Degree project

How fractions are introduced in compulsory school
A comparative study of Grade 6 textbooks in Turkey and Sweden

Author: Nazlı Irmak Yıldırım
Date: 2012-01-09
Subject: Mathematics
Level: Bachelor
Course code: 2MA11E
Abstract

The purpose of this study is to provide a detailed description of the way fractions are processed in Turkish and Swedish mathematics textbooks. For our investigation of textbooks, we used several theoretical notions such as different ways of understanding fractions (part-whole, ratio, measure, quotient, decimal and operator), representations (treatment, conversion), making distinctions between routine tasks and problems, classifying contexts of examples and problems in terms of real-life situations, semi-realities, and pure mathematics, and about texts narrative and paradigmatic styles.

Through our analyses of these three textbooks, we have come to a result that shows similarities and differences among the textbooks, regarding the topics included and their sequencing, how the concept of fractions is represented in exercises and examples. The findings are emphasized with figures, tables and diagrams that illustrate the similarities and differences between mathematics textbooks in Turkey in Sweden as well as the textbooks’ development in Turkey.

Abstrakt

Syftet med denna studie är att ge en detaljerad beskrivning av hur bråk behandlas i turkiska och svenska läroböcker för matematik. I vår undersökning, använde vi flera teoretiska begrepp såsom olika sätt att förstå bråk (del-helhet, förhållande, mått, kvot, decimaltal och operator), representationer (behandling, omvandling), att skilja på rutinuppgifter och problem, att klassificera exempel och uppgifters kontexter i termer av verkliga situationer, semi-verkligheter, och ren matematik, och om texters berättande och paradigmatiska stilar.

Genom våra analyser av dessa tre läroböcker, har vi kommit fram till ett resultat som visar både likheter och skillnader mellan dem, både vad gäller de ingående ämnen och deras sekvensering och hur begreppet bråk har representerats i övningar och exempel. Resultaten beskrivs med bilder, tabeller och diagram som illustrerar likheter och skillnader mellan matematikläroböcker i Turkiet och Sverige och dessutom visar läroböckernas utveckling i Turkiet.
# Table of Contents

1. Introduction .................................................................................................................. 4

2. Research questions......................................................................................................... 5

3. Theoretical background ................................................................................................. 6
   3.1 Different ways to understand fractions ..................................................................... 6
       3.1.1 Part-whole comparison ................................................................................. 6
       3.1.2 The ratio perspective ..................................................................................... 6
       3.1.3 The decimal subconstruct ........................................................................... 6
       3.1.4 The operator ................................................................................................. 7
       3.1.5 An indicated division (quotient) ................................................................... 7
       3.1.6 The measure ............................................................................................... 8
   3.2 Different ways to represent mathematical objects ..................................................... 9
       3.2.1 Description of mathematical objects ............................................................ 9
       3.2.2 Representations, treatment, and conversion ................................................... 9
       3.2.3 Relationships between mathematical objects and representations ............... 10
   3.3 Different types of examples and exercises ............................................................... 10
       3.3.1 References to pure mathematics ................................................................. 10
       3.3.2 References to semi-reality ........................................................................... 11
       3.3.3 Real-life references ...................................................................................... 12
       3.3.4 Generic examples ......................................................................................... 12
       3.3.5 Routine tasks ............................................................................................... 13
       3.3.6 Problems ..................................................................................................... 13
   3.4 Different styles of texts .............................................................................................. 14
       3.4.1 Narrative style ............................................................................................... 14
       3.4.2 Paradigmatic style ......................................................................................... 14

4 Method .......................................................................................................................... 15

5 Results and Analysis ..................................................................................................... 16
   5.1 Background information about textbooks ............................................................... 16
   5.2 Structure of the textbooks ....................................................................................... 17
       5.2.1 Structure of the Turkey 2010 textbook ......................................................... 17
       5.2.2 Structure of the Sweden textbook .................................................................. 20
       5.2.3 Structure of the Turkey 2005 textbook ......................................................... 24
       5.2.4 Preliminary comments on the textbooks’ structures ..................................... 29
       5.2.5 The textbooks’ introduction of addition of fractions .................................... 30
5.3 Character of the textbooks’ examples and exercises

5.3.1 The first examples of fractions in each textbook
5.3.2 Examples – use of representations
5.3.3 Examples – different ways to understand fractions
5.3.4 Exercises – use of references with routine task and problems
5.3.5 Additional observations and association between criteria

6 Discussions

6.1 Methodological discussion
6.2 Result and analyses discussion
6.3 Further research

References
1 Introduction

Comparative studies between countries are always providing to improve people and to learn new techniques. Concerning education, such studies are particularly useful and provides benefits for new generations, teachers and curriculum designers. Moreover, comparative work between developed and developing countries will help to find solutions for their educational problems (Tezcan, 1985).

Mathematics education has a significant role in education systems all around the world, for every country. Since mathematics is always present in our life and has its own language, symbols; we could say it is important. However, mathematics is sometimes hard to learn.

Critical mathematics education emphasises that mathematics as such is not simply a subject to be taught and learnt (no matter whether the processes of learning are organised according to a constructivist or a sociocultural approach).

(Skovsmose, 2001, p.123)

Beside the teacher, textbooks have an essential role when we improving our skills in mathematics.

Swedish secondary and undergraduate mathematics students normally spend the main part of their study time trying to solve textbook exercises. This is the way students are supposed to practice and learn mathematics in order to be able to apply their knowledge in other situations, for example in their further studies, in their future professional life, or in their everyday life as members of the modern society.

(Lithner, 2003, p.2)

This study represents one 6th grade mathematics course subject “Fractions” and examines both general and particular perspectives of the textbooks from two countries, Sweden and Turkey. In addition, we examine how Turkish textbooks have developed over time. The reasons why we choose fractions are, fractions are making link with other number types and algebraic operations in later years of students’ school experience.

Many student difficulties in algebra can be traced back to an incomplete understanding of earlier fraction ideas.

(Behr et al., 1993, p.3)

And at the same time, this course subject was clearly included in the 6th grade school curricula for both countries. Using one course subject in textbooks from two different countries and its development over time in one country will highlight specific differences and similarities in textbooks. To identify differences and similarities we have chosen a wide variety of criteria. These criteria will help us examine textbooks in several different respects.
2 Research questions

The research goal for this thesis is to describe how fractions are introduced and presented in textbooks in Turkey and in Sweden, and account for similarities and differences regarding this introduction which occurs in Grade 6 in both countries. Furthermore, we will consider also how the textbooks in the Turkish curriculum have developed over the past five years by comparing two different textbooks.

Specifically, the study tries to answer the following questions;

- What kinds of exercises and examples in terms of different representations of fractions have been used in Turkish and Swedish textbooks?

- What similarities and differences can be observed in the introductory presentation of fractions in Grade 6 mathematic textbooks in Sweden and in Turkey?
3 Theoretical background

Textbooks are indispensable elements of education. And they are also multi-faceted educational tools. The textbook is an important resource in support of teaching and learning. Every student has different type of perception for mathematics. Due to this reason, mathematics textbook needs to include different types of representation. For fractions, beside visual representations, there are different forms that are important part of learning fractions with all aspects. In this section we will introduce theoretical views of fractions.

3.1 Different ways to understand fractions

Concepts of fractions are examined by many authors. Most of them has identified and discussed different subconstructs of rational numbers. According to Behr et al. (1993) rational numbers can be interpreted, as a part-whole comparison, a decimal number, a ratio, an indicated division (quotient), as an operator and as a measure. Moseley (2005) also accounts for several different ways of categorizing schema for the understanding of fractions. To consider these ways in textbook yield us explanatory information about textbooks’ teaching techniques. In the next sections each of them are discussed.

3.1.1 Part-whole comparison

The part-whole comparisons are generally introduced in the early grades in primary schools. This model is the most natural for young children (Behr et al., 1993). We can easily introduce fractions this way. For instance, with concrete objects, whole apple, half apple and quarter apple. This perspective of rational number emphasizes situations which are related to a part comparison with total amount (Moseley, 2005).

► In the city zoo, there is a lion. He had 12 kilos of food all to himself and ate 8 kilos. What part of his food did he eat?

3.1.2 The ratio perspective

The ratio perspective of rational numbers is based on comparing separate quantities. While part whole fraction seldom represents a counting of the partitions which would complete the whole, a ratio shows a relationship between two quantities that is less anchored to a direct counting of the number of parts (Moseley, 2005). For instance,

► There are 20 students and 25 desks in the classroom. What is the relation between the number of desks and students in the classroom?

3.1.3 The decimal subconstruct

The decimal subconstruct of rational numbers concerns properties associated with the base ten numeration system (Behr et al., 1993). We can interpret a decimal number (with finitely many
decimals) as a ratio with denominator ten or multiples of ten, such as $0,37 = \frac{37}{100}$. We can also translate from ratio to decimal representation, such as $\frac{1}{4} = \frac{25}{100} = 0,25$ (However, in some cases we get infinitely many decimals.)

Joe and Mary are friends. They are both good at basketball. They made competition each other. Joe threw 25 shot and 21 of them were accurate shot. Mary threw 20 and 16 of them were accurately shot. Calculate their success of basketball shots.

Joe $\frac{21}{25} = \frac{84}{100} = 0,84 \rightarrow 84 \%$

Mary $\frac{16}{20} = \frac{80}{100} = 0,80 \rightarrow 80 \%$

### 3.1.4 The operator

As an operator, we can identify the rational number with a function that transforms geometric figures to similar geometric figures $\frac{a}{b}$ times as big or small when $\frac{a}{b}$ is a rational number. This interpretation of rational number is suitable and useful in studying equivalence of fractions and the operation of multiplication (Behr et al., 1993).

Moseley (2005) also narrates that a student sees a rational number as a tool that will change the properties of a set in the operator perspective. While the relation between two sets defines a ratio, the relation between two states of set describes an operator. With an example,

One-third of 18 apples is 6. We multiply 18 with $\frac{1}{3}$. However we divided 18 by 3. If we have said two-thirds of 18 apples then a student may think “divide by 3 and take 2 of them”, as an operation which gives the same result as multiplication by $\frac{2}{3}$.

With a similar example in Behr et al. (1993) the authors interpret that “The general fractional operator appears to require the coordination of the partitioning of two subsets of numbers with a multiplicative operation, in this case doubling.” (p.5)

### 3.1.5 An indicated division (quotient)

The interpretation of $\frac{a}{b}$ as $a$ divided by $b$ is the subcontract of a rational number as an indicated quotient (Behr et al., 1993). For explaining the differences between interpretation of rational numbers the authors state that

According to the part-whole interpretation of rational numbers, the symbol $a/b$ usually refers to a fractional part of a single quantity. In the ratio interpretation of rational
numbers, the symbol \(a/b\) refers to a relationship between two quantities. The symbol \(a/b\) may also be used to refer to an operation. That is, \(a/b\) is sometimes used as a way of writing \(a \div b\). This is the indicated division (or indicated quotient) interpretation of rational numbers. (Behr et al., 1993, p.4).

Thereupon, it could also indicate with a situation like;

- There are 2 watermelons and 7 people. If the watermelons are shared equally by the 7 people, how much watermelon does each person get?

3.1.6 The measure

The fractional measure subconstruct of a rational number represents how much there is of a quantity relative to a specified unit of that quantity. Because of its relationship to measurement it is useful in developing an understanding of addition. It is reported that measurement interpretations have a positive effects on children’s ability to learn fraction concepts (Behr et al., 1993).

For the measure subconstruct of rational number, it is important to measure the distance from zero (Moseley, 2005). To use this subconstruct is the best way to indicate that the rational numbers are a subset of the real numbers. When rational numbers are interpreted as points on a number line, it is easy to realize that.

Fractions can be represented on a number line. First the integers 0 and 1 are assigned to points on a line. Every other rational number is assigned to a specific point. For instance to represent \(\frac{1}{2}\), the segment is divided from 0 to 1 into 2 segments of equal length (Figure 1).

The number line aids us to concretize the measure subconstruct. And also, the number line model adds an attribute of generality as it is more than one unit long (Behr et al., 1993).

![Figure 1. The number line model](image)

- We have 5 fishes. And my father is going to put 5 gallons of water into our empty 10 gallon tank. How much fuller will the tank be after he puts the water in it?

- Alberto drives a car to his job everyday and he makes 3 stops to take 3 friends of him. The distance from his home to his job is totally 50 km. First stop is 4 km later from his home, the second is \(\frac{1}{5}\) of the whole way, away from the first stop. And the third stop is \(\frac{6}{10}\) of 50 km far away from Alberto’s home. What is the distance between the third stop and his job?

To use the number line in the second example can help students substantially. It is useful to make the data more concrete.
3.2 Different ways to represent mathematical objects

3.2.1 Description of mathematical objects

_Pictures_ have an essential role for translating either calculations or abstract subjects to concrete in mathematics education. In textbooks, pictures help students understand explanations and if the pictures describe exercises, they are also help to answer questions.

_Language_ is also underpinning all mathematical activity (Sollervall, 2011). We will consider written natural language in textbooks. Generally, written language is used to explain new topics in mathematics. Therefore, the first sentences affect students’ minds directly and they have big role about this new topic. We are also using natural language for the topics in textbooks. It makes students’ to see first picture of this subject. In this respect, the choice of words is significant in the presentation of topics. It is also important that if the text supported by other representations or not.

_Diagrams_ are simple drawings which consist mainly of lines and/or graphs. They are useful tools for mathematics especially to describe relationships between variables. The number line is an example of a diagram.

_Symbols_ are for items in a calculation or a scientific formula: numbers, letters or shapes that represent those items. They support abstract thinking and generalization in mathematics.

In addition, besides formal mathematical symbols, textbooks contain _reminders_ with can be symbolized with different colors or markers that could mean a hard question, an important note or a different type of question.

3.2.2 Representations, treatment, and conversion

Mathematical activity needs to have different semiotic representation systems that can be freely used according to the task to be carried out, or according to the question that is asked. (Duval, 2006)

The transformations of representations in mathematical activities can be distinguished with two types. They are distinguished with how they have transformations of representation.

_Treatment_ concerns transformations within one system to the same system such as

\[
\frac{3}{5} + \frac{7}{9} = \frac{27}{45} + \frac{35}{45} = \frac{62}{45}
\]

This calculation translates symbols to symbols. Treatments can also interpret between diagram representations or between picture representations (Sollervall, 2011; Duval, 2006).

_Conversion_ is a transformation between representations in different systems, for example between a symbolic representation and a diagram (Sollervall, 2011; Duval, 2006).

► Draw a number-line and mark the approximate position of \(\frac{5}{7}\).
3.2.3 *Relationships between mathematical objects and representations*

Treatments of abstract representations can be supported by conversions, as below;

\[
\frac{2}{5} + \frac{1}{5} = ? \quad \text{Treatment} \quad \frac{3}{5}
\]

![Conversion Figure](image)

*Figure 2. Illustration of treatments and conversions*

Changing representation register is the threshold of mathematical comprehension for learners at each stage of the curriculum. It depends on coordination of several representation registers and it is only in mathematics that such a register coordination is strongly needed. (Duval, 2006)

Representations make mathematical activity more concrete and understandable for students. Particularly it is useful with generic examples or the introduction of topics. We will consider these two aspects in our analysis of the textbooks.

On the other hand, mathematical objects can be more complex than representations occasionally or the opposite. The objects may appear more concrete when they are shown with different representations, however all these representations can confusing for students.

The progressive development of the use of different representations undoubtedly enriches the meaning, the knowledge and the understanding of the object, but also its complexity. In one sense the mathematical object presents itself as unique, in another as multiple. (D’Amore, 2007)

3.3 *Different types of examples and exercises*

Many mathematical examples and questions are presented with pure mathematics, just involving symbols and no use of natural language. When language is involved, the content may pretend to describe a situation in real life, but sometimes the situation does not appear real to the students. Ole Skovsmose (2001) has made a distinction between semi-real and real-life references, which we will describe in the sections below.

3.3.1 *References to pure mathematics*

By means of this reference, mathematical questions and activities refer to mathematics and mathematics only (Skovsmose, 2001). It is the traditional mathematics teaching method. Pure mathematics exercises can be of these forms;
\[
(38x + 15y) - (21x - 4y) - 10z =
\]
\[
(\frac{2}{9} - \frac{1}{5}) - (\frac{2}{7} + 3 \frac{3}{5}) =
\]

These exercises are presented with numbers, fractions, and variables, but neither with described objects nor figures. Therefore teaching mathematics subject with pure mathematics can be abstract for young students. This reference to pure mathematics provides to improve mathematical skills however students will not learn how to use these skills in life. This could lead to negative effects on learning unless the students know that why they are learning these calculations.

Skovsmose (2001) describes mathemacy that is referring to different concerns in critical mathematics education. He states that

Mathemacy refers not only to mathematical skills, but also to a competence in interpreting and acting in a social and political situation structured by mathematics.  
(Skovsmose, 2001, p.123)

This latter aspect of mathemacy, as preparing the students for practical life in society, is addressed in the next two sections about semi-real and real-life references.

### 3.3.2 References to semi-reality

This reference considers a reality which we cannot actually observe, but a reality constructed by for instance an author of a mathematical textbook (Skovsmose, 2001). The following example illustrates such a semi-real situation;

- Greengrocer X sells carrots for 15 kr per kilogram. Y sells them 1.3 kg for 25 kr.  
Which greengrocery is cheaper? What is the difference between the prices charged by the two greengrocers for 15 kg of carrots?

This artificial situation located in a semi-reality does not consider some circumstances, such as taste of carrots, distance between grocers or who is going to buy the carrots. This exercise refers to a reality that is only imagined by the author. However it is useful for practicing mathematics.

Solving exercises with reference to a semi-reality is an elaborated competence in mathematics education, based on a well-specified contract between teacher and students.  
(Skovsmose, 2001, p.126)

This contract usually includes principles that students should know that the given information is enough to solve the exercise and there is one and only one answer which is correct.
3.3.3 Real-life references

This reference shows us how mathematics is operating in real-life situations with actual observations (Skovsmose, 2001).

In real-life exercises, all figures are real-life figures and all incidents from life that you could imagine easily with possibility to come face to face with this situation. Nevertheless the activities are settled in the exercise frame (Skovsmose, 2001). At the same time, there is not any claim that real-life exercises have one and only one answer.

For instance;

► There are 15 books in the bookshelf in picture. The brown ones are $\frac{2}{5}$ of 15. How many books are brown?

► There are 2 water bottles in our house. They are both 12 liters. My family has already used $\frac{5}{6}$ of the first bottle. How much water is left in our house?

Examples are used to introduce new topics and to emphasize something important. They also represent typical features of the topics that are treated in the textbook. In particular mathematics education, it is necessary for students to understand situations with examples. We will count a presented problem as an example if its solution appears in the textbook.

3.3.4 Generic examples

In mathematics textbooks, trying to understand topics without generic examples would be almost incomprehensible for all students. A generic example is presented with numbers that concretize the example, but the solution allows for changing the number so the student can interpret the solution method as general.
In the case of fractions’ addition, the following example may be considered as generic for the purpose of adding fractions with the same denominator;

\[
\frac{3}{5} + \frac{4}{5} = \frac{7}{5}
\]

Even this example seems particular at first; it is explaining how to add fractions when we have same dominator. However, it is not a generic example for adding arbitrary fractions. Then we could instead use the following generic example;

\[
\frac{3}{5} + \frac{4}{9} = \frac{3 \cdot 9}{5 \cdot 9} + \frac{4 \cdot 5}{9 \cdot 5} = \frac{27}{45} + \frac{20}{45} = \frac{47}{45}
\]

The generic examples explain what we are going to do next step of problems or calculations; besides showing with mathematical symbols, the examples could also add some explanations step by step and describe it in text or in pictures.

**Exercises** are generally short activities in schools or in textbooks which are designed to help students learn a particular skill. In mathematics education, exercises have an essential role to practice and understand subjects or concepts from different aspects. Textbooks contain many different types of exercises, for example creative problems and routine or procedure-practice tasks (Lithner, 2003).

### 3.3.5 Routine tasks

This task type is generally considered as ‘easy questions’ by students. Solving a routine task requires well-known procedures or well-defined procedures. It can also include simple calculations. Therefore, when the solver encounters a routine task in a textbook, to imagine the complete solution method is readily in mind within a short period of time (Lithner, 2003). To enhance understanding of more general methods and ideas, then you solve routine task for yourself even if they are ‘problems’ for many others.

For instance, the exercise 12+3 =? should be routine task for Grade 6 because they already know addition since earlier grades. On the other hand even 12+3 =? can be a problem if the learner is not familiar with symbolic addition. But if the learner knows numbers 1-20 and the idea of addition, then the learner can approach the addition through representations such as

\[
\begin{array}{cccc}
\bullet & \bullet & \bullet & \bullet \\
& & & +
\end{array} = ?
\]

If the learner engages in such an exploration to find the solution, then the exercise can count as a creative problem for this learner.

### 3.3.6 Problems

A **problem** is a puzzle that requires logical thought or mathematics to solve it. Lithner (2003) explains that an exercise can have different meanings for different people. It is connected with the discrepancy between solvers and does not concern how much work the learner has to get involved in to solve the exercise. Instead, the notion of problem has to do with if the learner knows the solution method in advance or not. Lithner (2003), who quotes Schoenfeld (1985), describes this discrepancy; an exercise can be a creative problem for some students, for others
it can be a routine task that they already know how to solve, or for some mathematicians it is only recall. In this way, the notion of problem is not an inherent quality of a mathematical task that distinguishes it from a routine task (Lithner, 2003). A problem is as an exercise that poses intellectual challenges for the individual who is trying to solve it. For most Grade 6 students the following exercise will be such a ‘problem’;

► Carl, Johan and Martha start at the same time walking around a circular track in the same direction. Carl takes \( \frac{1}{2} \) hour to walk around the track. Johan takes \( \frac{5}{12} \) hour, and Martha takes \( \frac{1}{3} \) hour. How many times will each person go around the track before all three meet again at the starting line?

3.4 Different styles of texts

There are two modes of thought, which have each different ways of sorting experience or combining reality and their procedures of verification are in a complete other way. But both modes of thought can be used to convince other people; one by a good story and the other one by a clearly defined argument.

In each mode of thought the term ‘then’ functions different:
- In narrative style the term ‘then’ leads to likely special connections between events:
  e.g. ‘The man died, and then his wife died.’
- In paradigmatic style the term ‘then’ leads to a search of fundamental truth conditions:
  e.g. ‘if A, then B’. (Bruner, 1986)

3.4.1 Narrative style

Narrative style is a story-like task of a series of events, extended in time. It is built upon concern for the human condition: stories reach sad or comic or absurd consequences, while theoretical arguments are simply conclusive or inconclusive. A good story must consist of two different landscapes, which are essential and distinct. One is the landscape of action, where the subject is the argument of action: intention, situation or agent. The other one is the landscape of consciousness, what the involved in the action know, think or feel. In these stories psychic realities dominates narrative and have no need for testability. (Bruner, 1986)

3.4.2 Paradigmatic style

Paradigmatic style describes a very clear or typical collection of tasks, which tries to accomplish the ideal of a formal, mathematical system of description and explanation. This mode of thinking deals in general causes, in their setting and makes use of procedures to assure verifiable reference and to test for empirical truth. Different powerful prosthetic devices have been developed from paradigmatic mode: logic, mathematics, sciences, etc. The imaginative application of the paradigmatic mode leads to good theory, tight analysis, logical proof and empirical discovery. ‘Paradigmatic imagination’, or intuition, is the ability to see possible formal connections before one is able to prove them in any formal way. (Bruner, 1986)
4 Method

We found “fractions” as an important topic in primary grades to connect abstract algebraic systems. Fractions is one of the main topics in mathematics which is a necessary prerequisite many further topics’. Furthermore, textbooks have an essential role in students’ education. That is why every country considers it important to evaluate textbooks’ development.

It is important to consider many aspects of researches for teaching and describing different kinds of exercises and examples with attention to all sides of topics in the textbooks and to follow international trends in mathematics textbooks also. Countries should profit from each other, and then it helps to develop each of them.

Having said this, it is coming to light why I have chosen as the topic of this thesis to investigate how fractions are introduced and treated in mathematics textbooks. After deciding to do comparative work between Swedish and Turkish textbooks, we started with investigating educational researches on the topic.

In this study, it was important to compare two countries’ textbooks with some principles. To choose criteria for comparison, we examined steering documents, textbooks and teaching way of fractions. After considering Swedish and Turkish textbooks from many different perspectives, we decided to use four main criteria;

- Different ways to understand fractions
  - Part-whole comparison
  - The ratio perspective
  - The decimal subconstruct
  - The operator
  - An indicated division (quotient)
  - The measure

- Different ways to represent mathematical objects
  - Natural language, pictures, diagrams, symbols
  - Treatment and conversion

- Different types of examples and exercises
  - References to pure mathematics, semi-reality, real-life
  - Generic examples
  - Routine tasks and problems

- Different styles of texts
  - Narrative style and paradigmatic style

We have examined the sections in each of the three Grade 6 text books where the topic of fractions was introduced and treated. The data from this examination has been structured to prepare for using the above criteria in the analysis of the data.
5 Results and Analysis

In this section, after given information about publication and writers of textbooks; we will examine topic sequence of each mathematic textbook. By doing this, it might reflect how cultural differences affect mathematical thinking.

Then we will consider how textbooks introduce addition of fractions, because it was the common and important topic for all textbooks.

It will be shown with samples from textbooks, how we partitioning textbooks’ examples and exercises in deference to our criterions. And how are tables created with association between them.

5.1 Background information about textbooks

We provides some basic information about publication of the analyzed mathematic textbooks:

Swedish textbook’s writers are;
Synnöve Carlsson, Gunilla Liljegren, Margareta Picetti, publisher; Bonnier Utbildning, published in Sweden, year of publication 2004, selling about 15 €.

Turkish textbook’s (2010) writer is;
Yeşim Göğün, publisher; Özgün Matbaacılık, published in Ankara, Turkey, year of publication 2010, given for free.

Turkish textbook’s (2005) writers are; Şehnaz Bilgi, Hilal Ekmen, Nedim Gürsoy, publisher Milli Eğitim Bakanlığı, published in İstanbul, Turkey, given for free.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Textbooks analyzed in this study</td>
<td>İlköğretim Ders Kitabı Matematik 6</td>
<td>İlköğretim Matematik 6 Ders Kitabı</td>
<td>MatteDirekt Borgen 6A</td>
</tr>
<tr>
<td>Number of pages for fractions</td>
<td>37 Pages</td>
<td>16 Pages</td>
<td>28 Pages</td>
</tr>
<tr>
<td>Curriculum</td>
<td>National</td>
<td>National</td>
<td>National</td>
</tr>
<tr>
<td>Percentage of pictures or diagrams on the pages</td>
<td>~16 %</td>
<td>~37 %</td>
<td>~42 %</td>
</tr>
</tbody>
</table>

Table 1. Basic facts about the textbooks
5.2 Structure of the textbooks

We will examine textbooks page by page in this section. Investigating will help us to understand the structure of the textbooks, organized texts and differences between them. Every textbook has their own way to describe topics and define them. For instance, one textbook may have first a theory part then example and exercises. Another textbook may start with examples then theory and exercises at the end a summary. These factors affect how the students perceive the topic. At the same time, the page by page examination gives information about the representations that appear on each page.

5.2.1 Structure of the Turkey 2010 textbook

Page 76: Heading: “2. Chapter: Fractions”
A box: title: “What we will learn?” (4 Aims):

- We will learn, to compare fractions, to line up fractions, to show them on a number line,
- Using strategy to guess calculations’ results which made with fractions
- Addition, subtraction, multiplication, division with fractions,
- And, to be able to answer and set up problems about fractions. “

Subheading: “Compare Fractions”
Picture (4 children are telling ideas with speech bubbles)
Introduction Example 1 (Figure 6): “Students are trying to find which one is larger of $\frac{5}{9}$ and $\frac{7}{8}$. Which idea is more logical for you?”

Activity: Comparing fractions and line them up. (Tools and materials: fractions kit)
Example 1
Page 77: Example 2
Subheading: Exercises
Exercise 1,2,3,4,5,6,7,8,9,10,11,12

Page 78: Subheading: “Estimation with fractions”
Picture (Child with music player)
Introduction Example 2: “Some songs take little more than 4 minutes; some others take little less than 4 minutes. If we listen all songs, how many minutes will it take? Explain your guess strategy.”
Table: (with 4 song and their minutes such as $3 \frac{4}{5}, 4 \frac{1}{10}, 3 \frac{7}{8}, 4 \frac{3}{11}$)
A box: title: “Career” (information about cooks that they are also using ratio, percentages and fractions when they are calculating meals for how many people)
Activity: Which one is more closely? (Tools and materials: paper and pencil)

Page 79: Subheading: “Examples”
Example 3, 4
Subheading: “Exercises”
A box: title: “Attention!” (A hint about fractions)
Exercise 13,14,15,16,17,18,19,20,21,22

Page 80: (Figure 3)
Heading: “Addition and Subtraction Calculations with Fractions”
Picture (a band, playing musical instruments)
Introduction Example 3: “Stroke of musical notes is expressed by fractions. Please calculate the value of each and every part of slices, below.”
Music line
Activity: Addition with fractions kit (Tools and materials: fractions kit)
A box: title: “Attention!” (A hint about fractions)
Subheading: “Examples”
Example 5

Page 81: Example 6, 7
A box: title: “Attention!” (A hint about example 6)
A box: title: “Attention!” (A hint about fractions)
Subheading: “Exercises”
Exercise 23, 24, 25,26,27,28

Page 82: Heading: “Multiplication with fractions”
Picture (Orchestra)
Music line
Introduction Example 4
A box with title: “Historical Angle” (The early Egyptian numeration system had symbols for fractions with numerators of 1. Most fractions with numerators other
than 1 were expressed as a sum of different fractions with numerators of 1, for example;
\[ \frac{8}{15} = \frac{1}{3} + \frac{1}{5}, \quad \frac{5}{6} = \frac{1}{2} + \frac{1}{3}, \quad \frac{7}{12} = \frac{1}{4} + \frac{1}{3} \]

Activity: Multiplication with fractions. (Tools and materials: fractions kit)

Page 83: Activity: Paper folding. (Tools and materials: paper, dye)
Subheading: “Examples”
Example 8, 9, 10

Page 84: Example 11
A box: title: “Attention!” (A hint about example 11)
Subheading: “Exercises”
Exercise 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41

Page 85: Heading: “Division with fractions”
Introduction Example 5: “how can children in the picture decide about the right side, in your opinion?”
Picture (4 children are telling ideas in speech bubbles)
Activity: Division with fractions. (Tools and materials: fractions kit)
Subheading: “Examples”
Example 11

Page 86: Example 12, 13, 14
A box: title: “Attention!” (A hint about fractions)
Subheading: “Exercises”
Exercise 42, 43, 44, 45, 46, 47, 48

Page 87: Heading: “Let’s solve and set up problem”
Example Problem
A table about problem
A box: title:” strategies of problem solving”
Subheading: “Understanding problem”
Subheading: “Make a plan”
Subheading: “Solve the problem”
Table (about problem)
Picture (two football players)

Page 88: Subheading: “Let’s check”
Table (about same problem)
Exercise Problem 1: “A biscuit factory produces and sells many products. Sales department organized relevant datas for the most sold products between 2000 and 2005 years, as follows below. Create a problem which requires these datas processing.”
Graph (about data of exercise problem)
Example problem: “In year 2000, how much was sold biscuits with cocoa more than biscuits with sesame?”

Exercise problem 2
Table

Exercise problem 3: (“Make a problem with need to be calculate \( \frac{1}{2}, \frac{3}{8}, \frac{1}{12} \) these fractions.”)

Page 89: *Heading*: “Chapter’s Assessment”
Exercise 49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70

Page 90: *Heading*: “Unit Assessment”
Exercise 71,72,73,74,75,76,77,78,79,80

Page 91: Exercise 81,82,83,84,85,86,87,88,89,90

5.2.2 Structure of the Sweden textbook

Page 38: *Heading*: “Fractions”
*Picture*: (Hat shop with colorful hats)
*A box*: title: “Aims”

“When you have worked on this chapter you will,
- able to read and write fractions
- know what is meant by numerator and denominator
- know and be able to use concepts of fractions
- able to add and subtract with same denominator
- able to work out a certain part of series
- able to compare fractions”

*A box*: (4 discussion questions about hats)
“How much of the hats are brown?”

Page 39: Picture (a map with countries flags)
A box: (3 discussion questions about flags)
“How much of Bolivia’s flag is yellow?”

Page 40: *Heading*: “Parts of the whole”
*Picture*: (Peru’s flag)
Introduction Example- (Figure 7): “The flag is divided into three equal parts. Each part is called one-third and is written \( \frac{1}{3} \). Two-thirds of the flag is red. \( \frac{1}{3} \) and \( \frac{2}{3} \) are called fraction.

Talking about how many parts there are
Talking about what type of parts these are
\[ \frac{\text{numerator}}{\text{denominator}} = \frac{2}{3} \]

A speech bubble: “Sometimes fractions can show with slash: \(\frac{2}{3}\)”
Exercise 1, 2, 3, 4, 5, 6

Page 41: Exercise 7, 8, 9, 10, 11, 12

Page 42: Heading: “Whole”
Example 1 - “Tortillas can be divided in several ways.”
Picture (three tortillas)
Whole = three-thirds = four-fourths
Exercise 13, 14, 15, 16, 17, 18

Page 43: Heading: “More than a whole”
Example 2 - “When there are more than whole we can write various ways”
Picture (2 tortillas)
\[
\frac{5}{4} = 1 \frac{1}{4}
\]
Exercise 19, 20, 21, 22, 23, 24

Page 44: (Figure 4)
Heading: “Addition and Subtraction”
Diagrams (explaining addition and subtraction with generic examples)
Introduction Example
Exercise 25, 26, 27, 28, 29, 30

Page 45: Example 3
A speech bubble: “Count integers first, and then parts”
Exercise 31, 32, 33, 34
Example 4
A speech bubble: “I write the answer in a mixed response form”
Exercise 35, 36

Page 46: Heading: “How many is the part?”
Example 5
Picture (12 parrots)
Exercises 37, 38, 39, 40, 41, 42

Page 47: Example 6
Picture (12 butterflies)
Exercises 43, 44, 45, 46, 47, 48, 49, 50, 51
**Page 48:** *Heading:* “Comparing Fractions”

Table of fraction kit

Diagrams

Introduction example: \(\frac{1}{2}\) is colored. \(\frac{1}{4}\) is colored. The colored areas are equal.

\[
\frac{1}{2} = \frac{2}{4}
\]

A speech bubble: “This is what I call a fractions kit.”

Exercise 52, 53, 54, 55, 56

---

**Page 49:** *Heading:* “Problem with fractions?”

Exercise 57, 58, 59, 60, 61

*Heading:* “True or False?”

Exercise 62, 63, 64, 65, 66, 67, 68, 69, 70, 71

---

**Page 50:** *Heading:* “Diagnosis” (kind of test that tells which level the students’ knowledge is)

Exercise 72, 73, 74, 75, 76, 77, 78, 79, 80, 81

---

**Page 51:** Exercise 82

Picture (A fish)

*Heading:* “Trick Questions”

3 Exercises, One of them;

“Daniel says that \(\frac{1}{8}\) of the largest square is red. What do you say?” (This question was in English in the textbook)

---

**Page 52:** *Subheading:* “Blue Course”

*Heading:* “Parts of the whole”

Picture (1 waffle is divided 5 parts)

Introduction Example: “Waffle is divided into five equal parts. Each parts is called **one-fifth**. We write

\[
\frac{1}{5} \quad \text{Numerator, tells you how many parts there are} \\
\frac{5}{5} \quad \text{Denominator, tells you what type of parts these are}
\]

A speech bubble: “\(\frac{1}{5}\) is a fraction.”

Exercise 83, 84, 85

---

**Page 53:** Example 7: “Waffle is divided into 5 equal parts. Lena gets two of the 5 parts. She gets \(\frac{2}{5}\).”
Picture (waffle with 2 parts and a girl with a speech bubble: \( \frac{2}{5} \) is a fraction.)

Exercise 86, 87, 88, 89, 90

**Page 54:** *Heading:* “Whole”

Introduction Example
Exercise 91, 92, 93, 94, 95

**Page 55:** Exercise 96, 97, 98, 99, 100, 101, 102, 103

**Page 56:** *Heading:* “Counting with fractions”

Diagrams (which is showing addition and subtraction)
Example 8
A speech bubble: “Denominators must be equal.”
Exercise 104, 105, 106, 107, 108, 109, 110

**Page 57:** *Heading:* “How many is the part?”

Picture (8 apples)
Example 9
Exercise 111, 112, 113, 114, 115

**Page 58:** *Subheading:* “Red Course”

Here you will work
- with different units of time
- with problem solving
- more with the addition and subtraction of fractions

*Heading:* “Time”

Example Explanation: “1 year = 12 months = 52 weeks; 1 week = 7 days”
Picture (Calendar of one year)
Exercise 116, 117, 118, 119, 120, 121

**Page 59:** Picture (a few clocks)

Example Explanation: “1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 second”

A speech bubble: “Hour shortened with h, minute with min and second with s.”
Exercise 122, 123, 124, 125, 126, 127, 128

**Page 60:** *Heading:* “Fractions- home and away”

Picture (several 1 kronor)
Example 10
Page 61: Exercise 135,136,137,138,139

Page 62: *Heading*: “More addition and subtraction”
Example 11
Exercise 140,141,142,143,144,145

Page 63: Example 12
Exercise 146,147,148,149,150,151

Page 64: *Heading*: “Summary”
Diagrams
*Subheading*: “Fractions in different ways”
Explanation: “A fraction written with two numbers and a fraction bar:
\[
\frac{3}{4}
\]
this fraction called “three-fourths”.
Fractions greater than 1 can be written in two ways:

<table>
<thead>
<tr>
<th>Fraction form</th>
<th>Mixed form</th>
</tr>
</thead>
</table>
| \[
\frac{13}{4} = 3 \frac{1}{4}
\] |            |

*Subheading*: “How many is the part?”
Explanation
*Subheading*: “Addition and subtraction of fractions with same numerators”
Diagram (fractions kit)
*Subheading*: “Different fractions – same value”
For example, \[
\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}.
\]

Page 65: *Heading*: “Challenge”
*Subheading*: “fun with numbers”

5.2.3 *Structure of the Turkey 2005 textbook*

Page 80: *Heading*: “Fractions and Fractions types”
*Subheading*: “1. The concept of fractions”
*Introduction example*: (Figure 9) (Story about a girl who is going picnic with her friends and her mother cut a watermelon and share to all children)
*Explanation*: “We indicate parts that children ate from watermelon with fractions. It shown with this fraction \(\frac{1}{8}\) and it is called, one- eighth. 1 is numerator and 8 is denominator.”
Diagram (circle with divided 8 parts and 1 of them colored red)
Example 1

Page 81: Theory text with diagrams
Subheading: “2. Unit fraction”
Explanation
Definition
Warning

Page 82: Heading: “Exercises”
Exercise 1, 2, 3, 4, 5

Page 83: Heading: “3. Fraction types”
Subheading: “a. Proper fraction”
Explanation
Definition
Subheading: “b. Improper fraction”
Explanation
Definition

Page 84: Subheading: “c. Mixed numbers”
Explanation
Definition
Heading: “4. Show fractions on the number-line”
Warning

Page 85: Example 2, 3, 4, 5

Page 86: Heading: “5. Change an improper fraction to a mixed number”
Explanation
Example 6
Warning
Heading: “6. Change a mixed number to an improper fraction”
Explanation
Example 7
Warning

Page 87: Heading: “Exercises”
Exercise 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16
**Page 88:** *Heading:* “Relation between Fractions”  
*Subheading:* “1. Equivalent Fractions”  
Explanation  
Definition  
Warning  
Example 8

**Page 89:** *Subheading:* “a. Fraction expansion” (A fraction can be expanded, by multiplying the denominator and the numerator with the same number)  
Diagrams  
Explanation  
Definition: “The value of fraction does not change if its numerator and denominator are multiplied by the same nonzero whole number. We call this fraction expansion. If we expanded fraction with nonzero whole number, this fraction and first fraction are equivalent”  
Picture (an apple divide 4 parts)  
*Subheading:* “b. Simplifying Fractions”  
Explanation

**Page 90:** Simplifying example  
Definition  
Explanation  
Warning

**Page 91:** Example 9

**Page 92:** *Heading:* “Exercises”  
Exercises 17,18,19,20,21,22,23

**Page 93:** *Heading:* “2. Ordering fractions”  
*Subheading:* “a. the comparison of fractions with like denominators.”  
Explanation  
Warning  
*Subheading:* “b. the comparison of fractions with like numerators.”  
Explanation  
*Subheading:* “c. the comparison of fractions with unlike denominators and numerators.”  
Explanation
Page 94: Picture (an orange and half orange)
Example 10, 11, 12
Warning

Page 95: Heading: “Exercises and problems”
Exercise 24, 25, 26, 27, 28, 29, 30, 31

Page 96: (Figure 5)
Heading: “Addition with fractions”
Subheading: “1. Addition of fractions with like denominators”
Diagrams
Explanation
Example 13, 14
Warning
Example 15
Warning

Explanation example
Example 16, 17

Page 98: Example 18, 19, 20

Page 99: Example 21, 22

Page 100: Heading: “Exercises and problems”
Exercise 32, 33, 34, 35, 36, 37, 38, 39

Page 101: Heading: “Subtraction with fractions”
Subheading: “1. Subtraction of fractions with like denominators”
Diagrams
Example 23
Subheading: “2. Subtraction of fractions with unlike denominators”
Diagrams

Page 102: Example 24, 25, 26

Page 103: Example 27, 28

Page 104: Heading: “Exercises”
Exercise 40, 41, 42,43,44,45,46,47,48

**Page 105:** *Heading:* “Multiplication of fractions”
- Diagrams
- Explanations
- Example 29, 30

**Page 106:** Example 31
*Heading:* “1. Finding a fraction of a whole”
Example problem: “Serdar has read $\frac{5}{8}$ of the book which has 112 page in total.
How many pages did he read?”
*Warning*

**Page 107:** Example 32, 33
*Heading:* “Number 1 and 0 in Multiplication of fractions”

**Page 108:** *Heading:* “Exercises”
Exercise 49, 50, 51,52,53,54

**Page 109:** *Heading:* “Division of fractions”
- Explanations
- Warnings

**Page 110:** *Heading:* “1. Finding the whole from a part or finding the whole when given a known part”
Example problem: “If $\frac{3}{4}$ of a bag of rice is 24 kg, how much would the whole bag of rice weigh?”
*Heading:* “2. Finding a fraction of another fraction”
Example problem: “A farmer is planting wheat for $\frac{4}{5}$ of $\frac{2}{3}$ of a field. What part of the whole field is planted?”

**Page 111:** Example 34, 35

**Page 112:** *Heading:* “Exercises”
Exercise 55, 56,57,58,59

**Page 113:** *Heading:* “Exercises and problems”
Exercise 60-72
Page 114: Exercise 73-81

Page 115: Heading: “Test 4”
Exercise 82-90 (Multiple-choice questions)

Page 116: Exercise 91-101 (Multiple-choice questions)

5.2.4 Preliminary comments on the textbooks’ structures

As we saw in the data, every textbook has their own way to teach subjects to students. In each textbook, similarities and differences can be observed. After headlines, all of the textbooks start with generic example which has narrative style or paradigmatic style. These generic examples are used for introduction to a new topic. All textbooks have tips or warnings for students with different eye-catching symbols.

All textbooks have a lot of exercises about fractions but they all do not include answers. This means the authors designed their textbooks, so the teachers could explain when students could not find the solutions to the exercises.

They have differences when starting with new topics or giving some definition. In Turkey 2010 and Sweden, textbooks start with aims. However we do not see this in Turkey 2005. Although both Turkish textbooks included multiplication and division of fractions, Swedish textbook had only included addition and subtraction of fractions.

In Turkey 2005, there are long explanations written in one example and definitions or warnings which have academic language and complicated way to express without any representations. In Turkey 2010, there are still long explanations about one example but the academic definitions are shorter. These examples were explained with diagrams and pictures. Swedish textbook has neither long explanations nor complicated definitions. We see really short explanations, sometimes generic examples without any explanations but with representations.

And it is also important that how textbooks are representing them. For this reason three specific pages were selected to examine. These specific pages from all textbooks are representing common important subject that is introduction of addition of fractions. Presently, these pages will be displayed with explanation and include some examples and exercises.
5.2.5 The textbooks’ introduction of addition of fractions

Turkey 2010, Page 80;

Beats of musical notes is expressed by fractions.

Calculate how many beat are in each pieces of music track, below.
Activity: Addition with Fraction kits
- Let’s put $\frac{1}{2} + \frac{1}{3}$ fractions together as we see in the picture.
- Use $\frac{1}{2} + \frac{1}{3}$ properly and make whole.
- With $\frac{1}{2}$, how many $\frac{1}{2}$ did you add then you generate whole?
- With $\frac{1}{3}$, how many $\frac{1}{3}$ did you add then you generate whole?
- How you can make a whole if you use both $\frac{1}{2}, \frac{1}{3}$ together?
- With same way, let’s do $\frac{1}{2} - \frac{1}{5}$ with fraction kit.
- Why we have to make equal denominators?
- Discuss with your friends that how can we use equalization denominator’s methods
- Let’s express this calculation mathematical.

Tools and materials
- Fraction kits

ATTENTION!
While calculating fractional questions, it is wise to estimate result before, because it helps us to understand whether result is reasonable or not.

Examples
1. Let’s modeling $\frac{1}{3} + \frac{1}{4}$.

This page starts with introduction example 3. This example makes a relation between musical notes and fractions. There is a conversion from music symbols to mathematical symbols. We use notes in real life. However some students may not know much about notes sufficiently then this example turns as a handicap for students when the teachers did not explain. We can count this example as a real-life example.

Then we have a box with title of “Attention!” We see them in this textbook very much. These boxes are warning students and making some explanations for them.

There is “Activity” box about addition of fractions. There are a lot of these types of activities in the textbook too. They are asking questions and encourage students to discuss with each other and at the same time they are useful tools to make subjects more concrete.

Then we see example 5. This example is explaining addition of fractions with part-whole perspective. As we see in diagrams, it is a conversion symbols to diagrams and diagrams to symbols. It is counted as a pure mathematics references.
Addition och subtraktion

Räkna ut [4]

25 a) tre femtedel + en femtedel b) tre fjärde delar – två fjärde delar
26 a) \( \frac{3}{5} + \frac{1}{5} \) b) \( \frac{1}{4} + \frac{1}{4} \) c) \( \frac{2}{6} + \frac{3}{6} \)
27 a) \( \frac{2}{7} + \frac{4}{7} \) b) \( \frac{2}{9} + \frac{5}{9} \) c) \( \frac{3}{10} + \frac{6}{10} \)
28 a) \( \frac{4}{5} - \frac{2}{5} \) b) \( \frac{3}{4} - \frac{2}{4} \) c) \( \frac{7}{9} - \frac{5}{9} \)
29 a) \( \frac{7}{10} - \frac{4}{10} \) b) \( \frac{5}{7} - \frac{2}{7} \) c) \( \frac{7}{8} - \frac{6}{8} \)

30 Malin håller \( \frac{1}{8} \) liter koncentrerad saft i \( \frac{3}{8} \) liter vatten. Hur mycket saft får hon då?

Figure 4

[1] Addition and Subtraction
[2] Two-fifths + one-fifth = three-fifths
[4] Calculate
[5] 30 Malin pours \( \frac{1}{8} \) liters of concentrated juice in \( \frac{3}{8} \) liters of water. How much juice can she do?

This page starts with example 4. This example is generic and reference as a pure mathematics. Because it starts with diagrams then symbols it is also called conversion. In diagrams two big rectangles are divided to 5 equal rectangles. These smaller rectangles are parts of these big two rectangles. Consequently, this is a relation between parts and whole and its called part-whole comparison. Moreover, this example describes us “how addition and subtraction of fractions is” for this reason it is paradigmatic style.

Then it continues with exercise 25, 26, 28, 29. They all include pure mathematics. And for sixth grade, this should generally be routine task.

Then we see exercise 30 with related picture. This picture does not help students to understand this problem exactly. We called it problem because, it is not a routine task for sixth grade in general. And it can be referenced as a semi-reality.
ADDITION WITH FRACTIONS

1. ADDITION OF FRACTIONS WITH LIKE DENOMINATORS

Let's show \( \frac{3}{8} + \frac{4}{8} \) with diagrams;

\[
\frac{3}{8} + \frac{4}{8} = \frac{7}{8}
\]
Let’s find this sum with calculations:

Example

(Warning) when we finding sum of two fractions; first we write nominators’ addition as a nominator, then common denominators write as a denominator.

Let’s find fractions’ sum with calculations.
   First solution way:
   Second solution way:
   We can change these fractions first an improper fraction then do calculations.

(Warning) When you add two mixed fractions with same denominators, you simply first add integers then add the numerators and keep the same denominator.

(Symbol of Turkish 2005 textbook which is giving tips for students)

This page started with a generic example and addition of fractions shown with diagrams.

And as we see in this page there is different representation about addition of fractions. These symbolic examples have intermediate step such as \( \frac{3+4}{8} \). These calculations that show additions of numerators only present in Turkish textbooks.

Following examples are example 13, 14, 15. They are without representations as a picture or diagram so we called them treatment symbol to symbol and they are all references to pure mathematics.

There are two warning with special symbol. These warnings are explaining addition of fractions in paradigmatic way.
5.3 Character of the textbooks’ examples and exercises

In this section, we will evaluate examples and exercises. And we will show some of them.
Although it is separated by categories, all criterions are related to each other which we have explained in theory part. Therefore we will explain all property of examples or exercises below figures which will be shown. Then it will be explained with tables and graphs for whole examples and exercises of textbooks.

5.3.1 The first examples of fractions in each textbook

It will be reasonable to start with examining first examples because the first examples, mostly causing the students to determine their views on the topic. Accordingly, these introduction examples will give us ideas about style of textbooks. What is more, the explained property of examples in this section also will help us to explain further sections.

Turkey 2010, from page 76:

![Figure 6](image)

[1] Unit 3

  Compare Fractions

[3] \( \frac{5}{9} \) is closer to half, so it is smaller.

\[ \frac{7}{8} \] is closer to 1 that is why it is larger one
I would check denominators
If we equalize denominators of given fractions?

[4] What will we learn? (Aims)
  - We will learn, to compare fractions, to line up fractions, to show them on a number line,
- Using strategy to guess calculations’ results which made with fractions,
- Addition, subtraction, multiplication, division with fractions,
- And, to be able to answer and set up problems about fractions.

[5] Students are trying to find which one is larger of $\frac{5}{9}$ and $\frac{7}{8}$. Which idea is more logical for you?

This is (figure 6) first page and first introduction example of Turkey 2010 textbook. In introduction example 1, children discussing about ordering fractions so it is references as a pure mathematics. Because the picture of this example is displayed just children and all discussions are explained with the text; it is treatment. Sweden, from page 40,

Figure 7

[1] Parts of the whole

[2] Flag is divided into three equal parts. Each part is called one-third and is written $\frac{1}{3}$. Two-thirds of the flag is red. $\frac{1}{3}$ and $\frac{2}{3}$ are called fraction.

nominator $\rightarrow 2 \rightarrow$ Talking about how many parts there are
denominator $\rightarrow 3 \rightarrow$ Talking about what type of parts these are

[3] Sometimes fractions can show with slash: 2/3

[4] Symbol of Swedish textbook which is giving tips for students

This introduction example in Swedish book is from real-life references. They are using Peru’s flag. Each equal part can be called by a fraction in this flag. As the same time it is part-whole comparison because flag was compared its parts and whole. Each part is called one-third. This
conclusion can be found from picture, and then we can symbolize them with fractions, therefore it is *conversion*.

This first introduction example is not on first page of Swedish textbook. First pages are without examples but they have aims and some questions to make students think about fraction. It looks like this;

**Sweden, Page 38 and 39,**

![Image of a page from a Swedish textbook](image)

*Figure 8*

In first two pages of Swedish textbook have aims and some questions about hats and countries flags related to fractions. These pages are truly colorful with pictures. This property in Swedish textbook is not similar to Turkish textbooks.
Özen, her mother and friends of her decided to go to picnic together. They make preparations for it. At the weekend, they go to the forest near their house all together.

Özen and her friends are running around and playing then they get tired. Özen’s mother cuts eight equal parts the watermelon that she brought. After they ate watermelon, they came back home.

In this case, the watermelon that Özen’s mother bought is a whole. We indicate parts as a fraction which Özen and her friends ate. Each one of these parts that everybody ate;

Showing with $\frac{1}{8}$ and it is called “one-eighth”

In $\frac{1}{8}$ this fraction 1 called nominator, 8 called denominator.
This introduction example is describing fractions with a story, therefore it is narrative style. It is in reference to semi-reality as well. And because of it starting with a story then we show parts with symbol and then with diagrams in example. It is conversion. Additionally, this textbook’s example was used part-whole comparison.

5.3.2 Examples – use of representations

Examples are given with pictures, symbols or diagram in these three textbook. As we saw, Figure 3 is given with symbol with small picture, Figure 4, 5 and 9 are given with only a diagram. And Figure 6 and 7 with picture. End of this section we will see table of whole textbooks examples’ representation. At the same time there are some interesting and distinctive representations of fractions in each textbook.

In Turkish textbooks’ examples; in order to equate denominators both Turkish textbooks used finding least common multiple method. However they have different representations and explanation styles.

Turkey 2010, from page 81,

4. Let’s use equalize denominators model then do \( \frac{2}{9} + \frac{5}{12} \) this calculation.

Least common multiple of 9 and 12.

This example 8 (Figure 10) in Turkey 2010, the picture of example shows 9 cm and 12 cm. It may looks like measure but it is part-whole comparison. We just have small parts and bigger parts of one whole. This picture or we better say diagram for addition, helps students to find least common multiple of 9 and 12. Students first see diagram then turn it number 4 then we can call this example as a conversion. It is also reference as a pure mathematics.
2. ADDITION OF FRACTIONS WITH UNLIKE DENOMINATORS

Let’s do $\frac{3}{4} + \frac{1}{5}$ this calculation.

Because of they do not have same denominators we should equalize the denominators of $\frac{3}{4}$ and $\frac{1}{5}$. Denominator that we looking for is least common multiple of 4 and 5.

Since 4 and 5 relatively prime numbers,

Least common multiple of 4 and 5 =20. In this case $\frac{3}{4}$ and $\frac{1}{5}$ can be expanded so that the chosen denominator is 20.

This introduction example related to addition with unlike denominators, we see information is as ordered. This information is explaining “how it is” (how the operation addition is applied with different denominators). Therefore it is paradigmatic style. It is references as a pure mathematics and this example starts with symbol. Then since it has also shown symbol for solution, it is called treatment.
Differences between explanations about using least common multiples for equalizing denominators are that we have diagram in Turkey 2010 and we have long academic explanations in Turkey 2005.

In both examples we see that this have intermediate step such as \( \frac{15 + 4}{20} \) for addition. We do not see this step in Swedish textbook. Instead there is a different representation about addition in Swedish textbook.

**Sweden, page 62,**

![Figure 12](image)

[1] More addition and subtraction
[2] Calculate
[3] Then the parts
[4] Count first integers
[5] \( \frac{7}{5} \) is \( \frac{2}{5} \)

This *example* related to addition of mixed numbers. It is a reference as *a pure mathematics* and it is *treatment*. Swedish textbook uses same colors with simply and short explanation for calculations. However this clear expression is with same denominators. We do not see calculation with unlike denominators in Swedish textbook. On the other hand we can see ordering fractions with different denominators in Swedish textbook. To order fractions, equalizing denominators method is not used in this textbook. This textbook is encouraging student that they can use fractions kit or equivalent fractions for ordering.

And only Turkey 2005 textbook used the method that is finding the greatest common divisor for simplifying fractions.
Figure 13

[1] Let’s write \( \frac{24}{36} \) ’s simplest form.

[2] Let’s find greatest common divisor of 24 and 36.

[3] It is \( \frac{24}{36} = \frac{2}{3} \). Because of nominator 2 and denominator 3 are relatively prime numbers, they cannot divide with any number, and hence, the simplest form is \( \frac{2}{3} \).

\[ \frac{24}{36} \] can be simplify with successive division operations too.

[4] or

[5] To write a fraction in the simplest form, divide both nominator and denominator with greatest common divisor of fraction’s numerator and denominator.

This example is a reference as a pure mathematics and treatment which has shown three ways for simplifying fractions in Turkey 2005 textbook. Third way has interesting representation. It may useful students to simplify fractions however it is a bit misleading representation for
shown in textbook. This representation is not present in Turkey 2010 textbook and Swedish textbook.

Turkey 2005 is also only textbook that has shown addition and subtraction of fractions in number line
Turkey 2005, part from page 102;

Example

Let’s show $\frac{7}{8} - \frac{3}{8}$ this calculation in number line.

Has found.

Turkey 2005, part from page 102;

Mathematics 6 • Unit 4

Fractions

Example

Let’s show $\frac{4}{8} + \frac{5}{8}$ this calculation in number line.
Since $\frac{4}{8}$ and $\frac{5}{8}$ have the same denominator, let’s divide into 8 equal lengths between consecutive numbers. Then starting from 0 and count 4 segments, then point A, from A count 5 segment then point B. And B corresponds to $\frac{9}{8}$ or $1\frac{1}{8}$. This calculation can be written; \[
\frac{4}{8} + \frac{5}{8} = \frac{9}{8} = 1\frac{1}{8} \] like that.

These two examples are references as pure mathematics. They are also conversion example with symbol and diagram relation. And we count them as a measurement because in these examples, distance from zero is important for calculation of fractions.

In addition, second example (Figure 15) is paradigmatic style. It is explained with long explanation and this style of explanations is typical mathematical collection of tasks. It also can count as an illustrative generic example.

Some representations with examples we have seen with previous figures. And Graph-1 will explain whole example of textbooks what we see about representations.

![Graph 1](image)

They are some pictures in Turkey 2005 textbook too. But these pictures are not related to any example or exercises. They are in theory parts, with some explanations or randomly placed in the textbook. We see in figure 3, figure 4 and figure 5 examples with diagram from all textbooks.

5.3.3 Examples – different ways to understand fractions
We will focus on the six ways to understand fractions in this section. We have examined all examples in textbooks and count them with considering which understanding they are giving to students. At the end of this section three graphs will help us to see whole example of textbooks considering these six ways.

It has shown with previous figures some part-whole comparisons such as figure 7, 9 and 10. In order to have examples about measurement subconstruct see figure 14 and 15.

To give an example for operator subconstruct from Swedish textbook, we can check figure 16. In operator subconstruct, fractions like a tool, which have a different property of a set that is what we see in this example, figure 16. It is also references as a semi-reality.

**Sweden, part from page 57,**

Figure 16

[1] How many is the part?

[2] There are 8 apples. A quarter the apples are green. How many are 1/4 of 8?

\[
\frac{1}{4} \text{ of } 8 = \frac{8}{4} = 2
\]

Calculate 8 divided by 4.

2 apples are green.
Six ways to understand fractions are used in textbooks’ examples

**Turkey 2010**
- **part-whole**: 12; 47%
- **ratio**: 4; 15%
- **decimal**: 8; 23%
- **operator**: 4; 15%

**Graph 2**

**Turkey 2005**
- **part-whole**: 16; 39%
- **ratio**: 5; 13%
- **decimal**: 8; 20%
- **operator**: 11; 28%

**Graph 3**

**Sweden**
- **part-whole**: 15; 75%
- **ratio**: 4; 21%
- **decimal**: 0%
- **operator**: 0%

**Graph 4**
We firstly see in these graphics that Swedish textbook used just two of them, which are part-whole comparison and fractions as operator. It can be a reason that the Swedish textbook includes neither multiplication nor division of fractions in the chapter.

It is clear that the most common subconstruct is part-whole for all three countries.

There are not any decimal representations in examples of these three textbooks. It comes just after from this chapter in the Turkish textbooks but it does not appear in the whole of the Swedish textbook.

The ratio is also one of the subconstructs which have not appeared in the three textbooks. Additionally, no subconstruct was specified in some different examples. However there are some examples using part-whole comparison which was accompanied by the operator subconstruct.

5.3.4 Exercises – use of references with routine tasks and problems

- Pure mathematics references for exercises;

Turkey 2005, page from 113:

1. \( \frac{a}{b} \) is proper fraction. Since, a is element of counting number set, write the set of numbers that substitutable instead of a.

2. \( \frac{b}{c} \) is improper fraction and b is element of natural numbers. Which is the lowest substitutable natural number that we can write instead of b?
[3] 3. \( \frac{3}{8} \) write this fraction as a unit fraction.

[4] 4. How many unit fraction \( 2 \frac{3}{5} \) this fraction has?

[5] 5. What we should write instead of \( a \) then \( \frac{3}{8} \) and \( \frac{a}{24} \) they can be equivalent fraction?

[6] 6. If \( \frac{a}{12} = \frac{20}{48} \) then what is \( a \)?

[7] Fractions

[8] EXERCISES AND PROBLEMS

[9] Mathematics 6 • Unit 4

[10] (picture: As you see there is a Picture in Turkey 2005, but as we mentioned before it is not explaining any exercises or examples.)

These six exercises are references to pure-mathematics. We count exercise 4, 5 and 6 as a problem and others as routine tasks because the other exercises require basic concepts of fractions.

Turkey 2010, page 86;

Figure 18

[1] Exercises

[2] Please first guess the calculations’ value, below. And then compare your guess with solution.

[3] 5. Aynur brought \( \frac{42}{4} \) meters rope in the classroom. She wants to give rope with cutting equal length to 5 groups of her classmates which has divided groups for jump on this rope. How many meters should she give each group?
In figure 18, the first four exercises referenced as *a pure-mathematics* too. And they are routine tasks. Even though they can be problems for some students, we generalize them as routine tasks.

*Exercise* 25, 26, 28, and 29 from figure 4 were *pure-mathematics* which is in Swedish textbook and they were also routine tasks.

- **Semi-reality references for exercises;**

In the figure 18, exercise 5 is referenced as a *semi-reality*. And we counted it as a *problem*. As we mentioned before it can be routine task for some students.

**Turkey 2010, page 88;**

![Figure 19](image)

**Figure 19**

[1] A biscuit factory produces and sells many products. Sales department organized relevant datas for the most sold products between 2000 and 2005 years, as follows below. Create a problem which requires these datas processing.

- pink: Classic biscuit
- green: Biscuit with cream
- white: Biscuit with sesame
- purple: Biscuit with cacao
- orange: Biscuit with orange

- pink: Classic biscuit
- green: Biscuit with cream
- white: Biscuit with sesame
- dark orange: Biscuit with cacao
- orange: Biscuit with orange
This exercise refers to a **semi-reality**. Although it may seem to be real, it is still an artificial situation. This exercise aims to make students create new problems with the given information. We regard this exercise as a **problem**.

- *Real-life references for exercises:*

**Sweden, Page 57,**

![Figure 20](image1)

[1] The mother in the picture weighs 50 kg. The baby weighs \( \frac{1}{10} \) of 50 kg. How many kilograms does the baby weigh?

[2] Fractions

This exercise refers to **real-life exercises**. It is generally **problems** for 6\(^{th}\) grade students. At the same time, we may see real-life references in routine tasks such as;

**Sweden, Page 40,**

![Figure 21](image2)

[1] How much of Paraguay flag is

\[ a) \text{ red} \quad b) \text{ blue} \]

This type of **real-life exercise** is **routine task** for 6\(^{th}\) grade students in general. We have reserved exercises which are generalized in this way. All exercises are counted in each textbook.
As we may see in the table above, in 2005 there are no real-life exercises in the Turkish textbook. When we consider the total sum of exercises, the Swedish textbook has the most with the total amount of 155. Whereas, the Turkish textbooks have the total sum of exercises are 101 in 2005 and total sum of 93 in 2010.

In the exercises, there are common styles in both the Swedish and Turkey 2010 textbooks. For instance, there are true-false exercises. These exercises begin from 71 to 75 in the Turkish 2010 textbook and begin from exercise 62 to 71 in the Swedish textbook and they involve true-false questions about fraction. We count them also as a pure mathematics reference and routine tasks.

5.3.5 Additional observations and association between criteria

Turkish 2010 and Swedish textbooks have parts besides examples or exercises. They are called “Activity” and “Challenge”. They look like similar parts. However the Turkish textbook has 6, while Swedish textbook has only 1 of them.

Only the Swedish textbook has a summary at the end of each chapter and it also has one exercise in the trick questions part, which was in English.

It is also significant to consider the exercises which are pure mathematics, semi-reality or real-life references with representations. The total sum of exercises has been shown in table 3. In this table, we may also see treatment or conversion examples. Representations of these examples are given in Graph 4.

We may also see that both Turkish textbooks exercises are devoid of pictures in Table 3. It does not mean, they do not have pictures at all. However any pictures which have been shown with the exercises are described. In contrast, we may see a lot of pictures with the exercises in the Swedish textbook.

As it was previously mentioned, we may consider treatment or conversion examples which have shown in Table 3. After counting total sum of examples, we may see that in 2005, the Turkish textbook has more treatment, than it does in 2010. From this we may conclude that
the new textbook in Turkey (2010) has given more prominence for representations. The Swedish textbooks almost have equality between treatment and conversion. It may be said that even though the Turkish 2010 textbook has incorporated more example with representations than in 2005, the sum is still less than the Swedish textbook.
<table>
<thead>
<tr>
<th>Exercises</th>
<th>Representations</th>
<th>Conversion</th>
<th>Treatment</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Turkey (2010)</td>
<td>Sweden</td>
<td></td>
</tr>
<tr>
<td>with Pictures</td>
<td></td>
<td>Turkey (2008)</td>
<td>Sweden</td>
<td></td>
</tr>
<tr>
<td>with Diagrams</td>
<td></td>
<td>Turkey (2006)</td>
<td>Sweden</td>
<td></td>
</tr>
<tr>
<td>Pure mathematics</td>
<td></td>
<td>Turkey (2005)</td>
<td>Sweden</td>
<td></td>
</tr>
<tr>
<td>Semi-reality</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real-life</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total exercises</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3
6 Discussion

6.1 Methodological discussion

We have used some criteria to categorize examples, exercises and the texts in textbooks. And this has helped us to identify disparate features. With this, we may have an idea about which textbooks could structure different learning opportunities for students. Choosing these criteria was useful to analyze the textbooks.

6.2 Result and analyses discussion

In the result and analysis part, the findings explain how the Turkish and Swedish introductory presentation of fractions concepts in mathematics curriculum, and how the Turkish textbooks are developed in five years.

The purpose of this study was to examine textbook differences and similarities with all aspects of fractions. This thesis has analyzed the textbooks and has presented some pages, exercises and examples from them, and also used criterions to categorize them.

The interesting and unexpected thing was the big difference between Turkish textbooks. This may be because Turkey is a developing country or because the textbook designers have considered new international trends in education. The Turkish 2005 textbook has a very formal way to explain topics and examples. This indicates that the Turkish 2005 textbook has an academic language and could be considered as complicated and unreadable for students. Also, this textbook has no real-life situations and does not use representations to make students interested in them and the links between topics could be said to be insufficient.

In developing countries, education systems are not stable. Occasionally they take education systems as an example from other developed countries directly. Then it needs to be changed in accordance with countries own education system. And they also should consider their cultural differences. (Tezcan, 1985) Maybe that is the reason of the differences between the two Turkish textbooks. However this may not always have a positive result. In Turkey 2010 textbook, multiplication of fractions’ first representation could be considered as a simply repeated addition. Although repeated addition is a tool to calculate the product, multiplication is much more. In the 2005 Turkish textbook, we may see that this has been shown with understandable diagrams with all its aspects.

To make a comparison between two countries; we have chosen the Turkey 2010 textbook which is currently still being used in Turkey and the Swedish Matte Direkt Borgen textbook which is currently being used in Sweden now. And to examine developing in Turkey we have chosen Turkey 2005 textbook which is not in use now. The developed new Turkish textbook has a more different kind of representations than the older version. The Swedish textbook has more representations than all. However we should also consider that the Swedish textbook does not include multiplication and division of fractions. It is obvious from our tables in results and analysis part that the most amount of exercises in Swedish textbook. Real-life situations are represented more in Swedish textbook as well. Moreover, examples and exercises which refer pure-mathematics were the most abundant in all textbooks.
Furthermore, subcontracts of fractions such as part-whole, ratio, measure, quotient, decimal and operator has not been shown with all, in any textbooks. It was obvious that all three textbooks used part-whole comparison much more than other subcontracts. Also it has indicated that Swedish textbook only has two of them. The Swedish textbook has the narrowest scope in fractions. Maybe that is why it included just two of the understanding concepts of fractions.

The analyzed textbooks present us how it can be different in two different countries, and how they may be different even in same country within the period of five years. Comparing studies about textbooks between different countries provide students with developing educational materials.

6.3 Further research

For further research, it would be beneficial to also include a comparative study of mathematic teaching from teacher’s perspective to see how they are using textbooks between Sweden and Turkey. My study has been constructed to analyze the mathematical material, in three textbooks. Looking from a student’s perspective and how they practice these materials in classrooms could also be an interesting suggestion for further research.
References


