

Fig 3.9: SNR improvement by increasing the number of receive antenna.

![BER for QPSK modulation with Alamouti STBC (Rayleigh channel)](image2)

Fig 3.10: Bit error performance comparison between Alamouti scheme, MRRC and SISO.
CHAPTER 4: CONCLUSION AND FURTHER WORK

4.1. Conclusion

From simulated result in chapter-3 one can conclude the following points:

1. The Alamouti’s channel capacity very close to Shannon capacity channel that means Alamouti’s code is maybe good code
2. Alamouti’s coding technique we get better error probability performance than SISO. However MRRC with the same diversity order with transmit diversity gives the same error performance if there is no power limitation

3. In transmit diversity using different kind of modulation and coding technique we can achieve with almost zero probability error whereas in receiver diversity we can achieve it by increasing SNR which can be improved by increasing the number of antenna element at receiver side.
4.2. Further Works

This thesis project limited only on simple space time block code (Alamouti’s coding) which is focus only to enhance diversity. However, further works could be done in other schemes which are enhancing both the capacity of channel and diversity. Moreover, MIMO Channel modeling the key area in wireless system and it needs more work.
Reference


[7]” Introduction to multiple Antennas: from SISO to MIMO:” <http://www.conniq.com/WiMAX/mimo-02.htm>


Appendix 1.0:- MATLAB Code for Channel Capacity over Rayleigh Fading
Channel and Probability Error vs. SNR for Alamouti Code.

%%---------------------------------------------------------------------------------
%A Matlab code that simulates a Rayleigh fading channel and implements
the Alamouti’s coder and decoder
%--------------------------------------------------------------------------------- -----
Clear all;

%initialization
SNRv=-20:1:40; %the range of SNR in dB
Cv=zeros(length(SNRv),1); %initialization
pe=zeros(length(SNRv),1);
N=1000; %Number of realization
M=2; %Number of antenna
%
for k=1:length(SNRv)
    SNR=SNRv(k);
    ptot=10^(SNR/10); %total power
    A=sqrt(ptot/2); %the relation b/n amplitude and total power
    count=0; %counter
    C=0; %initialization for channel capacity
%
    for m=1:N;
        m1= floor(4*rand); % random message b/n 0&3
        m2= floor(4*rand);

        sn= A*exp(j*pi/4)*exp(m1*j*pi/2); %modulated signal from antenna 0
        sn1= A*exp(j*pi/4)*exp(m2*j*pi/2); %modulated signal from antenna 1

        H=randn(M,M)+j*randn(M,M); %Rayleigh fading channel with MxM matrix

        %Noise
        v1n=(randn(2,1)+j*randn(2,1))/sqrt(2); %white Gaussian additive noise
        v2n = (randn(2,1)+j*randn(2,1))/sqrt(2);

        cn=[sn;sn1]; %coded signal sent at time t
        cnl=[-conj(sn1);conj(sn)]; % copy of original signal sent at t+T

        r = H*cn+ v1n;
        r1 = H*cnl + v2n;

        %-----------------decoded signals-----------------------------

45
\[ S_n = \text{conj}(H(1,1)) \cdot r(1,1) + H(1,2) \cdot \text{conj}(r_1(1,1)) + \text{conj}(H(2,1)) \cdot r(2,1) + \text{conj}(H(2,2)) \cdot r_1(2,1); \]
\[ S_{n1} = \text{conj}(H(1,2)) \cdot r(1,1) - H(1,1) \cdot \text{conj}(r_1(1,1)) + \text{conj}(H(2,2)) \cdot r(2,1) - H(2,1) \cdot \text{conj}(r_1(2,1)); \]

% -------

% determines m1hat and m2hat which are estimated value of the transmitted messages m1 and m2
% obtained by checking in which quadrant the complex symbols Sn And Sn1
% are -----------------------------------------------
%-------

%---------demodulation for second signal---------

\[
\text{if} \ (\text{real}(S_{n1}) \geq 0 \ \&\& \ \text{imag}(S_{n1}) \geq 0) \quad \%\text{decoding for second signal} \\
\quad \text{m2hat}=0; \\
\text{elseif} \ (\text{real}(S_{n1}) \leq 0 \ \&\& \ \text{imag}(S_{n1}) \geq 0) \\
\quad \text{m2hat}=1; \\
\text{elseif} \ (\text{real}(S_{n1}) \leq 0 \ \&\& \ \text{imag}(S_{n1}) \leq 0) \\
\quad \text{m2hat}=2; \\
\text{else} \ (\text{real}(S_{n1}) \geq 0 \ \&\& \ \text{imag}(S_{n1}) \leq 0) \\
\quad \text{m2hat}=3; \\
\text{end} \\
\]

%---------demodulation for first signal---------

\[
\text{if} \ (\text{real}(S_n) \geq 0 \ \&\& \ \text{imag}(S_n) \geq 0) \\
\quad \text{m1hat}=0; \\
\text{elseif} \ (\text{real}(S_n) \leq 0 \ \&\& \ \text{imag}(S_n) \geq 0) \\
\quad \text{m1hat}=1; \\
\text{elseif} \ (\text{real}(S_n) \leq 0 \ \&\& \ \text{imag}(S_n) \leq 0) \\
\quad \text{m1hat}=2; \\
\text{else} \ (\text{real}(S_n) \geq 0 \ \&\& \ \text{imag}(S_n) \leq 0) \\
\quad \text{m1hat}=3; \\
\text{end} \\
\]

%---------channel capacity---------

\[ C = C + \log_2(\text{det}(\text{eye}(2) + A \cdot A^H \cdot H^H)); \]

%---comparing the transmitted message and received message to calculate error---

\[
\text{if} \ (m_2 \neq m_{2\text{hat}}) \\
\quad \text{count} = \text{count}+1; \\
\quad \text{if} \ (m_1 \neq m_{1\text{hat}}) \\
\quad \quad \text{count} = \text{count}+1; \\
\quad \text{end} \\
\text{end} \\
\]

end
pe(k)=count/(2*N) ; %average probability error
Cv(k)=C/N; % average channel capacity

end

%----------------plot channel capacity vs SNR--------
subplot(2,1,1)
plot(SNRv,Cv,'-go')
xlabel('SNR in dB')
ylabel('Average channel capacity')
title('Average Channel Capacity vs SNR')
grid
hold off
pause(0.1)

% ----------------plots Probability error as function of SNR--------
subplot(2,1,2)
plot(SNRv,pe,'-y*')
xlabel('SNR in dB')
ylabel('Probability Error')
title('Probability Error vs SNR')
grid
hold off
pause(0.01)
Appendix 1.1 MATLAB Code to determine Performance of Alamouti Code

------------------------------------------------------------------------
% Alamouti coder and decoder and comparing the performance of Alamouti code channel with the Shannon capacity for Rayleigh fading channel.
------------------------------------------------------------------------

clear;

%-------------------initialization-----------------------------
SNRv=-20:1:40; %the range of SNR in dB
s2=0.00001;   %variance
m1=1; m2=1;   %mean
Cv=zeros(length(SNRv),1); %initialization
pe=zeros(length(SNRv),1);  %initialization
N=1000;       %Number of realization
M=2;  %Number of anttena
H=randn(M,M)+j*randn(M,M); % Rayleigh fading channel

%H=sqrt(s2)*randn(2,2)+m1*ones(2,2)+j*sqrt(s2)*randn(2,2)+m2*ones(2,2);  % Rice fading channel

for n=1:4
    % n is number of bit per symbol
    for k=1:length(SNRv) % for-loop for SNR
        SNR=SNRv(k);
        ptot=10^(SNR/10);  %total power
        A=sqrt(ptot/2);
        count=0;
        C=0;
        for m=1:N; %for-loop for Number of realization

            switch n

            case 1|3|4
                tet1= floor(2^n*rand(1,1))*(pi/(2^(n-1))); %angles for n= 1 or 3 or 4
                tet2=floor(2^n*rand(1,1))*(pi/(2^(n-1)));
            case 2
                tet1=(pi/4 + floor(4*rand)*pi/2); %angles for n=2
                tet2=(pi/4 + floor(4*rand)*pi/2);
            end

            sn= A*exp(j*tet1);
            snl= A*exp(j*tet2);

        end

    end

end

------------------------------------------------------------------------
%------------------------Noise----------------------------------------------
v1n=(randn(2,1)+j*randn(2,1))/sqrt(2);
v2n=(randn(2,1)+j*randn(2,1))/sqrt(2);

%-------------------coding-------------------------------------------------
cn=[sn;sn1];      % signal transmitted at time t
cln=[-conj(sn1);conj(sn)];      % copy of original signal transmitted at
time t+T

%-------------------received signal----------------------------------------
r = H*cn+ v1n;
rl = H*cln + v2n;

%-------------------decoded signals----------------------------------------
Sn = conj(H(1,1))*r(1,1) +H(1,2)*conj(r1(1,1)) + conj(H(2,1))*r(2,1) +
    conj(H(2,2))*r1(2,1);
Sn1= conj(H(1,2))*r(1,1) -H(1,1)*conj(r1(1,1)) + conj(H(2,2))*r(2,1) -
    H(2,1)*conj(r1(2,1));

%-------------------modulation--------------------------------------------
switch n
    case 1
        %-------------------modulation for n=1 at antenna 0----------------------
        if (real(sn)>= 0 )
            tet11=0;
        elseif (real(sn)<= 0 );
            tet11=1;
        end

        %-------------------modulation for n=1 at antenna 1----------------------
        if (real(sn1)>= 0 )
            tet22=0;
        elseif (real(sn1)<= 0 );
            tet22=1;
        end

    end

%-------------------demodulation------------------------------------------

if (real(Sn)>= 0 )
    theta1=0;
else (real(Sn)<=0 );
    theta1=1;
end

%decoding
if (real(Sn1)>= 0 )
    theta2=0;
else (real(Sn1)<=0 );
end
tetha2=1;

end

case 2
%----------------modulation--------

if (real(sn)>=0 && imag(sn)>=0)
    tet11=0;
elseif (real(sn)<=0 && imag(sn)>=0)
    tet11=1
elseif (real(sn)<=0 && imag(sn)<=0)
    tet11=2;
elseif (real(sn)>=0 && imag(sn)<=0)
    tet11=3;
end

if (real(sn1)>=0 && imag(sn1)>=0)
    tet22=0;
elseif (real(sn1)<=0 && imag(sn1)>=0)
    tet22=1;
elseif (real(sn1)<=0 && imag(sn1)<=0)
    tet22=2;
elseif (real(sn1)>=0 && imag(sn1)<=0)
    tet22=3;
end

%-----------------demodulation--------

if (real(Sn)>=0 && imag(Sn)>=0)
    tetha1=0;
elseif (real(Sn)<=0 && imag(Sn)>=0)
    tetha1=1;
elseif (real(Sn)<=0 && imag(Sn)<=0)
    tetha1=2;
elseif (real(Sn)>=0 && imag(Sn)<=0)
    tetha1=3;
end

if (real(Sn1)>=0 && imag(Sn1)>=0)
    tetha2=0;
elseif (real(Sn1)<=0 && imag(Sn1)>=0)
    tetha2=1;
elseif (real(Sn1)<=0 && imag(Sn1)<=0)
    tetha2=2;
elseif (real(Sn1)>=0 && imag(Sn1)<=0)
    tetha2=3;
end

case 3
%----------------modulation--------------------

if (-pi/8<=angle(sn) <=pi/8)
    tet11=0;
elseif (pi/8<=angle(sn) <=3*pi/8);

tet11=1;

elseif(13*pi/8<=angle(sn1) <=15*pi/8)
tet22=7;

end

%------demodulation---------

if (-pi/8<=angle(Sn) <=pi/8)
tetha1=0;
elseif (pi/8<=angle(Sn) <=3*pi/8);
tetha1=1;

end

case 4
%-----------modulation ---------

if (-pi/16<=angle(sn) <=pi/16)
tet11=0;
elseif (pi/16<=angle(sn) <=3*pi/16);
tet11=1;

elseif(27*pi/16<=angle(sn1) <=29*pi/16)
tet22=14;
elseif(29*pi/16<=angle(sn1) <=31*pi/16)
tet22=15;
end

%-------------demodulation----------

if (-pi/16<=angle(Sn) <=pi/16)
tethal=0;
elseif (pi/16<=angle(Sn) <=3*pi/16);
tethal=1;
elseif (27*pi/16 <= angle(Sn1) <= 29*pi/16)
    tetha2 = 14;
elseif (29*pi/16 <= angle(Sn1) <= 31*pi/16)
    tetha2 = 15;
end

otherwise
    disp('the number of bit less than 4')
end

%----------------channel capacity-------------------

C = C + log2(real(det(eye(2) + A*A*H*H')));

%------comparing the transmitted message and received message to calculate error--------

if (tet22 ~= tetha2)
    count = count + 1;
    if (tet11 ~= tetha1)
        count = count + 1;
    end
end

%----------------probability error and channel capacity---------------------

pe(k) = count / (2*N);
Cv(k) = C / N;
if (pe(k) == 0.001)
    Ca = SNRv(k);
end

end

% --------plots Probability error as function of SNR---------------------

subplot(2,1,1)
plot(SNRv, pe, '-y*')
xlabel('SNR in dB')
ylabel('Probability Error')
title('Probability Error vs SNR')
grid on
hold on
pause(0.01)

%-------------The plot compare the average channel capacity suggested by Shannon with alamouti--------------------------
subplot(2,1,2)
plot(SNRv,Cv,'-go')
plot(Ca,n,'-y*')
xlabel('SNR in dB')
ylabel('channel capacity')
hold on
end