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Theoretical loops in Mathematics education: Theory-practice relationships, revisited

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ABSTRACT. Relationship between theory and practice is a recurrent theme in educational research. This is so also in fields, which are primarily concerned with particular academic disciplines (e.g. science or mother tongue education). In this article I discuss relationship between theory and practice in mathematics education, but present my argument in rather more general terms. The point is that the expectation that there is or should be an immediate impact of educational research on classroom practices is misguided and to some extent inconsistent with important theoretical premises of the field itself these years. This means neither that there is no impact at all, nor that mathematics education research should abandon its general commitment to institute change. However, it does suggest that expectations should be more modest than what is often the case.

Keywords: *theory and practice, theoretical loops, mathematics education, theory-practice relationships*

INTRODUCTION

The relationship between theory and teaching-learning practices is a recurrent theme in mathematics education research, and it is generally expected that the former is to significantly influence the latter. Anna Sfard (2005), for instance, suggests not only that most mathematics education research is committed to improving classroom instruction, but even defines the field as “an exploratory discourse that aims to understand and enhance the practice of teaching and learning” (p. 399).

Elsewhere, Sfard argues that at best theory is suggestive relative to practice (Sfard, 2003). In spite of that, the expectation that mathematics education research is aimed to improve classroom teaching and student learning is widely shared in the research community. Also shared, however, is the worry that in general, actual influence runs short of the expectations (e.g. Kilpatrick, 1992; Lester & Wiliam, 2002; Philipp, 2007). In other terms, the potential and actual influence of the field of research has been a main concern ever since it was established half a century ago, but in retrospect most research initiatives seem to have had disappointing results in terms of impact on classroom practice.

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A variety of reasons have been suggested for the dismal state of affairs. Most relate to the role of the teacher in the mathematics classroom. At times teachers have been expected to incorporate research findings in instruction. From this perspective, however, teachers are often found to be incapable or reluctant to change, especially if their current priorities and practices from an observer's perspective appear incompatible with research recommendations (e.g. ICMI, 1986). This sometimes leads to models of teachers, who lack knowledge or beliefs in comparison with what is suggested by research.

Viewing the theory-practice relationship differently and blaming research rather than the teacher, Bishop (1998) argues that researchers are often engaged in developing local theories and critiquing each others' contributions on the basis of analyses of a few individual cases. He worries that these self-contained theoretical discussions are of little practical value, because of "researchers' difficulties of relating ideas from research with the practice of teaching and learning" (p. 33). This includes their reluctance to take seriously that there are institutional and other contextual factors that significantly influence classroom practice.

And Wiliam (2003), also attending to context, suggests that teaching is not a rule-based endeavour in which universals are brought to bear on a particular situation. According to Wiliam, it rather requires what Aristotle called phronesis, i.e. the practical wisdom to act in the moment on the basis of general principles. Consequently, educational research should not seek eternal truths to be disseminated to teachers. Instead it should develop principles of a certain generality that teachers may integrate into their contextual decision making as they engage in the act of teaching. Wiliam maintains that the main aim of educational research is the improvement of practice, but that a research endeavour that seeks to establish universals is misguided.

In spite of the differences, explanations like the ones above share not only their disappointment with the current lack of impact of research, but also an underlying optimism. The moral seems to be that there are ways of making teachers and researchers change their current approaches and solve the problem.

In the present article I revisit the relationship between theory and practice in mathematics education. My main argument is that although expectations have been modified and explanations for the general lack of influence of theory on practice have changed in recent years, the field is still overly optimistic with regard to the potential impact of research. This argument is based on historical developments in mathematics education, but I suggest that it may also be pertinent to other fields. My intention, therefore, is to phrase the argument in terms that invite a discussion of theory-practice relationships more generally. I do so in two ways. First, I refer to two interconnected developments in mathematics education since the early 1980s, which are shared at least by some other fields of educational research. One of these is a challenge to the notion of the teacher as implementer of research results; the other is the tendency to adopt still more social perspectives when researching teaching and learning. Second, I draw a parallel to developments in curriculum theory in order to relate to a more general perspective on theory-practice relationships.

Before I do so, however, I shall elaborate on the relationship between the two dominant intentions of the theory of mathematics education (cf. Sfard 2005), i.e. the ones of understanding educational processes and contributing to their further development.

Two intentions of educational research

The two intentions of understanding and enhancing mathematics teaching and learning are generally considered highly compatible. Several scholars, for instance, have used Stokes' notion of use-inspired basic research to characterise the field (e.g. Schoenfeld, 2002; Wiliam, 2003).

Stokes (1997) discusses the dimension of basic versus applied research with examples from the natural sciences. Bohr was interested in the structure of the atom, but not concerned with how knowledge of that structure may be applied. Edison wanted to electrify America, but was largely uninterested in theoretical issues. Bohr and Edison, then, are at opposite ends of the basic versus applied spectrum. According to Stokes, however, this one-dimensional description of research has severe limitations. His main example is Pasteur. Pasteur aimed to develop fundamental understandings of micro-biological processes, and should therefore be placed at or near the 'basic research'-end of the spectrum. However, he became equally committed to control these processes for instance in fermentation, and for this reason he should also be placed at the opposite, applied end of the axis linking the two poles.

Stokes' solution to this quandary is to suggest that research should not be described by its position on the single dimension of basic versus applied. Rather, we need to consider two dimensions. One of these concerns the extent to which a particular piece of research is committed to improving practice; the other describes the extent to which it is in quest of fundamental, theoretical understandings (see fig. 1). In this two-dimensional picture Bohr and Edison are placed in the 2nd and 4th quadrant respectively. Pasteur's research takes on positive values on both dimensions, and is an example of what Stokes calls use-inspired basic research. Consequently Stokes places Pasteur in the first quadrant, which Stokes names after him: Pasteur's quadrant.

The suggestion that mathematics education is an example of use-inspired basic research and should go in Pasteur's quadrant seems to dissolve a possible conflict between the two intentions of theorising practice and contributing to its further development. In spite of the apparent compatibility between these intentions, I shall point to a certain tension between them. My main argument is that although they are both laudable aims, the expectation that new theoretical understandings may easily be transformed into improved teaching practices and student learning may be

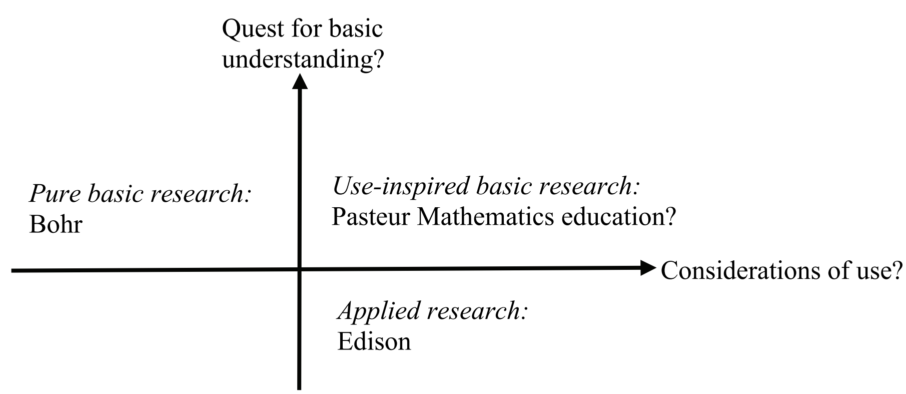

***Figure 1*. Stokes' (1998) two-dimensional view of research (adapted from p. 73).**

overly optimistic. Indeed this expectation appears, somewhat ironically, to be based on an assumption that dominated mathematics education research a couple of decades ago and that run counter to more social theoretical underpinnings of current research in the field. This is the assumption that teachers are primarily to function as implementers of educational reform.

From linear relationships to theoretical loops

The late 1970s, a time for contemplation and retrospective analysis of the new maths era, marked a shift of focus in mathematics education research and development. This is so also as far as the relationship between theory and practice is concerned.

New maths was inspired by two sets of theoretical developments. First it was informed by the more general structures-of-the-disciplines approach to education at the time. The intention was to teach the scientifically fundamental structures of the subject also at the school levels. Consequently, subject matter specialists played a key role by conducting the content analyses that became the mathematical basis of new maths. Second, new maths was informed by clinical studies of learning. Two aspects of Piaget's work became especially important in this respect, namely the stage theory and what appeared to be a remarkable resemblance between the structures of children's thinking and the structures of the mathematical content as advocated by the proponents of new maths. Between them the analyses of mathematics and the clinical studies of students' learning were transformed into sets of curricular materials, which in turn were expected to structure or at least inform life in mathematics classrooms in ways that ensured the students' learning of fundamental mathematical structures.

In the 1970s there was considerable discussion and bewilderment about why learning as intended did not occur in new maths: If the best scholars in the fields of mathematics and learning theory had been involved, what went wrong? Bauersfeld (1979) suggested that the disappointing results could be explained exactly by the exclusive interests in the mathematical contents per se and in clinical studies of student achievement. This concern with what he called "the matter meant" and "the matter learnt" left out the link between the two in institutional contexts, i.e. "the matter taught". In Bauersfeld's own terminology,

> both research and development had focussed on only one of two main determinants of the learning process: the pupil or the curriculum. They did not consider the influence of the teacher nor of the general context of instruction. (p. 200.)

Bauersfeld's point reflects the two intentions of understanding practice and contributing to its further development. On the one hand he has a theoretical interest in understanding the role of teachers and contexts for student learning in mathematics classrooms. On the other he is also and possibly primarily concerned with a developmental aspect: how may student learning of mathematics be supported by improved teaching methods?

Since Bauersfeld wrote this article, teaching and teachers have become pivotal concerns of mathematics education research, and there seem to have been two major responses on the part of the research community to the call for a stronger emphasis on the roles of the teachers and contexts. One of them concerns teachers as the missing link between curricular intentions and student learning, i.e. as implementers of educational reform. The other investigates the role of teachers from a more social perspective and views teachers as participants in emerging practices. I shall discuss the two of them in turn.

Teachers as implementers of reform

In 1986 the International Commission on Mathematical Instruction (ICMI) published a study on *School Mathematics in the 1990s* (ICMI, 1986). Like other ICMI-studies, it sums up the views of a substantial part of the research community, this time on the relationship between visions for school mathematics and how they may be realised. The aim of the book is summarised as follows:

> This ICMI Study is intended to help those who wish to form a vision of what school mathematics might be in the 1990s and who want to work towards the fulfilment of specific goals. In doing this it draws on the experience of the past thirty years which have taught us that what is desirable might not be attainable; and that goals must be set which acknowledge the existence of constraints. (ICMI, 1986, the text on the back of the book).

In line with this, it is a main concern of the study to discuss the problematic relationship between visions for the school subject and the limitations on their realisation, i.e. what is often referred to as the problems of implementation.

School Mathematics in the 1990s presents a set of themes that may be considered when forming a vision for the subject. These include mathematics in a technological society, mathematics and general educational goals, and the aims and contents of school mathematics. In relation to each theme, one or more key problems are outlined, and a number of alternative ways of addressing the problems are listed. Finally a set of possible consequences of each alternative are described. To some extent, then, the group behind the study avoids prioritising their own suggestions for reform. Instead they discuss possibilities and potentials.

The section on *Classrooms and teachers in the 1990s* begins with the regret that the bulk of mathematics classrooms are dominated by "a stereotyped form of teaching which relies heavily on the textbook and the traditional teaching pattern of exposition-examples-exercise" (p. 75). The teacher struggles hard under different types of pressure, and due consideration should be given to the circumstances under which teachers work. Also, some of the suggestions for revised curricula and new forms of evaluation may increase the pressure on teachers. Finally, it is acknowledged that teachers' conceptions and solutions to the problems of mathematics teaching and learning may be useful. The study, then, to a large extent avoids condemning the ways in which teachers handle their obligations.

In spite of this, the study seems based on a technical rationality that sees teachers as the key obstacle to educational reform, i.e. as the most significant "constraint" for the realisation of a new vision for school mathematics (cf. the quotation above). This is evident from the concluding chapter on *The processes of change*. In the last section of this chapter the authors do not formulate different solutions to the problems raised and outline the consequences of each. Instead they acknowledge the key role of the teacher for the realisation of educational reform, and raise one question related to that role:

> […] the vast majority of teachers of the 1990s are already in post and have firm ideas about their role in the school and clear expectations regarding both the curriculum and their students. Significant changes in school mathematics will only be achieved if there are marked changes in the perceptions and attitudes of these teachers and if they are assisted to develop necessary new skills.

> How can one attempt to change attitudes, values, skills, teaching styles, etc. and develop confidence in the use of new methods and technology? (ICMI, 1986, p. 94)

Teachers, then, are primarily seen as implementers of reform. This is highly compatible with traditional development-implementation approaches to educational change. It focuses on teachers as the missing link or even the barrier between what in Bauersfeld's terminology is the matter meant and the matter learnt, when the latter is considered in the institutional context of the school rather than in clinical situations. As a consequence it also focuses on teachers as the main 'problem of implementation' (cf. fig. 2).

Teachers as participants in emerging practices

The other major response on the part of the research community to the challenge of understanding the matter taught (cf. Bauersfeld, 1979) adopts a different perspective on classrooms and teachers. From this perspective teachers are not primarily seen as implementers of curricular initiatives, but as participants in practices that emerge in the classroom. Bauersfeld himself is a dominant advocate of this approach.

Ten years after he wrote the article quoted previously, Bauersfeld looks back on his own work from the 1970s onwards. At this point he claims that his interest in the role of the teacher and the general context of learning has generally been replaced by one in teacher-student relationships and in the social interaction in the classroom. Elaborating on this last point he says:

> […] we learned about the relative symmetry of classroom actions: Both teacher and students contribute to the classroom processes. It is a jointly emerging 'reality' rather than a systematic proceeding produced or caused by independent subjects' actions […] Teacher and students jointly create the reality of the classroom. (Bauersfeld, 1988, p. 29-30)

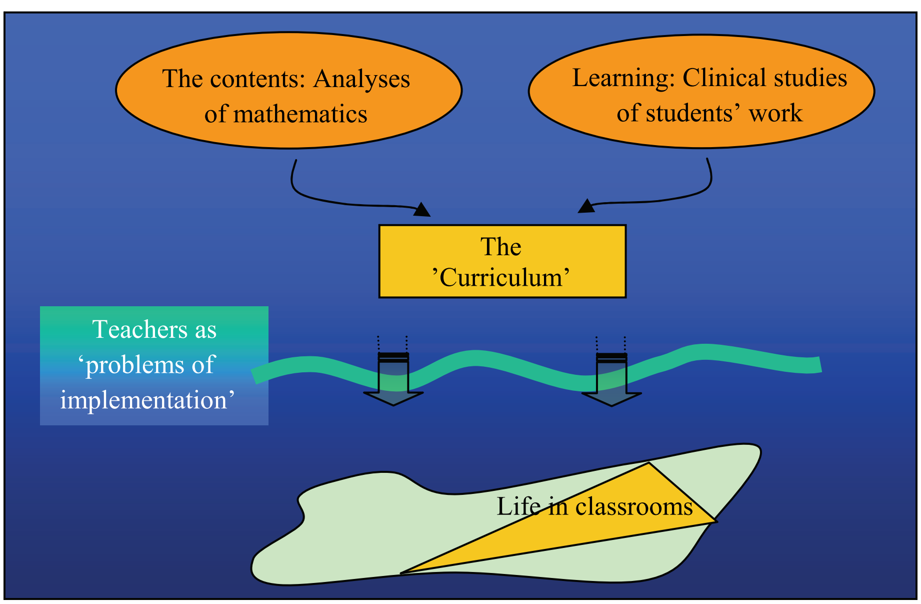

***Figure 2*. Teachers as "problems of implementation"**

Bauersfeld's point, then, is that in order to understand classroom interaction one has to perceive it as such. One has to focus not on the alternating actions of teachers and students as cause and effect respectively, but on the evolving patterns and the "intersubjective constitution for norms for action" (p. 32). This is a perspective that views interactions as both influenced by and continually (re-)generating local contexts. And it does not see teachers as the main problem of implementation, but implies a shift from problems of implementation to problems of "implementation": It questions the very notion of the term if understood in the traditional sense outlined previously.

Bauersfeld's remarks in the late 1980s are – as are the ones from 1979 – indicative of broader developments in mathematics education research. Inspired by Blumer (1969) and symbolic interactionism, they may be considered both a signal of and a contributing factor to the more social orientation of the field from then onwards. Since the late 1980s, then, research has to an ever-increasing extent moved into the classroom in order to theorise emerging classroom practices and more specifically to investigate how learning opportunities evolve for the students, and what roles teachers' knowledge and beliefs may play for them.

This research is often based on small-scale, qualitative and sometimes collaborative studies of the interactions in one or a few classrooms, and it has contributed with novel understandings of for instance classroom communication and student reasoning (e.g. Lampert & Blunk (eds.), 1998; Cobb, Stephan, McClain, & Gravemeijer, 2001), the character of mathematical knowledge for teaching (e.g. Ball & Bass, 2000; 2003; Rowland, 2008), and relationship between current reform recommendations, teachers' beliefs and the classroom practices (e.g. Leder, Pehkonen & Törner (eds.), 2002; Skott, 2009). Many of these studies are meant to inform teacher education practices and subsequently to improve instruction in schools.

Cobb, Boufi, McClain and Whitenack present an example of the intention to theorise emerging practices (Cobb et al., 1997). They describe an instructional sequence in a grade 1 classroom, in which the teacher uses a table to symbolise the students' different solutions to the task of how many monkeys could be in either of two trees, if there are five monkeys altogether. The table becomes the means to shift the students' attention from finding answers to the initial question to more interesting meta-concerns of whether all possible solutions have been found, and how one can be certain to have found them all. In this shift the initial solutions to the task become the object of investigation and discussion in a process that Cobb and his colleagues describe as reflective discourse. This is a communal counterpart to Piaget's notion of reflective abstraction, as the students' activity as well as the results of this activity become objects of discussion in the classroom community. Although Cobb and his colleagues do not explicitly suggest that the notion of reflective discourse is made an object of study in mathematics teacher education, it is tempting to do so. For instance, prospective teachers could work with how locally derived symbolisations that facilitate meta-cognitive shifts may support reflective discourse and influence student learning.

Jaworski's notion of *the teaching triad* (1994) is another example of the intention of theorising practice. Jaworski developed the concept as a description of how a constructivist teacher may balance three interdependent aspects of his or her activity. The three aspects are the ones of managing students' learning, reacting sensitively to their needs and challenging them mathematically. The relative emphasis on each of these depends at any point in time on the evolving classroom practices. Later Potari and Jaworski suggested using the teaching triad as a reflective tool in teacher education (Potari & Jaworski, 2002).

As a further example, one may consider Skott's notion of critical incidents of practice or CIPs (2001). These incidents are characterised by the emergence of multiple and often conflicting goals of the teacher's activity, as he or she interacts with the students. Also, CIPs both challenge the teacher's school mathematical priorities and are critical to the further development of the classroom processes and the students' learning. Elsewhere Skott suggests using CIPs from prospective teacher's own practice as focal points in teacher education (Skott, 1999).

These examples illustrate that mathematics education has become an empirical field aiming to understand the learning opportunities that emerge in the classroom. Each of the studies goes through a theoretically informed process of theorising practice, i.e. one of developing constructs that contribute with novel ways of understanding issues emerging from the interactions of the classroom in question. In turn, new theoretical constructs gain at least part of their legitimacy from their ability to guide instruction.

The approach to research in these studies differs fundamentally from the tradition described by Bauersfeld in 1979. The latter was directed towards practice in the sense that it intended and expected to have significant impact on instruction and student learning. As pointed out earlier, however, it was distant from practice in its theorising. In the studies referred to above, this linear movement from theory into practice has at least in part been reversed. Classroom based research is now concerned with theorising, i.e. it is directed towards theory development, but the theorising is grounded in practice (though not necessarily in the technical sense of grounded theory). Generally, however, it is still expected that research is to influence practice. Indeed, the main reason why mathematics education research moved into the classroom was to ensure that its recommendations proved valid in the sense of referring to mathematics teaching and learning in relevant institutional contexts. This means that the previous linear movement from ends to means to outcomes is replaced by a theoretical loop from practice to theory to practice (cf. fig. 3). As indicated above, the latter half of the loop is to be facilitated primarily as the results become part of the education of prospective and practising teachers.

A social turn in mathematics education

To the extent that the above examples are indicative of more general developments in mathematics education, it is apparent that the two intentions of theorising practice and contributing to its further development still orient the field. Also, the emphases of the former of these two intentions seem based on the premise that it is necessary to take context seriously to fulfil the latter intention. Phrased differently, the premise is that mathematics education research is only likely to have an impact on instruction in ordinary classrooms, if the contextual character of teaching and learning is acknowledged. However, there is an inherent conflict or at least a certain tension between the increasing attention to context and social interpretations of classroom processes on the one hand and the expectation of a smooth and conflict-free run of the second half of the theoretical loop on the other. This is the case if the necessary condition of considering context in research is also regarded as sufficient to ensure the suggested impact on practice. To

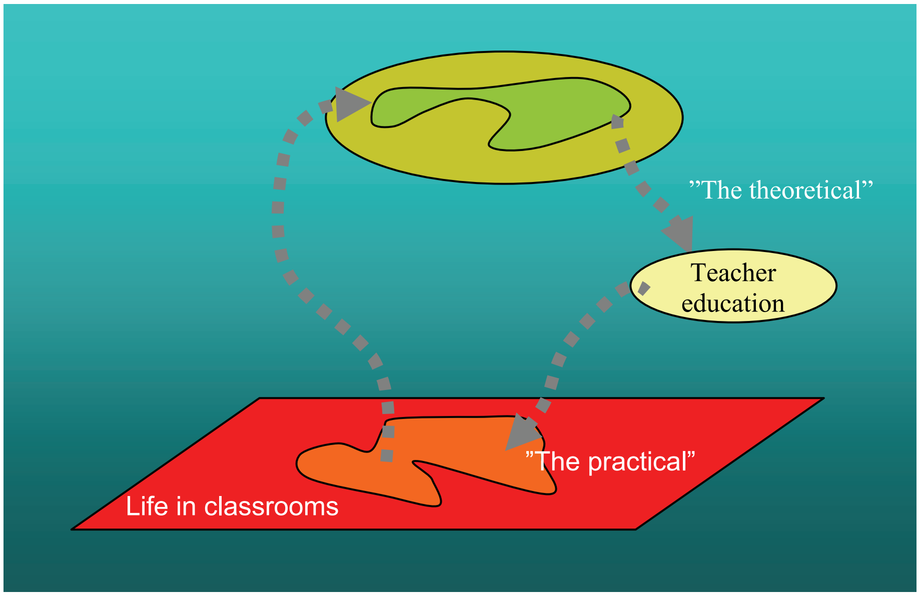

Figure 3. **The theoretical loop.**

make this point, I shall briefly discuss the notion of context.

The attention to context is an aspect of what Lerman (2000) calls *the social turn* in mathematics education. Lerman defines the social turn as "the emergence into the mathematics education research community of theories that see meaning, thinking, and reasoning as products of social activity" (p. 23). He links the social turn to the interest in Soviet psychology (especially Vygotsky) and social practice theory (especially Lave and Wenger) from the mid 1980s. The increasing number of references to symbolic interactionism at the same time may be seen as yet another part of the tendency to look beyond the individual to notions of context in an effort to account for the social character of mathematics teaching and learning in the classroom.

The notion of context has been used in a variety of different ways (Skott, 2009). Lave (1996) points to a major distinction between the ways in which the concept is used, when she says that socio-culturally informed studies tend to emphasise "how it is that people live in *history*", while studies more in line with symbolic interactionism prioritise "how it is that people *live* in history" (p. 21, emphases in original). In either case, however, context is viewed not merely as time and space as unproblematic and external frames of the event or activity in question. Context is considered intrinsically related to practice and mutually constitutive of it.

This understanding of context contrasts with the everyday usage of the term, when context is viewed as "a container into which things are placed […] [as] the 'con' that contains the 'text', the bowl that contains soup" (McDermott, 1996, p. 282). This latter sense of context sits well with the traditional view of implementation. In such a view the teacher is expected to pursue curricular intentions in the classroom and ensure that they are realised in practice. As the bowl should not alter the substance of the soup (cf. McDermott), the school and the classroom are expected not to significantly influence the substance of 'the teacher's practice'.

It is somewhat ironic, however, if the expectation remains that the teacher is to carry research findings into the classroom, also when the findings are the result of attempts to theorise the emerging classroom practices. In this case, the appreciation of the role of context that informs the research process is mysteriously lost when it comes to expectations of impact. Or at the least it is an implicit assumption that the contextual character of the classroom practices that are the object of study correspond closely to the context also in classrooms in which the findings are to be 'implemented'. But a teacher's context, i.e. the framework that orients her actions in the classroom, is established and continually regenerated as she manoeuvres in relation to the multiple simultaneous commitments that evolve. It is not determined beforehand by time and place, but emerges as an element of and continually contributing to the jointly established practices in the locally social (Skott, 2009).

The examples in the previous section may illustrate the point. The notions of meta-cognitive shifts, of the teaching triad, and of critical incidents of practice were all based on contextually sensitive analyses of the role of the teacher for classroom interaction. They may also be valuable as reflective focal points for other teachers. For instance the idea of meta-cognitive shifts may enable teachers to ask themselves the question of whether a new symbolisation of the students' results may become an object of collective inquiry and discussion and may in turn facilitate a shift in their approach to meta-issues related to the task. However, it does not only depend on the teacher's knowledge and beliefs, whether this becomes an option in a particular situation. Rather, it depends on how the intention of supporting a meta-mathematical discourse relates to the context that evolves in the classroom and the school in question at the instant.

A parallel to curriculum theory?

The relationship between theory and practice has also been a topic for discussion and controversy in curriculum theory (e.g. Hlebowitsch, 1999; Pinar & Grummet, 1981; Schwab, 1997/1967). Somewhat surprisingly, however, these debates were largely unnoticed in mathematics education.

Curriculum theory prior the 1960s is often described as a technical and administrative endeavour aiming to develop and describe the aims, scope, sequence and assessment of educational experiences. Ralph Tyler's book on *The Basic Principles of Curriculum and Instruction* is paradigmatic in this sense (Tyler, 1949). In the 1960s the science centred approach of the time meant that these administrative functions were at least in part taken over by scholars in the different disciplines, for instance mathematicians and scientists. Consequently, traditional curricularists lost part of their managerial functions. This was one part of the background to Schwab's opening remark in the first of his seminal articles on 'the practical': "the field of curriculum is moribund" (Schwab, 1997/1969, p. 101). Referring to Schwab's comment and to Bobbitt's much older book on curriculum development (Bobbitt, 1918), Pinar and his colleagues declared the field of curriculum not only moribund, but dead: "Curriculum development: born: 1918; died: 1969" (Pinar et al, 1995, p. 6).

It has been suggested that the reconceptualist description of the traditional curriculum field as an administrative attempt to order and structure the functioning of schools does not do justice to intentions and accomplishments of early curricularists (Hlebowitsch, 1999). Also, the claim that curriculum theory is no longer engaged in considerations of how classroom teaching and student learning may be improved, i.e. in curriculum development, seems to be somewhat of an exaggeration. In spite of this, the 1970s marked a significant shift in the field. From then onwards, curriculum theory has often been more concerned with "understanding curriculum" than with curriculum development (Pinar et al., 1995). A significant part of the field of curriculum, then, is no longer primarily interested in organising the activity of schooling, but in using a variety of perspectives to theorise curricular texts as well as the experiences of students and teachers in school. Emphasising the latter of these interests Clandinin and Conelly (1992) state that

> Teachers and students live out a curriculum; teachers do not transmit, implement, or teach a curriculum and objectives; nor are they and their students carried forward in their work and studies by a curriculum of textbooks and content, instructional methodologies, and intentions. An account of teachers' and students' lives over time is the curriculum, although intentionality, objectives, and curriculum materials do play a part in it. (p. 365)

Theory-practice relationships in curriculum theory changed from the early 1970s in ways that at least in part resemble the changes in mathematics education that were to follow a decade later. One major objective is to understand life in classrooms, i.e. to theorise the curriculum in the enacted sense of term.

Almost simultaneously with Bauersfeld's recommendation that mathematics education research should move into the classroom to focus on the matter taught, Pinar and Grummet reflected on theory-practice relationships in curriculum theory (Pinar & Grummet, 1981).
Referring extensively to Aristotle they argue that in his view, theory is the result of contemplation on practice. It is the cultivation of a point of view. It is not, however, primarily legitimised through its possible prescriptions for practice. Theory is considered a separate and superior activity with its own values, possibly inspired by, but still distinct from the practical concerns of everyday life. According to Pinar and Grummet the alternative that theory is to guide practice is a much later notion inspired by Bacon and the scientific revolution of the 17th Century. From then on theory changes its role and status, and it becomes the answer to the question of how to control the future.

Pinar and Grummet suggest that this was the role of curriculum theory until the 1960s. It was to control life in schools and classrooms. From then onwards, however, what they see as an apolitical and technical rationality of the field was challenged by a more theoretical approach that aimed "to drive a wedge between theory and practice by suspending the instrumentalist intention" (p. 31). This does not mean that this part of curriculum theory necessarily positions itself as distant from the practices of teaching and learning in schools. On the contrary, a significant part of the field has moved into the schools in order to take context seriously and theorise emerging classroom processes. However, it maintains not only the distinction, but the separation between theory and practice and suggests that the main purpose of the former is not to inform the latter: theorising life in classrooms is an end in itself.

It seems, then, that curriculum theory and mathematics education share the intention of understanding emerging classroom processes. Reconceptualised curriculum theory, however, has also set a different agenda for itself than "keeping the curriculum ordered and organised" (Pinar, et al, 1995, p. 6). This may not be the sole purpose of mathematics education as a research domain either, but this field is still committed to educational development. In fact, the larger part of what is often referred to as *the reform* in mathematics education (e.g. Skott, 2004) may be seen exactly as an attempt to propose curricula that may inform the ways in which the subject is taught and learned in mathematics classrooms, while acknowledging that learning potentials are significantly influenced by practices that emerge in the locally social.

Ex ante and ex post evaluations of reform initiatives

As mentioned before it has been suggested that mathematics education may go into *Pasteur's quadrant* in Stokes' model of research. One may wonder, however, if *use-inspired basic research* is a proper description of the field, or whether its intentions may not be more appropriately described by reversing the order of the terms, i.e. by 'theoretically informed applied research'. Applicability seems to be the main argument legitimising the field.

One of the conceptual issues that Stokes analyses in relation to his quadrant model is whether the applied and/or basic character of research should be judged primarily on the basis intended goals or of the results. This discussion of the *ex ante* or *ex post* characteristics of research relates also to mathematics education. Development in the field over the last half century indicates that it is primarily an ex ante expectation rather than a result of an ex post analysis, if it is characterised as theoretically informed, applied research. Indeed, the general concern that research has not had the expected impact on practice indicates recognition of a regrettable discrepancy between the results of pre- and post evaluations.

The point is not that mathematics education research has not had an impact on the practices of mathematics teaching and learning in ordinary classrooms. Indeed, new maths in many ways significantly influenced the content of the school subject as well as the ways in which it was taught and learnt. Also, the constructivist movement that took off in the 1980s and that is still highly influential transformed the roles of students and teachers in many mathematics classrooms, including the types of investigative activities that involved students. Current reform initiatives, which are inspired for instance by Lakatos' fallibilist philosophy of mathematics and emphasise students' individual and communal engagement in processes of for instance reasoning, proving and refuting, have contributed to changing the dominant modes of communication in many mathematics classrooms. However, in retrospect it is seems that in none of the three cases, did the unquestionable impact resemble what was envisaged by the initiators of the reform.

CONCLUSIONS

At present mathematics education research generally aims to theorise the practices that unfold in mathematics classrooms as well as contributing to their further development. In order to do both, it has adopted still more social perspectives on mathematics classrooms. The point is that the attention to context inherent in the social turn is lost when it comes to expectations of impact. In other terms, there is an expectation that research results from one "context" may be carried over so as to dominate teaching-learning practices also in other classrooms, even in profoundly different social circumstances.

If school and classroom practices emerge in the locally social, as does any other form of practice, they should not be seen as 'implemented' or watered down versions of educational reform initiatives, even if the latter are the result of attempts to theorise what appears to be similar practices elsewhere. Rather, practices are continually regenerated as students and teachers negotiate the meaning of schooling, of mathematics, of teaching, and of learning as well as of other significant aspects of their ways of being together in the classroom. These socially constructed meanings come to form the social fabric of the mathematics classroom, and they are basically made from a different substance than any reform discourse, past or present. For the mathematics education research community, this means that the expectation of an almost immediate impact on classroom instruction and student learning through what I called a smooth and conflict-free run of the second half of the theoretical loop is overly optimistic.

I previously referred to Lakatos' and his inspiration for current reform initiatives. In his most frequently cited work, Lakatos gives a fascinating account of the historical development of the Euler formula for polyhedra (Lakatos, 1976). He does so by having a teacher in an imaginary classroom present the formula and a proof originally developed by Cauchy. After the teacher's presentation, however, both the proof and the theorem itself are challenged by the 'students' (called Alpha, Beta, Gamma, etc.), and over the next 100 pages proofs are modified, lemmas are introduced, and the concept of polyhedron is redefined in order to support the initial claim. At some point a student presents a counterexample to the formula. The teacher's reaction is to suggest that they need to find a way out of the quandary. The student vehemently denies a need for a further discussion of the theorem, which he has now proved wrong once and for all. The teacher, however, insists and says that he is interested in proofs, "even if they do not accomplish their intended task. Columbus did not reach India but he discovered something quite interesting" (p. 14). After Columbus, then, why does anybody ever expect to find what they look for? After Columbus, who would want to give up looking, even if whatever they do find is unlikely to be what they look for?

The preceding questions call for a parallel in education. Based on the experiences of the last half century, why does anyone in mathematics education ever expect to have the kind of impact they intend to have? Considering what most mathematics education looked like half a century ago and what it looks like today, who wants to give up trying to have an influence, even if whatever impact one may have unlikely to be what is expected?

The moral of the story, then, is not that mathematics education should give up its commitment to practice like the larger part of curriculum theory. Neither is it that the field should shift its emphasis away from theorising local classroom practices in order to do so. On the contrary, such theorising has proved essential both in order to develop significant new understandings and to provide some impetus for change. After all, one may be interested in supporting educational development, even if the social character of teaching and learning means that there are no technical means of ensuring that results comply with intentions.

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