Precoding in MIMO, OFDM to reduce PAPR
(Peak to Average Power Ratio)

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Abstract

One of the critical issues of systems utilizing Orthogonal Frequency Division Multiplexing (OFDM) is the high peak to the average power ratio of OFDM signals. We have used Precoding as a way to mitigate the PAPR problem. Furthermore, the performance of Precoded OFDM in fading multi-path channels has been studied. This thesis is based on an efficient technique for reducing the PAPR of OFDM signals. The proposed technique is data-independent and thus, does not require new processing and optimization for each transmitted OFDM block. The reduction in PAPR of the OFDM signal is obtained through a proper selection of a Precoding scheme that distributes the power of each modulated symbol over the OFDM block. The obtained results show that this Precoding scheme is an attractive solution to the PAPR problem of OFDM signals.
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Chapter 1

1 Introduction

World is moving very fast, and every day, there will be new deployments will occur no one have idea. What will be the next technology will emerge in the field of wireless communication, if we go back to our passed there was only one way of a medium to people connect to each other is post Letter. Now a day’s world moving towards on fourth generation of wireless technology. Orthogonal Frequency Division Multiplexing (OFDM) is promising technique to perform Multicarrier modulation with maximum utilization of bandwidth and high mitigation characteristic’s profile against fading in Multipath. MIMO in combination with other schemes can increase capacity, reliability, support to internet services and multimedia application. MIMO with OFDM reduces the equalization complexities by transmitting data on different frequency levels to gain spectral efficiency and error recovery features, it offers high spatial rate by sending data on multiple antennas and transmission in NLOS. Thus, MIMO-OFDM to achieve diversity it utilize three parameters frequency(OFDM), time(STC), spatial(MIMO). MIMO-OFDM is reproductive and famous for services of the Wireless broad band access. Combination of MIMO and OFDM accumulates benefits of each scheme, provides high throughput.

1.1 Background

The basic idea of multicarrier modulation is quite simple and follows naturally from the competing desires for high data rates and ISI-free channels. Multicarrier Modulation (MCM) is the principle of transmitting data by dividing the input data stream into some lower rate parallel bit streams and using these substreams to modulate a number of carriers. The individual signals are called sub-carriers, and the technique is coined as Frequency Division Multiplexing (FDM). Multicarrier modulation divides the high-rate transmit bit stream into L lower-rate substreams, each of which has $T S / L >> \tau$ and is hence effectively ISI free. These individual substreams can then be sent over L parallel subchannels, maintaining the total desired data rate. Orthogonal Frequency Division Multiplexing (OFDM) is a unique case of multicarrier transmission orthogonal is a unique case of multicarrier transmission. In OFDM, the carriers are arranged such that the frequency spectrum of the individual carriers overlap and the signals are still received without adjacent carrier interference. In order to achieve this, the carriers are chosen
to be mathematically orthogonal. The data rate on each of the subchannels is much less than the full data rate, so the corresponding subchannel bandwidth is much less than the total system bandwidth. The number of substreams is chosen to ensure that each subchannel has a bandwidth less than the coherence bandwidth of the channel, so the subchannels experience relatively flat fading. Thus, the ISI on each subchannel is small. Moreover, in the digital implementation of OFDM, the ISI can be completely eliminated through the use of a cyclic prefix. For a long time, usage of OFDM in practical systems was limited. Main reasons for this limitation were the complexity of real time Fourier Transform and the linearity required in RF power amplifiers. However, with the advent of Fourier transform eliminated the initial complexity of OFDM where harmonically related frequencies generated by Fourier and inverse Fourier transforms are used to implement OFDM systems [1]. Since 1990s, OFDM is used for wideband data communications over mobile radio FM channels, High-bit-rate Digital Subscriber Lines (HDSL, 1.6Mbps), Asymmetric Digital Subscriber Lines (ADSL, up to 6Mbps), Very high-speed Digital Subscriber Lines (VDSL, 100Mbps), Digital Audio Broadcasting (DAB), and High-Definition Television (HDTV) terrestrial broadcasting.

1.2 Advantages

OFDM has many advantages over single carrier systems.

1.2.1 Lower implementation complexity

The implementation complexity of OFDM is significantly lower than that of a single carrier system with equalizer. When the transmission bandwidth exceeds the coherence bandwidth of the channel, resultant distortion may cause inter symbol interference (ISI). Single carrier systems solve this problem by using a linear or nonlinear equalization. The problem with this approach is the complexity of effective equalization algorithms. OFDM systems divide available channel bandwidth into a number of subchannels. By selecting the subchannel bandwidth smaller than the coherence bandwidth of the frequency selective channel, the channel appears to be nearly flat and no equalization is needed.

1.2.2 Elimination of ISI

Furthermore, by inserting a guard time at the beginning of OFDM symbol during which the symbol is cyclically extended, inter symbol interference
(ISI) and intercarrier interference (ICI) can be completely eliminated, if the duration of the guard period is properly chosen. This property of OFDM makes the single frequency networks possible. In single frequency networks, transmitters simultaneously broadcast at the same frequency, which causes inter symbol interference. Additionally, in relatively slow time varying channels, it is possible to significantly enhance the capacity by adapting the data rate per subcarrier according to the signal-to-noise ratio (SNR) of that particular subcarrier.

1.2.3 Robustness against narrowband interference

Another advantage of OFDM over single carrier systems is its robustness against narrowband interference because such interference effects only a small percentage of the subcarriers.

1.3 Disadvantages

Despite all these advantages, OFDM has some drawbacks compared to single carrier systems.

1.3.1 Carrier phase noise and Frequency offset

Two of the problems with OFDM are the carrier phase noise and frequency offset. Carrier phase noise is caused by imperfections in the transmitter and receiver oscillators. Frequency offsets are created by differences between oscillators in transmitter and receiver, Doppler shifts, or phase noise introduced by nonlinear channels. There are two destructive effects caused by a carrier frequency offset in an OFDM system. One is the reduction of signal amplitude since sinc functions are shifted and no longer sampled at the peak, and the other is the introduction of ICI from the other carriers. The latter is caused by the loss of orthogonality between the subchannels. Sensitivity to phase noise and frequency offsets increases with the number of subcarriers and with the constellation size used for subcarrier modulation. For single carrier systems, phase noise and frequency offset’s only give degradation in the receiver SNR, rather than introducing ICI. That is why the sensitivity to frequency offsets and phase noise are mentioned as disadvantages of OFDM relative to single carrier systems.

1.3.2 Peak to Average Power Ratio Problem

The most important disadvantage of OFDM systems is that highly linear RF amplifiers are needed. An OFDM signal consists of a number of inde-
independently modulated subcarriers, which can give a large Peak-to-Average Power Ratio (PAPR) when added up coherently. When $N$ signals are added with the same phase, they produce a peak power that is $N$ times the average power. In order to avoid nonlinear distortion, highly linear amplifiers are required, which cause a severe reduction in power efficiency. Several methods are explained in the literature in order to solve this problem.

1.4 Previous techniques for PAPR reduction

Most widely used methods are clipping and peak windowing the OFDM signal when a high PAPR is encountered. However, these methods distort the original OFDM signal resulting in an increase in the bit error probability. There are other methods that do not distort the signal. Two of these methods can be listed as Partial Transmit Sequences [2][3] and Selected Mapping [4][5]. The principle behind these methods is to transmit the OFDM signal with the lowest PAPR value among a number of candidates all of which represent the same information. Coding is another commonly used method. In this case, the information bits are coded in a way that no high peaks are generated. Mainly, Golay codes, Reed Muller codes and linear block codes are used [6][7].
Chapter 2

2 Orthogonal Frequency Division Multiplexing

Orthogonal frequency division multiplexing (OFDM) concept first appeared in the 1950s. It has nearly 62 years of development history.

2.1 Single Carrier and Multicarrier System

2.1.1 Single carrier system

In a single carrier system, signals transmitted through a transmitter filter $Y_t(t)$ before being applied to a multipath channel. At the receiver side, the receiving signal is passed through a receiving match filter $Y_r(t)$ just like we apply at the transmitter side to maximize the signal-to-noise ratio (SNR). The basic structure of a single carrier system is shown in figure 1.

![Figure 1: Basic structure of a single carrier system](image)

2.1.2 Multicarrier system

In a multicarrier system, input signals which are divided by a multiplexer are applied to pulse-formed $Y_t(t)$ filters before being transmitted through multipath environment. Correspondingly, the receiving ends consist of $N$ parallel paths. Each one is passed through a respective match filter $Y_r(t)$ to realize maximum SNR. The basic structure diagram of a single multicarrier system is shown in figure 2. In a classical wireless communication system, the signal which is transmitted, when arrives at the receiver side via multipath. Thus, extracting the original signal at the receiving side becomes very difficult. If the signal is transmitted at time interval’s $T$, then the parameter concerning the multipath channel is the delay $\tau_{max}$ of the longest path with respect to the earliest path. The received signal can be theoretically influenced by $\tau_{max}$ previous signals, which must be considered seriously by the receiver [8].

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2.2 FDMA and OFDM

The main reason to chose the FDM (Frequency division multiplexing) to equalize the conflict with complexities and impulsive noise. It was first published in 60s [9][10] FDM requires a bank of subcarrier oscillator, coherent demodulator and bank of filters for each sub channel (parallel system). Between subcarriers guard band is introduced, which lowers the efficiency of the spectrum. Weinstein and Ebert [11] apply DFT and IDFT to parallel data stream instead of subcarrier oscillators and demodulators in FDM, which give rise to OFDM. Similarly, in MCM (multi carrier modulation) bandwidth divided into many non-overlapping sub channels (parallel subcarriers) [12] and not essential that all is sub carriers are orthogonal to each other [13]. Figure 3 shows the different view of spectrums normal, FDM and OFDM respectively,
2.3 Basic structure of OFDM system

A basic OFDM system architecture is shown in figure 4. Which is consisted of two parts first one transmitting side other one is receiving side.

- At the transmitting end, first of all, input binary serial data stream is first processed by serial to parallel converter.

- A single signal is divided into N parallel routes after N-point Inverse Fast Fourier transform (IFFT). Each orthogonal sub-carrier is modulated by one of the N data routes independently. By definition the N processed points constitute one OFDM symbol.

- Then copies the last L samples of one symbol to the front as cyclic prefix (CP) then converts modulated parallel data to serial sequence.

- After the process of digital to analog (D/A) conversion and radio frequency (RF) modulation the signal will be transmitted.

- Reception process is converse and self-explanatory to recover the information in OFDM system, demodulated signals are fed into an analog to digital (A/D) converter, sample output and take timing estimation to find initial position of OFDM symbol the digital down conversion is carried out.

- Removed the cyclic prefix and convert the data into the parallel to serial.

- Then applies the Fast Fourier Transform (FFT) and transformation will be conducted on the left sample points to recover the data in the frequency domain.

- The output of demodulation is passed to a decoder, which eventually recover the original data after passing through the parallel to the serial converter.

Orthogonal Frequency Division Multiplexing (OFDM) is deriving from technique based on multi carrier modulation (MCM) and frequency division multiplexing (FDM). It seems to be an optimal form of multicarrier modulation. It emerges modern digital modulation technique FFT (fast Fourier transform), which inherently avoids demodulators, filters and bank of oscillators. It supports smart antennas, directional and advance antenna techniques. OFDM is a good technique against interferences and multi path fading. OFDM uses subcarrier, which is mathematically orthogonal in which
information can be sent parallel overlapping sub channels inversely related to data rate for each individual subcarrier, in return increases the time laps of symbol, from which information or data can be extracted individually. Which solve the problem of delayed version of signals in multipath environment [13]. Each sub channel is orthogonal to each other and faces flat fading [13]. Orthogonality depends up on carrier spacing. Carrier space is to choose that it must be reciprocal of a symbol period. This is helpful in reducing inter-symbol interference (ISI) and good spectral efficiency of the receiver by correlation technique. In the digital domain, OFDM symbol is created before transmission so data is arranged through common methods i.e. BPSK or 16-QAM. The data is converted into $N$ parallel streams, which are to be converted into an OFDM symbol. An OFDM symbol generated by an $N$ subcarrier. In OFDM system, symbol consists of $N$ samples and then the OFDM symbol is

$$x_k = \sum_{n=0}^{N-1} a_n e^{j2\pi kn/N}.$$  \hspace{1cm} (1)

where $a_n$ is the data symbol on the $n^{th}$ subcarrier,

$$\frac{1}{T} \int_T x(t)y(t)dt = 0.$$ \hspace{1cm} (2)

Mathematically, where $x(t)$ and $y(t)$ are two different independent signals [14]. OFDM is an attractive transmission scheme, its wide support to high
data rate, and compatibility with multi path environment, makes it useful in high-speed modems.

Figure 5: OFDM bandwidth divided in many overlapping narrow strips (sub carrier)

In the simple narrow band scheme data transmitted sequentially in a serial form, each symbol occupies the whole bandwidth. While in parallel data transmission many symbols transmitted simultaneously on each sub channel.

In OFDM, whole bandwidth divided into many overlapping narrow strips (sub carrier) as shown in figure 5 above. Which are orthogonal to each other. Lower data rate provided to each narrow strip (total bit rate still high) which is useful to reduce the ISI (inter symbol interference) [15], and accurate data information can be extracted from each sub carrier [14]. Spectrum of OFDM contains many subcarriers. Where the wireless channel posses different frequency response to each subcarrier at a different time.

As data information distributed to each subcarrier. In case of deep fading or selective fading cause, some part of information received with error and other without error. By adding extra bits to transmitted information as error correcting code, there is high probability of correcting information. Because code related to corrupted information might be transmitted in a different sub carrier who might not suffer to fade. In OFDM transmission, as each subcarrier contains part of information so only particular, part of information destroys. While adjacent subcarrier suffer nearly flat fading because it occupies little space in bandwidth. Which makes equalization at the re-
receiver simpler or by introducing coding equalization and its complexities can be complete removed from OFDM receiver [15]. Encoded OFDM is called COFDM.

2.4 Mathematical Representation

The OFDM transmitted signal \( s(t) \) is sum of \( N \) OFDM symbols. Each OFDM symbol is multiplied by carrier \( c_k \). All these modulated symbols are multiplexed and finally multiplexed signal \( p(t) \) is multiplied by carrier signal.

\[
s^*(t) = \sum_{k=1}^{N-1} x_k(t)e^{j2\pi(f_k+F)t} \tag{3}
\]

Where,

\[
f_k = \frac{k}{NT}. \tag{4}
\]

\[
e^{j2\pi Ft} \sum_{k=0}^{N-1} x_k(t)e^{j2\pi f_k t} \tag{5}
\]

Where,

\[
c_k = e^{j2\pi f_k t} = e^{j2\pi \frac{k}{NT} t}. \tag{6}
\]

\[
e^{j2\pi Ft} \sum_{k=0}^{N-1} x_k(t)e^{j2\pi \frac{k}{NT} t} \tag{7}
\]

\[
e^{j2\pi Ft} p^*(t). \tag{8}
\]

Assuming amplitude of carrier signal is equal to 1 and phase is represented by \( C_k \), for symbol period \( C_k \) cannot change (amplitude and phase), for every symbol values of \( C_k \) would be different. Total no of subcarriers available is \( N \) subcarrier. In order to main orthogonality sinc-shaped pulses are use to define subcarriers in frequency domain. sinc-shaped pulses chooses so as it zero crossing occurs at the \( 1/T \) and multiple of \( 1/T \). Equation (9) below ”fi” is centre of carrier frequency and ”fc” is main carrier frequency. Maximum value of each sub carrier spectra occurs at its own frequency and zero on the centre of adjacent subcarrier frequencies [14].

\[
f_i = f_c + \frac{i}{T} \tag{9}
\]

Where,

\[
i = -\frac{N}{2}, \ldots, \frac{N}{2}. \tag{10}
\]
OFDM system is mainly used the division of the frequency selective channel into smaller subchannels. These subchannels can be equal to coherence bandwidth, in which the channel is behaved like flat fading channel, system has been correctly designed then OFDM symbol frequency selective fading channel and the subcarrier signals flat fading channel as shown in 7 [14].

2.5 Fourier Transform

Fourier transform is a mathematical method to convert a signal from the time domain to frequency domain, or from the frequency domain to time domain, Fourier transforms of practical and famous technique is DFT(discrete Fourier transform), which samples the signal in both temporal and frequency domain. FFT (Fast Fourier transform) is fast and efficient method of DFT used by computer application for analysis and signal manipulation. In OFDM, the incoming data is reshaping in order to parallel from serial bits of information, from serial and group of data bits in appropriate size according to design of OFDM and convert into complex numbers. Then these complex numbers is modulated by using IFFT (inverse fast Fourier transform) in a base band to reshape again from parallel to serial for transmission [14][15]. Zeros pad at the end and start of a composite spectrum of subcarriers, to avoid interferences between a next and previous composite spectrum.
2.6 Guard Band

Multipath propagation cause copies of a symbol to be a delay in different time and attenuation. For that reason, the result in inter symbol interference (ISI) and creates another problem is ICI (inter carrier interference), its cause the energy spread of one sub carrier into another sub carrier due to Doppler Effect, so creates the "Cross Talk". This problem resolved by inserting a guard band to symbol in OFDM. Guard band insertion to symbol time increase the temporal period of symbol ,i.e. $T_t = T + T_{gb}$. $T_t$ is total symbol duration and $T$ is original symbol time, while $T_{gb}$ is guard band extension time to symbol. $T_{gb}$ depends up on scenario and king of application. Usually it is $< T/4$. To reduce ISI guard band time $T_{gb}$ must be greater than channel impulse response and delay spread by multipath [14][15]. $T_{gb}$ could be kept adaptive with channel situation. Increasing the time of a symbol which is attained by using larger number of carrier, there is tradeoff between the number of carriers and FFT size, Doppler shift, latency, carrier instability etc [15]. Guard time inserted between consecutive symbols of OFDM [14]. Another way of extending symbol is to place end part of signal to start of signal, shown in figure 8.

2.7 Interleaving

Interleaving is a simple technique to get rid off errors in the burst. In OFDM symbol, information is distributed to many sub carrier frequencies. When
frequency selective fading occurs at different points. OFDM symbol while propagation through channel and this channel cause deep fades data loss in the burst. There is not good enough scheme to handle and recover data in burst error scenario. Interleaving technique rearranging data makes burst error as random error, which could be recovered at the receiver by simple coding. Interleaving is a method of rearranging bits in certain way, at receiver reverse of rearranging is performed to original shape, which makes error to appear in random. Interleaving method is also called block code, in which data is written in row by row and retrieve in column by column [14][15].

2.8 Windowing

FFT of square wave is sinc-function. Sharp transition of bits shapes cause spreading of signal into neighbor spectrum and leakage of energy. Windowing is a technique which causes a spectrum decreases rapidly to go down. Windowing technique is applied on each individual OFDM symbol. Time wave form is truncated by windowing scheme to make individual OFDM symbol. A periodic waveform is truncated to a single OFDM symbol length by applying windowing in the time domain optimum windowing scheme is raised cosine windowing technique. It accommodates channel bandwidth from certain minimum value($R/2$) to certain maximum value($R$) [14].
2.9 Cyclic Prefix of OFDM System

Cyclic Prefix (CP) is the use in OFDM system, can guarantee orthogonality of signals even when the moving through multi-path channels [8]. To avoid ISI, the condition; \( T_G > T_{\text{max}} \) should be satisfied, where \( T_G \) is the length of CP and \( T_{\text{max}} \) is the maximum delay spread [16]. As figure 9 shows that, a CP is a copy of the last part of an OFDM symbol moved to the front of a symbol. Assuming that \( T_G \) is the number of the extended OFDM, then the time period of a OFDM symbol is \( T + T_G \), where \( T \) is cycled for the FFT transform, \( T_G \) is the length of the guard interval, which is inserted to suppress ISI caused by multipath distortion. An OFDM symbol including CP can expresses as follows:

\[
s_n = s_n(t) |_{t=nT_s} = \sum_{t=0}^{N-1} d_i e^{j2\pi \frac{nt}{N}}, \quad n = N_G, ..., -1, 0, ..., N - 1 \quad (10)
\]

Operation between the signal and channel changes from linear convolution to cyclic convolution when CP is used with OFDM. In the frequency domain, linear weighing will be used. These changes avoid inter-symbol interference, while ensuring orthogonality among the sub-carriers all the time.

![Figure 9: OFDM symbols with cyclic prefix](image)

2.10 OFDM Design Issues

There are certain key factors needed to be taken under serious consideration when developing and designing OFDM system.

2.10.1 Useful symbol duration

The length of the useful symbol with respect of time affect the number of carriers and spacing between them. It is helpful in measuring. Larger symbol
duration is helpful in accommodation delay profile of channel and cause increment number of subcarrier, reduces subcarrier spacing, higher FFT size. In practice there may become issue of subcarrier offset and instability of OFDM symbol. Subcarrier spacing and number of carriers depend up on application and requirement. In mobile environment due to Doppler shift subcarrier spacing is chosen to be large [17].

2.10.2 Number of carriers

Number of Subcarrier selection depends on the channel bandwidth, data rate, through put requirements depends up to case (ruler, urban, etc). If the number of carriers is $N$ then it would be reciprocal of duration of a symbol in time $T$ i.e.

$$N = \frac{1}{T}.$$  

Number of carrier selection depends up FFT size, which is supported by FFT module. For higher the number of carrier there would be the higher number of complex point processing by FFT [17].

2.10.3 Modulation scheme

OFDM system having a capacity to the different modulation scheme can be applied to each sub channel depends up on channel condition, data rate, robustness, throughput, channel and bandwidth. There could be different modulation scheme applied i.e. QPSK, 16 QAM, 64 QAM [13]. Modulation to each sub channel can be made adaptive after getting information and estimation of channel at the transmitter [17].

2.11 Advantages and Drawbacks of OFDM

In this section try to summarize the strengths and weaknesses of orthogonal frequency division multiplexing (OFDM).

2.11.1 Advantages of OFDM

The following advantages of OFDM may be identified:

- As OFDM is a parallel transmission system which converts the problem of frequency selective fading to flat fading by distributing data to sub channels, OFDM good to fight again multipath fading and to randomizing the errors in the burst [15].
• As compared to Single-Carrier systems OFDM equalization is very simple.

• The OFDM transmitter is low cost as the design is simple because the modulation technique is simpler implementation based on a highly optimized FFT/IFFT block. Also OFDM transmitters poss the ability to implement the mapping of bits to unique carriers via the use of the Inverse Fast Fourier Transform (IFFT) [18].

• In a relatively slow time-varying channel, it is possible to significantly enhance the capacity by adapting the data rate per subcarrier according to the value of SNR for that particular subcarrier [19].

• OFDM can be used for high-speed multimedia applications with lower service cost.

• OFDM can also support dynamic packet access.

• It is attractive for broadcast applications by using single frequency [19].

• Subcarrier spacing could be adjustable according to the requirement of applications and data rate; it supports different modulation schemes for different sub channels [20].

• Smart antennas can be integrated with OFDM. MIMO systems and space-time coding can be realized on OFDM, and all the benefits of MIMO systems can be obtained easily [21].

2.12 Disadvantages of OFDM

2.12.1 Strict Synchronization Requirement

OFDM is highly sensitive to time and frequency synchronization errors. Demodulation of an OFDM signal with an offset in the frequency can lead to a high bit error rate. These are two sources of synchronization errors. One is caused by the difference between local oscillator frequencies in transmitter and receiver, while the other is due to the relative motion between the transmitter and receiver who gives Doppler spread. Local oscillator frequencies at both points must match as closely as they can. For the higher number of subchannels, the matching should be even more perfect. Motion of transmitter and receiver causes the other frequency error. So, OFDM may show significant performance degradation at high-speed moving vehicles [22]. To optimize the performance of an OFDM link, accurate synchronization is
therefore, of prime importance. Synchronization needs to be done into three aspects: symbol, carrier frequency and sampling frequency synchronization. A description of synchronization procedures is given in [19].

2.12.2 Peak-to-Average Power Ratio

Peak to Average Power Ratio (PAPR) is proportional to the number of subcarriers used for OFDM systems. The PAPR for an OFDM system is given by $10 \log(N)$ where $N$ is the number of subcarriers. For example, for a 48 subcarrier system, such as 802.11a where 48 out of 64 subcarriers are active, the PAPR is approximately 17 dB. Therefore, OFDM system with a large number of sub-carriers will thus have a very large PAPR when the sub-carriers add up coherently. A large PAPR of a system makes the implementation of Digital-to-Analog Converter (DAC) and Analog-to-Digital Converter (ADC) to be extremely difficult. The design of RF amplifier also becomes increasingly difficult as the PAPR increases. To moderate the effect of such a large PAPRs on performance degradation of the OFDM system, the design of the OFDM system needs to incorporate costly RF hardware, such as efficient and large linear dynamic range power amplifiers. Incorporating costly RF hardware, however, increases the cost of the OFDM system. There are basically three techniques that are used at present to reduce PAPR; they are Signal Distortion Techniques, Coding Techniques and finally, the Scrambling Technique which will be discussed later [21].

2.12.3 Co-channel Interference Mitigation in Cellular OFDM

OFDM system exhibit performance degradation due to frequency coherence of the channel. Coherence bandwidth is required to closer the spacing between the adjacent subcarriers. In many channels, adjacent subcarriers will fall within the coherence bandwidth and will thereby experience flat fading. In cellular communications systems, co-channel interference (CCI) is combining adaptive antenna techniques, such as factorization, directive antenna, antenna arrays, etc. Some are just prevention techniques but others may be truly interference cancellation methodologies. Using OFDM in cellular systems will give rise to CCI. Similarly, with the traditional techniques, with the aid of beam steering, it is possible to focus the receiver’s antenna beam on the served user, while attenuating the co-channel interferers. This is significant since OFDM is sensitive to CCI [21].
Chapter 3

3 Investigation of SISO PAPR Reduction

3.1 Precoding

As described in the previous chapter, most of PAPR reduction methods try to exploit the subcarrier symbols of the OFDM block by creating some correlation between them. As a result, the reduction in PAPR achieved by these techniques is relative and is obtained at the expense of either an additional complexity to the OFDM transceiver, a high coding overhead, and/or the need of some kind of transmitter/receiver symbol handshake. Signal companding is another method that has been proposed and studied in the literature [23][24]. Companding is a nonlinear transformation applied to the OFDM signal giving quite low PAPR values. However, such a nonlinearly operation destroys the OFDM orthogonality property and degrades its performance, especially in fading-multipath channels. Other possible alternative solutions are then to try to exploit other parameters of the OFDM signal. Exploiting the subcarrier waveforms of the OFDM signal appears as an attractive solution for reducing the PAPR of OFDM signals. This approach has the potential of reducing the PAPR of the OFDM signal without affecting the bandwidth efficiency of the system and, thus, leaves the chance to use coding for channel protection. Such an approach has been adopted in [25], where a set of subcarrier waveforms was proposed. It has been shown that the PAPR of OFDM signals can be reduced if the subcarrier waveforms have different shapes. It has also been shown in [25] that subcarrier waveform shaping in OFDM is a form of precoding scheme, where each OFDM block is linearly transformed by a shaping matrix before modulation and transmission. In the literature, precoding has been considered as a way of maximizing the diversity gain of OFDM signals and of trying to take advantage of the frequency selectivity of the multipath-fading channel [26]-[30]. For instance, in [26], linear constellation precoding (LCP), together with subcarrier grouping, has been designed to maximize both diversity and coding gains. It has been shown that subcarrier grouping can reduce the complexity of the receiver without affecting the maximum possible diversity and coding gains. However, Liu et al. [26] did not investigate the implication of LCP on the signal variations of the OFDM signal. Clearly, regardless of the LCP scheme used, the PAPR of the OFDM will increase as the number of subgroup increases. This has been shown in [28], where the numerical results showed that the lowest PAPR ratio is obtained when only one group is used. However, using
LCP with only one group will make the receiver too complex and limits the number of subcarriers that can be used. In fact, the precoder proposed in [27] was designed based on maximizing the diversity gain and minimizing the PAPR of the OFDM signal simultaneously. We consider precoding as a way of reducing the PAPR of OFDM transmitted signals. Precoding in OFDM systems consists of multiplying the modulated data of each OFDM block by a precoding matrix before OFDM modulation [inverse discrete Fourier transform (IDFT)] and transmission. A predefined precoding matrix is used in the OFDM system, and thus, no handshake is needed between the transmitter and the receiver. Having the same precoding matrix for all OFDM blocks will also avoid all the processing needed in block-based optimization methods. A design procedure for good precoding schemes is proposed and analyzed in this paper. We will show that it is possible to reduce the PAPR of OFDM signals through precoding without destroying the delectability property of the different symbols of the OFDM block. The obtained results show that, with a good precoding matrix, the PAPR of OFDM modulated signals can be made very close to that of single carrier signals.

3.2 Transmitter Model for a Precoded OFDM System (SISO)

An OFDM system with multiple phase-shift-keying (MPSK) modulation and a total of N baseband-modulated symbol per OFDM block is considered in this paper. As shown in figure 10, the transmitter consists of a baseband (complex) modulator followed by a precoder and the conventional OFDM modulator (IDFT). The incoming information data is first modulated in baseband using a bandwidth-efficient modulation (MPSK-type modulation). The baseband-modulated stream, with data rate 1/Ts, is grouped into blocks of length N symbols each.
Each block of symbols is then precoded by an $L \times N$ precoding matrix, denoted $P$, and defined as

$$
P = \begin{pmatrix}
p_{0,0} & p_{0,1} & \cdots & p_{0,N-1} \\
p_{1,0} & p_{1,1} & \cdots & p_{1,N-1} \\
\vdots & \vdots & \ddots & \vdots \\
p_{L-1,0} & p_{L-1,1} & \cdots & p_{L-1,N-1}
\end{pmatrix} \quad (11)
$$

Where $p_{i,j}$'s are the entries (complex numbers) of this precoding matrix, $L = N + N_p$ is the total number of subcarriers, and $N_p$ is the extra subcarriers (overhead) used with $0 \leq N_p < N$. When no precoding is used, the matrix $P$ reduces to an $N \times N$ identity matrix, and no overhead is used.

$$
X = [X_0, X_1, X_2, \ldots, X_{N-1}] \quad (12)
$$

Which after serial to parallel conversion becomes

$$
X = \begin{pmatrix}
X_0 \\
X_1 \\
\vdots \\
X_{N-1}
\end{pmatrix} \quad (13)
$$

Where

$$
X_i = \sqrt{E_s/T} e^{j\theta_0 + d2\pi/M} \quad d \in \{0, 1, \ldots, M - 1\} \quad (14)
$$

And $\theta_0$ is some initial phase.
The precoding process (matrix) transforms this vector into a new vector of length $L$ with

$$Y = PX = \begin{pmatrix}
Y_0 \\
Y_1 \\
\vdots \\
Y_{L-1}
\end{pmatrix}$$

(15)

Where

$$Y_i = \int_{m=0}^{N-1} P_{i,m} X_m \quad [i = 0, 1, 2, \ldots, L - 1]$$

(16)

These precoded symbols are then transmitted over the different subcarriers of the OFDM-modulation scheme. In this case, the equivalent low pass of the OFDM transmitted signal can be written as follows:

$$x(t) = \sum_{i=0}^{L-1} Y_i e^{j2\pi it/T} \quad -T_g \leq t < T$$

(17)

where $T = NTs$ is the duration of the OFDM block, and $T_g = GTs$ is a time guard interval introduced between consecutive OFDM blocks at the transmitter to prevent the possible problem of intersymbol interference, which can be caused by the communication channel, and to preserve the orthogonality between the OFDM subcarriers. This guard interval is, in general, ignored at the receiver before demodulation and signal detection.

The OFDM-band pass signal is related to its equivalent low pass by the following expression:

$$s(t) = \text{Real}\{x(t) e^{j2\pi f_c t}\}$$

(18)

Where $f_c$ is the carrier frequency.

In the linear case (when no power amplifier is used) and for large values of the number of subcarriers, the spectrum of conventional OFDM signals goes to an ideal band-limited rectangular spectrum. This means that the OFDM signal within each block appears as Gaussian with very high variations from one sample to the next. Thus, the power spectral density of the modulated signal will be broadened by the nonlinear distortions of a high-power amplifier. The PAPR is one way to measure such variation of the transmitted signal.

### 3.3 PAPR of precoded OFDM

The PAPR of the precoded OFDM transmitted signal of equation (17) can be defined as follows:
\[ PAPR = \max \left\{ \frac{|x(t)|^2}{E\{ |x(t)|^2 \}} \right\} \quad (19) \]

The value of the above expression will depend on the kind of precoder used at the transmitter. Our objective in this paper is to select a precoder that minimizes the above expression. Combining equations (16) and (17), the equivalent low pass of the OFDM transmitted signal can be rewritten as follows:

\[ x(t) = \sum_{i=0}^{L-1} Y_i e^{j2\pi it/T} \quad (20) \]

\[ = \sum_{m=0}^{N-1} X_m \left( \sum_{i=0}^{L-1} P_{i,m} e^{j2\pi it/T} \right) \quad 0 \leq t < T \quad (21) \]

where we have ignored the guard interval, for now, since it is just an extension by periodicity. Using the above equation in (19), we can relate the PAPR of the OFDM signal to the different entries of the precoding matrix. For OFDM systems with MPSK modulation schemes and uncorrelated symbols within each OFDM block, the PAPR of the OFDM signal at a given time instant \( t \) can be upper bounded as follows:

\[ PAPR(t) \leq \frac{1}{N} \left( \sum_{m=0}^{N-1} |(\sum_{i=0}^{L-1} p_{i,m} e^{j2\pi it/T})|^2 \right)^{1/2} \quad (22) \]

and the maximum PAPR is then obtained as

\[ PAPR_{\text{max}} = \max_{0 \leq t < T} PAPR(t) \]

\[ = \frac{1}{N} \max_{0 \leq t < T} \left( \sum_{m=0}^{N-1} |(\sum_{i=0}^{L-1} p_{i,m} e^{j2\pi it/T})|^2 \right)^{1/2} \quad (23) \]

Where we have assumed that \( |X_m|^2 = E_s \)

Where, \( m = 0, 1, \ldots, N-1 \), which represents the average-energy-per-transmitted symbol.

### 3.4 Selection Criteria Of Precoding Matrix

We notice that the PAPR of the OFDM signal is a function of the size of the OFDM block and the entries of the precoding matrix. Since the size of the OFDM block is fixed, one can reduce the PAPR of the signal by a proper selection of the precoding matrix \( P \). However, selecting the proper matrix is not an easy task because its entries are complex numbers and they can take any value, which makes computer-search methods very difficult to use.
Thus, before doing any precoding design, we should take a closer look at the PAPR expression given in equation (22).

From equation (22), we define a set of time limited (complex) function \( \{p_m(t)\} \) as follows:

\[
P_m(t) = \begin{cases} 
\sum_{i=0}^{L-1} p_{i,m} e^{j2\pi it/T}, & 0 \leq t < T \\
0, & \text{otherwise}
\end{cases}
\]  

for \( m = 0, 1, \ldots, N - 1 \). With the above definition, the PAPR of the precoded OFDM signal can be rewritten in terms of \( p_m(t) \) as

\[
\text{PAPR}(t) \leq \frac{1}{N} \left( \sum_{m=0}^{N-1} |p_m(t)| \right)^2 \]  

We notice that the PAPR ratio is now related to the sum of \( N \) positive functions within the time interval \( 0 \leq t < T \). This gives some hint on how to select the entries of the precoding matrix \( P \). A possible solution is to make sure that the peak amplitudes of the \( N \) functions \( |p_m(t)|, m = 0, 1, \ldots, N - 1 \) do not occur at the same time instant within the interval of definition. By ensuring that, the peak power of the OFDM signal can be reduced without altering the average signal power.

A possible set of functions that avoids having the peak amplitudes to occur at the same time instant can be obtained by selecting the different entries such that the different functions are cyclic shifts of each other within the time interval \( 0 \leq t < T \). In other words, we can impose the following relation between the different functions:

\[
p_m(t) = \begin{cases} 
p_0(t - mT_s + T), & 0 \leq t < mT_s \\
p_0(t - mT_s), & mT_s \leq t < T
\end{cases}
\]  

and solve for the entries of the precoding matrix \( P \).

By letting \( p(t) = p_0(t) \), we can relate all the different functions to this mother function \( p(t) \) with

\[
p_m(t) = \begin{cases} 
\sum_{i=0}^{L-1} p_{i,0} e^{-j2\pi im/N} e^{j2\pi it/T}, & 0 \leq t < T \\
0, & \text{otherwise}
\end{cases}
\]  

where \( p_{i,m} \) are the entries of the precoding matrix \( P \) in equation (11).

It is clear that if \( p_0(t) \) has only one amplitude peak, then all the other functions will also have one amplitude peak and all the amplitude peaks will not occur at the same time instant. Hence, this format will certainly reduce the peak-to-average ratio of the precoded OFDM scheme.

We also notice from the above expression that the entries of the precoding matrix are related to each other. Furthermore, all the entries of the different
columns of the precoding matrix are directly obtained from the entries of the first column. This indicates that we only need to find the first column of the matrix $P$, which will of course simplify the design of the precoder considerably. In fact, from equation (27), we notice that having a mother function $p(t)$, we can extract all the entries of the precoding matrix from $p(t)$ as

$$p_{i,m} = p_{i,0} e^{-j2\pi im/N} = e^{-j2\pi im/N} \int_0^T p(t) e^{j2\pi it/T} \, dt$$  \hspace{1cm} (28)$$

With $p(t)$ as any of the complex functions defined within the time interval $0 \leq t < T$.

3.5 Theorem 1

With a precoding matrix designed according to equation (28), the maximum PAPR of uncoded OFDM transmitted signals is upper bounded as

$$PAPR_{\text{max}} \leq N$$  \hspace{1cm} (29)$$

with equality when the precoding matrix is an $N \times N$ identity matrix (no precoding).

**Proof:**

From equation (25), the maximum PAPR of the precoded OFDM transmitted signal is upper bounded as

$$PAPR_{\text{max}} = \frac{1}{N} \max_{0 \leq t \leq T} \left( \sum_{m=0}^{N-1} |p_m(t)| \right)^2$$  \hspace{1cm} (30)$$

Using the relation given in equation (27), the maximum PAPR of the precoded OFDM signal becomes

$$PAPR_{\text{max}} = \frac{1}{N} \max_{0 \leq t \leq T} \left( \sum_{m=0}^{N-1} |p(\tau(t,m))| \right)^2$$

$$\leq \frac{1}{N} \left( \frac{T}{T_s} \int_0^T |p(t)| \, dt \right)^2$$  \hspace{1cm} (31)$$

where

$$\tau(t,m) = \begin{cases} 
(t - mT_s + T), & 0 \leq t < mT_s \\
(t - mT_s), & mT_s \leq t < T
\end{cases}$$  \hspace{1cm} (32)$$

Using Schwarz’s inequality in equation (31) and replacing $T_s$ by $T/N$, the upper bound of equation (29) is obtained.

Note that with this precoding scheme, the PAPR of the precoded OFDM signal is always reduced in comparison with that of conventional OFDM. This improvement in PAPR is valid for any number of subcarriers $N$. The
design of such a precoding scheme is also simple since all the entries of the precoding matrix are obtained from its first column.

A very important property of OFDM modulation is the separability of the $N$ transmitted symbols of each OFDM block at the receiver for detection. For regular OFDM, each symbol is transmitted over a different subcarrier, and hence, symbol separability at the receiver is ensured by the orthogonality property between the different subcarriers. In precoded OFDM, each symbol is spread by the precoding matrix over more than one subcarrier, as shown in equation (16). Hence, the orthogonality property between the different subcarriers alone is not enough to separate the different symbols of the block in precoded OFDM. As a result of precoding, crosstalk between the different symbols of the block will exist at the receiver, and symbol-by-symbol detection is no longer optimum. However, with a proper selection of the precoding matrix, we can eliminate this possible crosstalk and allow simple symbol-by-symbol detection for precoded OFDM signals. This can be done by selecting the precoding matrix as an orthogonal matrix, i.e., a precoding matrix that satisfies the following relation:

$$P^* P = I$$  (33)

where $I$ is the $N \times N$ identity matrix, and $P^*$ represents the Hermitian transpose of the matrix $P$.

Using equation (11), the above condition can be written as follows:

$$\sum_{i=0}^{L-1} p_{i,m}^* p_{i,k} = \begin{cases} 1, & m = k \\ 0, & m \neq k \end{cases}$$  (34)

Replacing $p_{i,m}$ by its expression given in equation (28) and with some manipulations, the symbol-separability condition of the precoded OFDM signal becomes

$$\frac{1}{T_s} \sum_{i=0}^{L-1} P(i/T_s)^2 e^{j2\pi(m-k)i/N} = \begin{cases} T, & m = k \\ 0, & m \neq k \end{cases}$$  (35)

Where

$$P(f) = \int_0^T p(t) e^{-j2\pi f t} dt$$  (36)

is the Fourier transform of the function $p(t)$.

For large number of subcarriers ($N \gg 1$), the above condition can be approximated by an integral as

$$\int_0^{(1+\beta)/T_s} P(f)^2 e^{j2\pi(m-k)f} df = \begin{cases} T, & m = k \\ 0, & m \neq k \end{cases}$$  (37)

with $\beta = Np/N$, which is just the Fourier inverse of $|P(f)|^2$ at time instant $t = (m - k)T_s$.

Denoting this function by $q(t)$
we conclude that the symbol-separability property of OFDM is preserved when $q(t)$ satisfies the following criterion:

$$q(mT_s) = \begin{cases} T, & m = k \\ 0, & m \neq k \end{cases}$$

which is simply the Nyquist criterion [31].

The above conclusion indicates that in order to preserve the symbol-separability property of the OFDM scheme, the function $p(t)$ should have a bandwidth of at least $1/T_s$. A bandwidth comparable to the bandwidth of the OFDM signal.

To summarize, a precoding matrix designed according to the following two criteria.

1. $$p_{i,m} = p_{i,0}e^{-j2\pi im/N} = e^{-j2\pi im/N} \int_0^T p(t)e^{j2\pi it/T} dt$$

   With
   
   i ∈ {0,1,……,L-1} and m ∈ {0,1,……,N-1}

2. $$\int_0^{(1+\beta)/T_s} P(f)^2 e^{j2\pi (m-k)f}df = \begin{cases} T, & m = k \\ 0, & m \neq k \end{cases}$$

   can reduce the PAPR of the OFDM signal without altering the symbol-separability property of the OFDM scheme.

Note that by selecting a precoding matrix satisfying equation (39), we basically obtain an OFDM signal similar to a single-carrier signal due to the wide bandwidth of the set of functions $\{p_m(t)\}$ and the cyclic shifting in time. However, this precoding scheme does not remove any of the properties of OFDM, where as discussed in the following sections, simple (single tap) equalization is still possible. It should also be mentioned that this scheme is also different from single-carrier signals with cycle prefix and frequency equalization [25]. In the latter, a cyclic prefix is added at the transmitter, and both fast Fourier transform and inverse fast Fourier transform (IFFT) operators are done at the receiver. The proposed precoded scheme in this paper is an OFDM scheme and does not alter any of the structure and properties of OFDM systems.

In the following example, we illustrate a possible precoding scheme designed based on the technique just described above and evaluate the PAPR of the corresponding precoded OFDM signal.
3.6 Examples of precoding schemes

A well-known function that satisfies the Nyquist criterion is the raised cosine function [31]. Consider the square root of a raised cosine function, which is denoted \( p_{src}(t) \), and having a Fourier transform [31]

\[
P_{src}(f) = \begin{cases} 
T_s \sin\left(\frac{\pi f T_s}{2\beta}\right), & 0 < f \leq \frac{\beta}{T_s} \\
T_s, & \frac{\beta}{T_s} < f \leq \frac{1}{T_s} \\
T_s \sin\left(\frac{\pi (f T_s - 1)}{2\beta} + \frac{\pi}{2}\right), & \frac{1}{T_s} < f \leq \frac{1+\beta}{T_s}
\end{cases}
\]  

(40)

with

\[
p_{src}(t) = \int_{0}^{(1+\beta)/T_s} P_{src}(f) e^{-j2\pi ft} dt
\]  

(41)

and

\[
\beta = \frac{N_p}{N} = \frac{L - N}{N}
\]  

(42)

defines the portion of extra subcarriers (overhead) used by the precoder.

With the above definition, the entries of the precoding matrix \( P \) can be computed according to equation (28) with the mother time function \( p(t) \) defined as follows:

\[
p(t) = \begin{cases} 
p_{src}(t - T/2), & 0 \leq t < T \\
0 & \text{otherwise}
\end{cases}
\]  

(43)

For large number of subcarriers \( N \), the entries of the precoding matrix can be written as

\[
p_{i,m} = p_{i,0} e^{-j2\pi i m N} = e^{-j2\pi i \frac{m}{N}} \frac{1}{T} \int_{0}^{T} p(t) e^{-j2\pi \frac{m}{N} t} dt
\]  

(44)

which are completely defined by the selected function. Using equation (40) in the above equation and rearranging terms, the entries of the precoding matrix become

\[
p_{i,m} = p_{i,0} e^{-j\frac{2\pi i m}{N}}
\]

with

\[
p_{i,0} = \begin{cases} 
\frac{(-1)^i \sin(\frac{\pi i}{2 N_p})}{\sqrt{N}}, & 0 \leq i < N_p \\
\frac{(-1)^i}{\sqrt{N}}, & N_p \leq i < N \\
\frac{(-1)^i \cos(\frac{\pi (i-N)}{2 N_p})}{\sqrt{N}}, & N < i \leq L - 1
\end{cases}
\]  

(45)

where, as indicated earlier in this paper, \( L = N + N_p \) and \( \beta \) was replaced by \( N_p/N \). It is observed that once we know the number of symbols per OFDM block \( N \) and \( L \), we can easily
Compute the entries of the precoding matrix $P$. The parameter $L$ is a design parameter and will depend on the extra subcarriers used, since $L = N + N_p$. It is easy to verify that this precoding matrix satisfies the orthogonality condition given in equation (33).

### 3.7 Simulation Results

Using equation (45), we plot the precoder matrix in the MATLAB and get the following results.

![Figure 11: Precoder](image)

It is clear from the simulation result that precoding matrix is a sinc function. The first graph shows the all column results and second one show the only first column of precoding matrix. It is also clear from the graph that precoding matrix distribute the whole power uniformly during the transmission time. Therefore it is better to reduce the peak to average power ratio. Using equation (23), we can now compute the maximum PAPR of the precoded OFDM signal and compare it to that of the conventional scheme.

When the number of subcarriers $N$ is large, the maximum of the PAPR will occur only very seldom, and thus, the measure of such a parameter may not give the whole picture about the dynamic variations of the OFDM signal. As shown in equation (19), the PAPR is a random variable and takes the values between zero and PAPRmax. A better measure of the PAPR of communication signals is then to consider the complementary cumulative distribution function defined as:

$$P_{P_{APR}} = P_r(P_{APR} \geq P_{APR_0}) \quad \text{(cumulative distribution Function)}$$

$$P_r(P_{APR} \geq P_{APR_0}) = 1 - P_{P_{APR}} \quad \text{(complementary cumulative distribution)}$$

where $P_{APR_0}$ is the PAPR threshold.
Figure 12 illustrates the complementary cumulative distribution function of the PAPR of the precoded OFDM signal for the case of \( N = 64 \) subcarriers and Precoder 1. It is observed that the proposed precoding scheme provides considerable gain in PAPR for the OFDM signal when compared to that of conventional OFDM. We also notice here similar results where the precoded OFDM scheme outperforms conventional OFDM in terms of PAPR. It compared to the case of the DFT matrix [27], we notice that with little overhead, the PAPR of the OFDM signal can be reduced with a reduction that depends on the amount of overheads. For instance, the PAPR can be reduced by about 1.5 dB with an overhead of 10% and by about 3 dB with an overhead of 20% at a complementary CDF value of \( 10^{-3} \).

To assess the effects of the number of subcarriers on the signal variations of the precoded OFDM signal, we have looked at the complementary cumulative distribution function of the precoded OFDM signal for the different number of subcarriers. This is illustrated in figure 13 as a function of the PAPR threshold. Similar to the results of the maximum PAPR, we notice that the complementary distribution function of the PAPR of the precoded scheme is not very sensitive to the number of subcarriers and especially at high values of the PAPR threshold.
Figure 13: CCDF for different block sizes
Chapter 4

4 System Performance In Fading-Multipath Channels

As already shown in the previous chapter, it is possible to design precoding schemes that reduce the PAPR of OFDM signals without altering the properties of symbol reparability of the OFDM block at the receiver. Thus, when the communication channel is ideal (additive interference only), the performance of the precoded OFDM system will be identical to that of conventional OFDM. However, any frequency variation of the communication channel will destroy this Orthogonality property and can degrade the system performance. We will treat this case in this section and see what kind of reliable low complexity detector(s) can be used.

4.1 Channel Model

The fading channel considered in this paper is a fading multipath channel with coherence bandwidth smaller than the total bandwidth of the OFDM system and, thus, seen as frequency-selective fading. The fading process is assumed to be stationary and slowly varying compared with the block duration of the OFDM signal, such that it is approximately constant during at least one block length. The complex baseband representation of the fading multipath channel impulse response can be described by [31]

$$h(\tau) = \sum_{l=0}^{P-1} h_l \delta(\tau - \tau_l)$$ (46)

where $h_l$ is a complex random-variable tap weight with variance $p_l$, $\tau_l$ is the time delay of the $l$th path, and $P$ is the total number of received paths. The tap weights are assumed independent of different paths.

4.2 Receiver Structure

Passed through the channel, the equivalent lowpass of the received precoded OFDM signal during the zeroth block interval is given by

$$r(t) = \sum_{l=0}^{P-1} h_l x(t-\tau_l) + z(t) \quad -T_g \leq t < T$$ (47)

where $z(t)$ is complex Gaussian random process with zero mean and power spectral density $N_0$, and $T_g$ is the time guard interval.
Assuming a guard interval larger than the maximum delay spread of the fading-multipath channel, the output sample of subcarrier $i$, after demodulation, is obtained as

$$D_i = \frac{1}{\sqrt{T}} \int_0^T r(t) e^{-j2\pi it/T} dt = \sqrt{T} H_i Y_i + Z_i$$

where $Y_i$ is as given in (16), $Z_i$ is a complex Gaussian random variable with zero-mean and variance $N_0$, and $H_i$ is the channel frequency response at subcarrier $i$ with

$$H_i = \sum_{l=0}^{P-1} h_le^{-j2\pi in/T}$$

which is a complex Gaussian random variable with variance

$$2\sigma^2 = \sum_{l=0}^{P-1} p_l$$

4.3 CSI (channel-state information)

To allow coherent detection at the receiver, channel-state information (CSI) needs to be known at the receiver. For wireless systems employing coherent OFDM modulation techniques such as HIPERLAN/2 and IEEE 802.11a

Figure 14: Precoded OFDM tranciever
systems, one or two complete OFDM symbols are provided in the preamble in order to support channel estimation. For the proposed precoded OFDM scheme, the pilot symbols are precoded the same way as the information data symbols. As shown in equation (48), a priori knowledge of the transmitted precoded pilot signal facilitates the generation of the CSI vector at the receiver. Denoting the pilot OFDM symbol by

\[ [C_0, C_1, \ldots, C_{N-2}, C_{N-1}] \]

the CSI at subcarrier \( i \) can be obtained from equation (48) as

\[ \hat{H}_i = \frac{D_i}{\sqrt{T \tilde{Y}_i}} = H_i + \frac{Z_i}{\sqrt{T \tilde{Y}_i}} i=0,1, \ldots, L-1 \]  

where

\[ \tilde{Y}_i = \sum_{m=0}^{N-1} p_{i,m} C_m i=0,1, \ldots, L-1 \]  

which is known at the receiver since it is completely defined by the pilot OFDM symbol and the precoding matrix. Considering all the received subcarrier samples during the zeroth OFDM block, the expression in equation (48) can be rewritten in a matrix form as follows:

\[ D = \sqrt{T} HP X + Z \]  

where \( X \) is as given in equation (13), \( Y \) is as given in equation (15)

\[ Z = [Z_0, Z_1, \ldots, Z_{L-1}]^T \]  

is the noise vector, and \( H \) is an \( L \times L \) diagonal matrix representing the channel coefficients of the different subcarriers with

\[ H = \text{diag}\{H_0, H_1, \ldots, H_{L-1}\} \]  

### 4.4 MMSE detector

Having the demodulated OFDM signal, the next step is to separate the modulated symbols \( X_m \)s using the received vector given in equation (53) and make a decision on the transmitted symbols. It is observed that when the channel is frequency selective, a direct multiplication of the vector \( D \) by the precoding matrix \( P^* \) does not give a perfect separation, and crosstalk between the different modulated symbols will remain. Several detection techniques that deal with this type of signal separation have appeared in the literature. The optimum detector is of course the maximum-likelihood sequence-estimation detector where all the \( N \) symbols are detected jointly. However, such a detector has a complexity that increases exponentially with \( N \) and is not
practical even for moderate number of subcarriers. Fortunately, much simpler detection schemes do exist and can provide very good performance in fading-multipath channels.

Here, we consider the minimum-mean-square-error (MMSE) detector, which is basically a one-tap equalizer per subcarrier. In this detector, each element of the received vector is first weighted using the following weighting parameter:

\[ G_i = \frac{\hat{H}_i^*}{\hat{H}_i^* + \sigma_z^2/\sigma_s^2}, \quad i=0,1,\ldots,L-1 \]  

(56)

where \( \sigma_z^2 \) is the variance of the additive white noise, and \( \sigma_s^2 \) is the variance of the data-transmitted symbol. The parameter \( G_i \) is used to compensate for the channel phase and to minimize interference between the modulated symbols of the OFDM block. The obtained vector is then multiplied by the precoding matrix \( P^* \).

The block diagram of the OFDM system using this receiver structure is illustrated in figure 15. In matrix form, the received vector after weighting and multiplication by the matrix \( P^* \) is written as follows:

\[ V = P^*GD = \sqrt{T}P^*GHPX + Z' \]  

(57)

where \( G \) is an \( L \times L \) diagonal matrix with

\[ G = \{G_0, G_1, \ldots, G_{L-1}\} \]  

(58)

and \( Z' \) is the complex Gaussian noise vector. Decisions on the transmitted symbols of the OFDM block are then carried out using the vector \( V \).

**4.5 Simulation Parameters and Results**

In this section, the performance of the precoded OFDM scheme is evaluated through computer simulations for different multipath radio channels. The system parameters were adjusted according to the HIPERLAN/2 standard [32] with an OFDM modulation using coherent QPSK and having a total of \( N = 64 \) subcarriers. The entries of the precoding matrix are obtained from equation (40) with an overhead of \( \beta = Np/N = 10\% \). This gives a total of \( L = 70 \) subcarriers for the OFDM system. The MMSE detector discussed in the previous section is employed at the receiver, where we have assumed that perfect CSI is available. The multipath radio channel considered is as specified in [33]. It contains different channel models, representing different environments, and with tapped delay lines with a total of 18 taps. Each tap
suffers independent Rayleigh or Rician fading with a mean corresponding to an exponentially average power-delay profile.

![Figure 15: Performance in fading multipath channels](image)

It is clear from the simulation results that bit error probability for precoding system goes to zero much earlier as compared to the conventional system. In terms of bit to noise ratio the precoding system gives the gains of more than 25 dB as compared to the conventional system. Precoding is good technique to increase the performance (in terms of bit error probability and Bit to Noise ratio) of the system in multipath fading channel.
Chapter 5

5 Multiple Input Multiple Output (MIMO)

5.1 Introduction

In 1984 Jack Winter writes an article in Bell Laboratories about MIMO signaling development in the field of wireless telecommunication [34]. Many of the applications in wireless communication require high data rates. Conventionally, for high data rate, more bandwidth is for transmission. On the other hand, due to bandwidth limitations, it is often impractical or sometimes very expensive to increase bandwidth. In this case, there is another solution for efficient transmissions by using multiple transmit and receive antennas. Multiple transmit antennas can be used either to obtain transmit diversity or to form multiple-input multiple-output (MIMO) channels. It is confirmed in that, compared with a single-input single-output (SISO) system [35]. For wideband transmission, MIMO system can improve the capacity by a factor of the minimum of the number of transmit and receive antennas [36].

5.2 Basic Structure of MIMO System

There are several wireless communication transmission systems, each system has different property's parameter and limits.

5.2.1 SISO System

SISO stands for single input and signal output. It system model uses only one antenna both at the transmitter and receiver. SISO systems are cheap to implement, as shown in figure 16.

![Figure 16: SISO Systems](image)
5.2.2 SIMO System

SIMO stands for single input and multiple outputs. Its system model uses only the single antenna at the transmitter side and multiple receiving antennas. For uplink purpose SIMO systems are preferably, as shown in figure 17 [17].

![Figure 17: SIMO System](image)

5.2.3 MISO System

MISO stands for multiple input and single output. Its system model has only multiple transmitting antennas and one receiving antenna. For downlink purpose MISO systems are preferably, as shown in figure 18.

![Figure 18: MISO System](image)

5.2.4 MIMO System

MIMO stands for multiple inputs and multiple outputs. Its system model uses multiple antennas both for transmission and reception. A number of antennas are placed at the transmitting and receiving ends. Their distances are separated far enough. For both uplink and downlink purpose, MIMO systems...
are effective. In MIMO system, multiple transmitting and receiving antennas will achieve antenna diversity without reducing the spectral efficiency, as shown in figure 19.

![Figure 19: MIMO System](image)

### 5.3 Channel Capacity

Channel capacity can be defined as the number of bits per channel per Hz. According to recent research on Shannon’s capacity has shown that there is huge channel capacity could be attained from the MIMO systems relatively single antenna system. Which is also depended up on different scenarios, fading, channel knowledge, impulse response of channel quality, correlation gain of channel on both antenna elements and transmitter receiver channel quantity knowledge [37]. Channel capacity for different antenna system can be analyses as.

#### 5.3.1 Channel Capacity for SISO

According to Shannon for SISO system, the channel capacity as shown in figure 20 [38]

![Figure 20: Channel Capacity for SISO system](image)
5.3.2 Channel capacity for SIMO

According to Shannon for SIMO system, the channel capacity as shown in figure 21 [38], where $M$ shows the multiple receiver, In which we have a single antenna on transmitting side and multiple antennas on receiving side the channel capacity of SIMO systems provides receiver diversity because of multiple antennas at the receiver side and capacity is slow logarithmic rising with increasing number of antennas. SIMO system model it is a kind of smart antenna in which there is Single Input at the Transmitter and on the receiver side. There are having multiple Outputs, it is used for different purpose as shortwave radio operators, military, amateur and at frequency below 30 MHz since the first world war [39].

![Figure 21: Channel Capacity for SIMO system](image)

5.3.3 Channel Capacity for MISO

figure 22 shows the channel capacity of MISO system. In which $N$ shows the multiple transmitter antennas, The MISO channel model provides transmits diversity because of multiple numbers of antennas at transmitter side, and slow logarithmic rise of capacity with increasing number of antenna. MISO is used for the improvement of transmission at a distance; the multiple transmitters are used to be combined and minimize errors and optimize data speed. It is also kind of smart antenna technology that uses multiple transmitters and single receiver. MISO technology is used widely in digital television, wireless local-area networks (WLAN’s) etc [40].
5.3.4 Channel Capacity for MIMO

In figure 23, shows the channel capacity of a MIMO system, in which we have multiple antennas on both sides either on receiving or transmitting side, $M$ and $N$ shows multiple transmitter and receiver antennas, and it is also in the category of smart antenna, and it improves the communication performance, MIMO technology is used in modern wireless communication Wi-Fi, WiMAX, HSPA, 4G and 3GPP LTE (Long Term Evolution).

In MIMO multiple antenna’s channel faces multiple input, and output are used, and its capacity is also determined by extended Shannon’s capacity [38]. Antenna with $N$ input from transmitter and $M$ output in a receiver channel is expressed as $N * M$ matrix of channel $H$ as shown in below equation (60):

\[
\begin{align*}
&c = \log_2 (1 + y) \\
&c = \log_2 (1 + N)
\end{align*}
\]
\[ C = \log_2 \det(I + \frac{1}{\sigma_n^2}HR_xH^H) \] (59)

Where \( H \) is \( N_\gamma \times M_\gamma \) channel matrix, \( R_x \) is covariance of input signal \( x \), \( H^H \) transpose conjugate of \( H \) matrix and is the variance of the uncorrelated and Gaussian noise [41]. Since equation (3.1) is obtained by large theoretical calculations, but practically it has never been achieved yet. To get precise results linear transformation at both transmitter and receiver can be performed by converting MIMO channel \((N_\gamma \ M_\gamma)\) to a SISO sub channel min \((N_\gamma \ M_\gamma)\). According to singular value decomposition (SVD) every matrix can be decomposed. Suppose the channel matrix \( H \) transformation is given by

\[ H = UDV^H \] (60)

Where the matrix \( U \) is \( N_\gamma \times N_\gamma \) matrix, \( V \) is \( N_c \times N_c \) matrix and \( D \) is a non-negative diagonal matrix of \( N_\gamma \times M_\gamma \). Therefore, capacity of \( N \) SISO sub channels is some of individual capacity and results from the total MIMO capacity [42].

### 5.4 Diversity

Diversity scheme is a technique which is used to improve the performance of the communication system by effectively transmitting the same information multiple times to improve the signal to noise ratio such that transmitted signal is detected correctly.

#### 5.4.1 Types of Diversity

To increase the capacity of MIMO system one of the methods is diversity technique. There are several types of diversities.

1. Frequency Diversity: In this type of diversity, the information is transmitted different frequencies. To achieve the uncorrelated diversity, the carrier frequency should be separated suitable.

2. Time Diversity: Time diversity can be explained like, different signal are transmitted on the different time interval which is depending on fading range. If the time interval between time slots is longer, then the fading becomes much low [43].
3. Space Diversity: It is a type of diversity in which there are two or more signals is sent over different propagation paths by using multiple antennas at both side transmitter and receiver. The spacing is carefully chosen to ensure the independence of possible fading events occurring in the channel [44].

4. Transmit Diversity: In transmit diversity multiple antennas are placed at a transmitter side, and data are sent through multiple channels.

5. Receive Diversity: In receive diversity of multiple antennas are placed in a receiver to pick the independent copies of transmit signal [45].

5.5 Advantages of MIMO

MIMO had some significant advantages over SISO, SIMO, MISO and other techniques:

- MIMO enhanced the result of QoS and coverage area due to array gain.
- Higher the multiplexing gain which in the result increase spectral efficiency.
- MIMO increase QoS service, higher diversity gain and fewer chance to lose information.
- Co-channel interference is minimized which is helpful in increasing cellular capacity [46].
Chapter 6

6 MIMO-OFDM

6.1 Introduction

In advance broad-band access in LAN and MAN OFDM (stands for frequency division multiplexing) is used in different combinations and techniques, e.g. MIMO. This can combat better against multipath fading (deep fading) and also supports high data rate. Over a radio link like HDTV, it supports multimedia applications. MIMO-OFDM reduces the receiver complexities and manipulations as they distribute over multiple sub carriers the data information and transmit at different frequency levels, which are helpful in spectral efficiency and error control transmission. All individual functions of OFDM system such as IDFT/DFT and CP are applied to individual transmit antennas and receiver antennas (MIMO), and then this makes the combination of MIMO-OFDM. Also for error-free transmission it supports Alamouti scheme and with the maximum degree of diversity. MIMO-OFDM sends the stream of independent data information to increase a spatial rate over different antennas [47].

In OFDM the bandwidth is divided into narrow band flat fading channels and data are transmitted on each channel. Thus, we can say that it is the technique which converts frequency selective channels to many flat fading channels and to each of sub channels the MIMO is applied [48].

Lipson wireless first introduced the MIMO-OFDM scheme. In NLOS, it allows transmission and successful communication. It performs communication on NLOS paths, like base station using MIMO-OFDM utilizes multipath scenario. Three techniques are used by MIMO-OFDM to achieve diversity of time, frequency and spatial. Consider MIMO-OFDM system having N transmitter antennas and M receiver antennas as in MIMO technique, spatial multiplexing is applied. Encoding can be performed collectively or per antenna. Individual encoding on each antenna branch of a transmitter system is called per antenna coding (PAC) [49].

In high-speed wireless communication, combining MIMO and OFDM technology, OFDM can be applied to transform frequency-selective into parallel flat MIMO channel to reducing the complexity of the receiver, through multipath fading environment can also achieve high data rate robust transmission. Therefore, MIMO-OFDM systems obtain diversity gain and coding gain by space-time coding, at the same time. The OFDM system can be realized with simple structure. Therefore, MIMO-OFDM system has become
a welcome proposal for 4G mobile communication systems.

6.2 Basic Structure of MIMO-OFDM System

At the transmitting end, a number of transmission antennas are used. An input data bit stream is supplied into space-time coding, then modulated by OFDM and finally fed to antennas for sending out (radiation). At the receiving end, in-coming signals are fed into a signal detector and processed before recovery of the original signal is made. Figure 24 shows the basic structure of a MIMO-OFDM system.

![Basic structure of MIMO-OFDM system](image)

Presently, most of the companies and research institutions developed MIMO OFDM experimental system. Airbust production of Iospan Company that first used MIMO and OFDM technology in the physical layer at the same time for wireless communication systems [50]. In MIMO OFDM system, the frequency response of $k^{th}$ subcarrier can be expressed as follows:

$$H_k^{(q,p)}(n) = \sum_{l=1}^{L-1} h_l^{(q,p)}(n)W_k^{kl}$$  \hspace{1cm} (61)

Where,

$$K = 0, \ldots, K - 1, h_l^{(q,p)}(n)$$  \hspace{1cm} (62)

is the impulse response, that is from $p^{th}$ transmitter antenna to $l^{th}$ channel of $q^{th}$ receiver antenna. $n$ is sequence number of the symbol and $K$ is the total number of subcarriers. Assuming that $W_K = \exp^{-2j\pi n/K}$, while $M$ and $N$ is the total number of transmitter and receiver antennas. The output response for the $q^{th}$ receiver antenna can be written as:
\[ y_k^q(n) = \sum_{p=1}^{M} H_k^{(q,p)}(n) X_k^p(n) + \zeta_k(n) \]  \hspace{1cm} (63)

For \( N \) transmit antenna, there would be \( N \) OFDM transmitter or \( N \) parallel branches of OFDM system for \( N \) antennas. Raw digital bits are multiplexed into to \( N \) branches. For each antenna, there is individual OFDM transmitter performing encoding interleaving, bit mapping (QPSK 16 QAM), IFFT, Guard interval or cycle prefix to each symbol and finally up convert the OFDM symbol to radio frequency then transmit over a radio link [49].

The receiver should have information and estimation about channels for reliable wireless channel transmission. The transmitter for this purpose transmits a training sequence periodically or with every packet to a receiver. So that according to channel’s variations the receiver could update. The training sequence depends upon applications and requirements sent by the transmitter to the receiver. In order to keep track of phase drift and amplitude variation the pilot symbols are used to insert it into OFDM modulator.

At the receiver multiple antennas receive information and on the basis of training sequence it performs estimation and correction in the preamble or performs forward error correction, which depends upon the encoding technique and equalization stages. In finding phase drift the Estimation or forward error correction, frequency offset and symbol timings. Guard band is removed, and information is presented to IFFT. Then per OFDM sub-channel MIMO detection is performed. Received signal of each sub channel is sent to the MIMO detector to retrieve \( N \) signal transmitted on a particular sub carrier. De mapping, de inter leaving and decoding is done per transmitter symbol, a resultant combined value is raw digital data, which was originally sent by transmitter and all these operations are performed over each individual branch of receiver antennas. To the fourth-generation communication system, the MIMO OFDM technology is the door step [49].

With MIMO OFDM, there are some certain limitations such as extra RF cost, antenna sizes and complexities at receivers [50]. For cell mobiles because of mutual coupling and power limitation it is still big issue to design and manufacture multiple antennas on cellular mobiles. MIMO-OFDM is a technique which arises for mobile and wireless broadband access in combinations of OFDM and MIMO. Due to its support to high-speed wireless broadband access, low complexities (in respect of equalization at a receiver) and spectral efficiency and flexibilities, it is considered to be the prominent and promising candidate for further wireless technologies. Examples are LTE,
4G, IEEE 802.16 (WiMAX), and IEEE 802.11n.

6.3 Advantages of MIMO-OFDM

- Less interference.
- Diversity Gain.
- Increase data capacity.
- Power efficiency.
- Bandwidth gain.
Chapter 7

7 PAPR Reduction in MIMO-OFDM

7.1 System Model for Precoding MIMO OFDM

There are three Major parts in our Precoding MIMO OFDM system model.

- Precoder
- Modulator
- Almouthi codes.

As shown in the figure 25 below,

![MIMO OFDM System Model](image)

**Figure 25: MIMO OFDM System Model**

7.2 Precoder

As discussed in the chapter 3, Precoder is an $L \times N$ precoding matrix, denoted $P$, and as defined in equation (12)

$$ P = \begin{pmatrix} p_{0,0} & p_{0,1} & \cdots & p_{0,N-1} \\ p_{1,0} & p_{1,1} & \cdots & p_{1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{L-1,0} & p_{L-1,1} & \cdots & p_{L-1,N-1} \end{pmatrix} $$

(64)
Where \( p_{ij}'s \) are the entries (complex numbers) of this precoding matrix, \( L = N + N_p \) is the total number of subcarriers, and \( N_p \) is the extra subcarriers (overhead) used with \( 0 \leq N_p < N \). When no precoding is used, the matrix \( P \) reduces to an \( N \times N \) identity matrix, and no overhead is used.

7.3 Modulator

We use QPSK modulator and Demodulator for our system. Digital data is transferred in an OFDM link by using a modulation scheme on each subcarrier. A modulation scheme is a mapping of data words to a real (In phase) and imaginary (Quadrature) constellation, also known as an IQ constellation. For example, 256-PSK (Phase Shift Keying) has 256 IQ points in the constellation, constructed in a square with 16 evenly spaced columns in the real axis and 16 rows in the imaginary axis. The number of bits that can be transferred using a single symbol corresponds to \( \log_2 (M) \), where \( M \) is the order of a modulation scheme used. Thus 256-PSK transfers 8 bits per symbol. Each data word is mapped to one unique IQ location in the constellation.

7.4 Almouthi Space Time Block Codes

Almouthi space time block coding is useful for 2x2 MIMO systems. There are two transmit and receive antennas for Almouthi codes. We assume in our whole discussion that transmitter does not have channelled knowledge, but receiver knows about the channel \( H \).

Suppose that we wish to send a message \( m = 0,1,2,3 \) (2 bits) using QPSK modulation. The modulated baseband signal at each antenna at time \( n \) is then given by

\[
s(n) = Ae^{j\frac{\pi}{4}}e^{j\frac{m\pi}{2}}, \quad m = 0; 1; 2; 3
\]

where \( A \) is the transmitted amplitude, and \( A^2 \) the transmitted power. The totally transmitted power by the two antennas is then

\[
P_{\text{tot}} = 2A^2
\]

We consider the following channel model for two consecutive channel uses at time \( n \) and \( n+1 \)

\[
\begin{align*}
y(n) &= Hc(n) + v(n) \\
y(n + 1) &= Hc(n + 1) + v(n + 1)
\end{align*}
\]

where \( c(n) \) are code words, and \( v(n) \) are zero mean uncorrelated additive noise with variance \( \sigma_v^2 \). The signal to noise ratio is defined by
\[ SNR = 10 \log_{10} \left( \frac{P_{\text{tot}}}{\sigma_n^2} \right) = 10 \log_{10} \left( \frac{2A^2}{\sigma_n^2} \right). \] 

(68)

The two code words \( c(n) \) and \( c(n + 1) \) in the Alamouti code are given by
\[ c(n) = \begin{pmatrix} s^{(n)} \\ s^{(n+1)} \end{pmatrix}, \quad c(n + 1) = \begin{pmatrix} -s^{*(n)} \\ s^{*(n+1)} \end{pmatrix}. \] 

(69)

By inserting the code words equation (69) in the signal model equation (67) we get the following signal model in explicit component form
\[ y_1(n) = H_{11}^r s(n) + H_{12}^r s(n + 1) + v_1(n) \]
\[ y_2(n) = H_{21}^r s(n) + H_{22}^r s(n + 1) + v_2(n) \]
\[ y_1(n + 1) = -H_{11}^r s^{*(n+1)} + H_{12}^r s^{*(n)} + v_1(n + 1) \]
\[ y_2(n + 1) = -H_{21}^r s^{*(n+1)} + H_{22}^r s^{*(n)} + v_2(n + 1) \] 

(70)

Now, the Alamouti decoding is given by
\[ \hat{s}(n) = H_{11}^r y_1(n) + H_{12}^r y_1^{*(n+1)} + H_{21}^r y_2(n) + H_{22}^r y_2^{*(n+1)} \]
\[ \hat{s}(n + 1) = H_{11}^r y_1(n) + H_{12}^r y_1^{*(n+1)} + H_{21}^r y_2(n) + H_{22}^r y_2^{*(n+1)} \] 

(71)

By inserting the signal model equation (70) into the decoding equations (70) we obtain finally
\[ \hat{s}(n) = (|H_{11}|^2 + |H_{12}|^2 + |H_{21}|^2 + |H_{22}|^2) s(n) + v'(n) \]
\[ \hat{s}(n + 1) = (|H_{11}|^2 + |H_{12}|^2 + |H_{21}|^2 + |H_{22}|^2) s(n + 1) + v'(n + 1) \] 

(72)

Or
\[ \hat{s}(n) = tr \{ H^H H \} s(n) + v'(n) \]
\[ \hat{s}(n + 1) = tr \{ H^H H \} s(n + 1) + v'(n + 1) \] 

(73)

where \( v'(n) \) and \( v'(n + 1) \) are noise terms which are linear combinations of the original noise terms \( v_1(n), v_2(n), v_1(n + 1) \) and \( v_2(n + 1) \).

Equations (72) and (73) show the action of the Alamouti coder/decoder. Note that the two equations are decoupled, i.e. the estimate \( \hat{s}(n) \) is proportional to \( s(n) \) (except for the noise) and is independent of the other message \( s(n + 1) \), etc. The gain factor for each decoded symbol is \( tr \{ H^H H \} = |H_{11}|^2 + |H_{12}|^2 + |H_{21}|^2 + |H_{22}|^2 \) which displays the spatial diversity obtained.

Eventhough there is strong fading, it is very unlikely that all channel components should fade at the same time.

The channel capacity in this case when the receiver knows the channel but the transmitter [51].
7.5 Simulation and results

Using above parameters and equations, we simulate whole system in Matlab and obtained the following results.

Figure 26: SNR Vs BER of MIMO OFDM System

The above simulation result shows that the precoded MIMO system gives 2dB SNR (signal to noise ratio) gain as compared to conventional MIMO system. We simulate whole MIMO system in a frequency domain at transmitter and receiver. Therefore, we do not have so appreciable difference between conventional MIMO system and precoded MIMP system. But still it has 2dB signal to noise ration advantage as compared to conventional MIMO system. If all the processing done in frequency domain and transmission and reception will be done in time domain then we get hopefully remarkable difference in SNR (signal to noise ratio) and BER (Bit Error Rate) between precoded MIMO system and conventional MIMO system.
8 Conclusion and Future work Suggestions

8.1 Conclusions

This thesis utilizes a newer technique to reduce the PAPR for OFDM transmission. The method is based on signal precoding, where each data block is multiplied by a precoding matrix prior to OFDM modulation and transmission. This method is data-independent and, thus, avoids blockbased optimization. It also works with an arbitrary number of subcarriers and any type of baseband modulation used. The PAPR distribution function of the OFDM transmitted signal was investigated for two suggested precoding schemes. The obtained results showed that precoding can reduce the PAPR of OFDM signals considerably. In terms of BER performance, it has also been shown that the proposed precoding scheme is similar to the previously proposed precoding schemes for OFDM as it takes advantage of the frequency variation of the fading-multipath channel and improves the BER of OFDM signals. It compared to conventional OFDM (no precoding), relative power gains of up to 15 dB for a bit-error probability of $10^{-4}$ can be obtained, depending on the frequency selectivity of the channel. The implementation complexity of the proposed technique is acceptable, since it does not require any optimization from one OFDM block to the next. The technique is also very flexible, and it can be used for OFDM and discrete multitone transmissions. It is proposed in the thesis that the implementation complexity can further be reduced by common subexpression elimination in the OFDM system employing precoding as a way of a PAPR reduction scheme. A precoded OFDM system has been used whereby constant multiplications involved in the precoding are decomposed into shifts and additions and then common subexpression elimination involving multiple variables is employed to reduce the number of operations required. It has been found that with common subexpression elimination almost 35% to 36% fewer adders can be used to implement the same precoding scheme. Which would result in improved performance, less area coverage and reduced power consumption.

8.2 Future Work Suggestions

1. The implementation of the OFDM system employing precoding as a way to reduce PAPR along with the common subexpression elimination to reduce the system complexity on hard ware is intended.

2. The thesis focused on the PAPR reduction of Single input single output OFDM system. Investigating the usage of the precoding scheme on
Multiple input multiple output system in intended.

3. Another possibility to combined pre-FFT smart antennas with pre-coded MIMO OFDM system to analyze bit error rate, signal to noise ratio and channel capacity of the system.
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