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ON THE RELATIONSHIP BETWEEN A NOVICE TEACHER’S MATHEMATICAL KNOWLEDGE AND TEACHING ACTIONS

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The aim of this paper is to examine the relationship between mathematical knowledge for teaching (MKT) and teachers’ actions when teaching mathematics, based on a qualitative case study of a novice mathematics teacher. The empirical data consist of interviews and an observed mathematics lesson during his first year of teaching mathematics. These data have been analysed by coordinating theories on teachers’ goal-oriented actions, action levels and MKT. The results show that at a classroom level, the lesson provides opportunities for using the whole spectrum of MKT. However, in the different situations the teacher mainly draws on common content knowledge (CCK) and aims at maintaining control over the situation.

The main point of departure for this study is a curiosity about what kind of knowledge teachers use when undertaking different actions for teaching in the classroom. Previous research clearly points at the insufficiency of simply knowing the mathematics being taught and emphasizes the need for a specific kind of knowledge for teaching mathematics (Ball & Bass, 2000; Silverman & Thompson, 2008; Shulman, 1986; Wood, Neubrand, Seago, Agudelo-Valderrama, DeBlois & Leikin, 2009). Based on Shulman’s work (1986), Ball, Thames and Phelps (2008) provide a categorisation of this specific mathematical knowledge for teaching (MKT), based on analyses of teachers’ work on different levels. This categorization has been used for evaluations of prospective and in-service teachers and for examinations of the relationship between MKT and students’ achievements (Hill, Rowan & Ball, 2005). Other studies have focused on how this specific kind of mathematical knowledge is developed (Rowland, 2007); on what kind of mathematical knowledge prospective teachers need in general (Hill, Sleep, Lewis & Ball, 2007) and in specific mathematical domains (Potari, Zachariades, Christou & Pitta-Pantazi, 2008); and on novice mathematics teachers’ use of different kinds of MKT (Rowland, Jared & Thwaites, 2011).

In this paper I want to take a narrower approach, scrutinizing the relationship between teachers’ actions and their knowledge. The way actions are related to knowledge is not obvious. A first step in elucidating this is to determine what kind of MKT teachers use for different actions. Novice teachers can make interesting contributions to our knowledge about the transition from teacher education to teaching practice. Novice teachers do not have the same experience and knowledge as their more seasoned colleagues. Still, the novice teachers’ goals and intentions are to help their students learn mathematics. Thus, the analysis of novice teachers can offer valuable insight into what mathematics teachers require in the area of mathematical knowledge for teaching, and what developments need to be put in place for mathematics teachers in general.

NOVICE MATHEMATICS TEACHERS

The main aim of teacher education is to prepare prospective teachers for their forthcoming teaching practice by providing them with the knowledge for teaching that is required in the
profession. The transition from teacher education to teaching in practice is a crucial step and highlights the relationship between pre-service teacher education and the practices of mathematics teachers in school. The character of the mathematical content that is addressed during teacher education seems to be significantly different from the mathematical content they are expected to teach at school.

In an ICMI study, Winsløw and his colleagues elaborate on novice mathematics teachers and their first years of teaching (2009). They recognize that students undergo a transition on an epistemological, institutional and personal level as they go from education to teaching practice. Even though all three levels are important, the epistemological level is particularly interesting for an investigation of mathematical knowledge for teaching, because it describes the shift from a more academic kind of mathematical knowledge to mathematical knowledge for teaching. It seems that a specific kind of mathematical knowledge for teaching is required, and that it differs markedly from academic mathematics. Thus, the epistemological transition from teacher education to teaching practice involves conflicting elements between the needs of schoolchildren to gain meaningful understanding of mathematics and mathematicians’ preferences for formal deductive mathematical reasoning (Moreira & David, 2008). Thus, novice teachers have to create relevant connections between the academic mathematics they have learnt during teacher education and the school mathematics they will have to teach. In other words, novice teachers have to transform their knowledge of academic mathematics into both school mathematics and how it should be successfully taught.

**MATHEMATICAL KNOWLEDGE FOR TEACHING**

Mathematical knowledge for teaching focuses on the relationship between different kinds of knowledge that teachers need in order to teach mathematics at school. Shulman’s work from 1986 can be regarded as a paradigmatic shift in that it stressed the fact that teaching requires more than academic skills and knowledge in the subject to be taught. He introduced the notions of curricular knowledge, content knowledge and pedagogical content knowledge as the three major parts of teachers’ knowledge for teaching.

Shulman’s categorization has been extensively referred to, further elaborated upon and developed. One of the most central and significant works in this direction is made by Ball et al. (2008), who give an account of a practice-based theory that describes mathematical knowledge for teaching, divided into six domains.

![Diagram of mathematical knowledge for teaching](image)

**Figure 1: Domains of mathematical knowledge for Teaching (Ball et al., 2008, p. 403)**
Common content knowledge (CCK) refers to mathematical knowledge that is used in settings other than teaching. Briefly, teachers must know the mathematics to be taught. They must be able to solve tasks, recognize errors and use the correct words and notions. Specialized content knowledge (SCK) concerns mathematical knowledge and skills that are specifically related to teaching. “In looking for patterns in student errors or in sizing up whether a nonstandard approach would work in general, […] teachers have to do a kind of mathematics work that others do not” (Ball, 2008, p. 400). There are many different things that teachers do when teaching that require specialized content knowledge. Describing how a task may be solved may not involve SCK, but if students ask “why questions” in relation to the solution it suddenly does. Teachers probably know that some textbook exercises are more difficult than others. This may be related to CCK, but knowing why it is difficult as well as how to present it to make it easier is certainly a matter of SCK.

The domain of knowledge of content and students (KCS) concerns how mathematical knowledge is related to students’ thinking and understanding of mathematics and its concepts. According to Ball and her colleagues, KCS is knowledge about the most common errors and miscalculations students are likely to make, the mathematical content with which students usually encounter difficulties, common conceptions and misconceptions, and how to facilitate and motivate students’ learning of a specific content. Knowledge of content and teaching (KCT) combines mathematical knowledge with knowledge about various teaching situations. Choosing a suitable sequence of the mathematical content to be treated during a lesson; choosing suitable teaching material, tasks and other tools and aids; judging whether a student’s question or comment should be further elaborated on or saved for a later occasion concern KCT.

Horizon content knowledge seems to be more tentative than the others (Ball et al., 2008). It involves the knowledge of how mathematical topics in the curriculum are related to one another. Knowing what students have done before and what they will encounter in their further studies concerns this kind of knowledge. The last category is knowledge of content and curriculum, which I interpret as an inheritance of Shulman’s original categorization.

This categorization is a result of research on teachers’ practices, which aimed to characterize and describe teachers’ use of mathematical knowledge when teaching. As Ball and her colleagues emphasize, their theory is not about teachers’ knowledge per se but rather their use of knowledge in and for teaching; the issue is mathematical knowledge for teaching, rather than teachers’ knowledge. A consequence of this is that from a methodological perspective, examining teachers’ mathematical knowledge for teaching actually has to be done in relation to a teaching activity or a teaching situation.

I will also draw on another of the authors’ comments in the paper. They state that the categories are derived from teaching practice. By researching teachers’ teaching of mathematics, they were able to identify the categories. However, a crucial issue also involves exploring teachers’ use of different domains of mathematical knowledge for teaching in relation to their teaching actions, or “how different categories of knowledge come into play in the course of teaching, needs to be addressed more effectively in this work” (Ball et al., 2008,
Further examining the relationship between mathematical knowledge for teaching and teaching actions is one attempt to respond to this request.

**TEACHERS’ ACTIONS AND DECISION MAKING**

To teach mathematics is to be constantly involved in, and undertake, actions. These actions are performed in neither isolation nor randomness. Rather, they are accomplished with an overall aim to teach mathematics, and consequently, to provide opportunities for students to learn mathematics. Actions take place in a complex, interactive interplay between teacher and pupils.

Schoenfeld (2011) has developed a theoretical framework aimed at guiding the understanding of teachers’ decisions and actions in the mathematics classroom. This framework builds on the assumptions that teachers are engaged in goal-oriented activities when teaching mathematics. The teacher undertakes observable actions. Because these actions are goal-orientated, goals influence actions. How an individual regards his or her environment and reacts to it are fundamentally shaped by that person’s orientations. On the other hand, teachers perform actions that often have an outcome that differs from their initial intentions. In this case the teacher may reconsider the goals, which in turn will affect his or her orientation. Thus, Schoenfeld claims that there is a mutual interplay between orientations, goals and actions.

Schoenfeld uses orientations as an inclusive term to encompass beliefs, dispositions, values, tastes and preferences. It is most plausible that teachers’ orientations are shaped by previous experiences of learning and teaching mathematics in specific social, cultural and contextual situations, but also from new experiences of the outcome from different actions. From a novice teacher’s perspective, the first years of teaching can be regarded as a point of convergence between his or her own experiences from school mathematics as a student, from teacher education and as an in service-teacher.

According to Schoenfeld (2011), to teach mathematics is to be engaged in goal-oriented activities. He defines a goal as something that an individual wants to achieve. Teachers’ different goals are influenced by their different orientations. In turn, their actions are influenced by their goals. Goals can be conscious or unconscious, and can be short-term or long-term. Teachers may have an overall goal to teach mathematics so that students can learn mathematics, but they can also have the goals of maintaining good relations with students, preparing students for assessment, and providing opportunities for students to have fun and enjoy themselves. For example, in a classroom situation with many students waiting for assistance from the teacher, guidance in small steps can be an effective way to achieve a short-term goal, namely to help as many students as possible. A more thorough explanation, which in the future could lead to the development of a deeper understanding of the mathematical content or problem, might be one way to achieve a long-term goal.

Another part of the framework is resources, the most important of which is knowledge. Schoenfeld defines knowledge as “the information that an individual has potentially available to bring to bear in order to solve problems, achieve goals or perform other such tasks” (2011, p 25). Though Schoenfeld does not make any further explicit statements about the relationship
between actions, goals and orientations on the one hand and resources on the other, my interpretation is that resources are present in actions, goals and orientations. In the description of his theory Schoenfeld gives an account of various characteristics of knowledge, for example facts or isolated pieces of knowledge, procedural and conceptual knowledge and problem-solving strategies. This categorization of mathematical knowledge is common in descriptions of the character of learners’ mathematical knowledge. However, to analyse the relationship between teachers’ knowledge and actions, we rather need a description of the content of the knowledge being used. Thus, if a teacher responds to a student’s question, it is more relevant to know if the answer relates to the teacher’s knowledge about the mathematical content, the student or both, than if the knowledge provided by the teacher is procedural or conceptual in its character. Therefore, we can coordinate Ball et al.’s categories of mathematical knowledge for teaching with Schoenfeld’s theory of goal-oriented actions and MKT as the main resource of knowledge used in teaching actions.

Goals, resources and orientations have an impact on a teacher’s actions, which in turn are consequences of constant decision making. Decision making involves monitoring and self-regulation. It refers to a person’s own awareness of the impact of actions and the ability to adjust them in relation to the goals and resources at hand. For example, if a teacher tries to explain a mathematical issue, monitoring involves noticing how the explanation is received. Self-regulation concerns adjusting the explanation to be understandable to a learner. The role of subjective valuations serves as an explanation for why a teacher acts in a specific way. Continuing to give an explanation to a student who does not understand can be due to a teacher’s subjective valuation. For example, a teacher may regard that specific explanation to be in line with mathematical rigour or correctness and therefore stick to it instead of giving a more understandable explanation that might be regarded as less mathematical.

Thus, Schoenfeld’s theory of goal-oriented decision making provides a comprehensive framework to analyse teachers’ actions when teaching, which involves their mathematical knowledge for teaching. However, actions for teaching can be accomplished on different levels. Concerning a lesson, teachers make decisions and actions that concern the overall arrangement of the whole learning situation (Sensevy, Schubauer-Leoni, Marcier, Ligozat & Perrot, 2005). Teachers perform actions when choosing tasks and activities, and create the overall organization of the teaching and learning situation. In the interaction with students a teacher performs actions that result in a distribution of roles and responsibilities between him- or herself and the students. With an even more fine-grained look, one can also refer to the actual development of students’ knowledge and a teacher’s self-monitoring process when teaching.

METHOD

In this study, I will use Schoenfeld’s (2008) theory of goal-oriented decision making as a starting point for analysing teaching actions, based on the relationship between orientations, goals and actions. While a teacher’s actions can be studied through observation, a teacher’s goals and orientations come to the fore through interpretations of interviews. According to Sensevy et al. (2005) the actions can be sorted into three different levels, i.e. actions for: the
overall organization of the teaching and learning situation; the distribution of roles and responsibilities; and the development of knowledge.

I will use Ball et al.’s categories of mathematical knowledge for teaching (2008). These categories will also be related to the transition and tensions between academic mathematics and school mathematics that were presented previously in the paper, as one aspect of mathematical knowledge for teaching. The approach of using these theories can be described as coordinating these theories “for a networked understanding of an empirical phenomenon or a piece of data” (Prediger, Bikner-Ahsbahs & Arzarello, 2008, p. 172).

To explore mathematics teachers’ actions and the knowledge they use when teaching, empirical data from a mathematics lesson taught by a novice mathematics teacher will be presented and analysed in this paper. Hill et al. (2007) stress the limitations of assessing teachers’ mathematical knowledge by analysing observations of a smaller number of mathematics lessons. However, for an initial study about a specific phenomenon, inductive case studies can be an appropriate way to gain an understanding of the basic phenomenon (Patton, 2002). The lesson was chosen from a number of observed lessons because it was representative of an ordinary lesson. This constitutes a case involving mathematics lessons by novice mathematics teachers, rather than this particular mathematics lesson.

The data presented in this paper constitute a small part of a more comprehensive study about novice mathematics teachers during their first years of teaching mathematics in school. Four teacher students, three female and one male, participated in this study. They were interviewed during their last semester of teacher education before graduation and were then frequently interviewed and observed during their first two years of teaching school. No specific arrangements were made because of the study, which means that the observations can be regarded as authentic. I will now give an account of a mathematics lesson, taught by the male teacher, that took place during his first semester as a newly-qualified teacher. During the lesson the teacher carried a small mp3-recorder with him. His writings on the whiteboard and the students’ tasks and solutions were photographed. The audio recordings were transcribed in full and analysed with regard to the frameworks presented above (Ball et al., 2008; Schoenfeld, 2011; Sensevy et al., 2005) by identifying empirical instances of crucial theoretical concepts in the study (Stadler, 2011).

**RESULTS AND ANALYSIS**

The teacher is in his first year as a newly-qualified teacher in mathematics and English at an upper secondary school in Sweden. As a student teacher he had done very well in mathematics. His self-confidence is strong, concerning both his own mathematical knowledge and his ability to teach mathematics. In an interview just before he graduated, he is asked if he sometimes worries about being able to answer questions from the students.

Teacher: No, because I know so much more mathematics than they do. I can give them answers about anything. They can call me in the middle of the night, and I can give them answers. I can get something five minutes in advance and deliver during the lesson without any problem. *(Interview before graduation)*
During the lesson, the students work with repetition of powers of integers, prime number factorization and reductions and calculations with fractions. The lesson begins with the teacher writing three tasks on the whiteboard and giving the students some time to work on these tasks on their own. After a while, the teacher asks the students to explain their solutions, and then gives them some new tasks. Some of the tasks are shown in Figure 2.

![Figure 2: The tasks on the whiteboard](image)

The teacher gives an account of his approach and the choice of exercises he presented on the whiteboard.

Teacher: I wanted the pupils to check themselves. We’ve worked with all these things before, but I know by experience that they forget it after some time. And also, I wanted to give them a chance before, and then talk about it as a whole class. That was my thought. (Interview after the lesson)

**Analysis:** The arrangement of the classroom setting and the setup of the lesson refer to the overall organization of the teaching and learning situation (Sensevy, 2005). On this level the teacher undertakes actions that concern, for example, the planning of the lesson and the choice of tasks. One specific goal of the lesson from the teacher’s point of view is to repeat the basic arithmetical rules for calculations with fractions so that the students can remember them. The quick and superficial planning of the lesson may be due to routinized actions, whereby the lesson is worked out in a standardized way and the content and its presentation are mainly based on tasks from the textbook.

According to Schoenfeld (2011), resources like knowledge are an essential part of teachers’ actions, goals and orientations. In this case, the teacher has a strong belief that a good teacher has to know a great deal of mathematics by heart and be able to deliver rapid answers. His quick preparation of lessons and his choice of tasks seem to be mainly influenced by his CCK. His responsibility when answering questions is to provide a mathematical solution for the task, while the student dimension is absent. However, in the interview after the lesson the teacher shows elements of KCS when expressing his insight into students’ ways of learning and which problems they may encounter.

The teacher’s planning of the lesson offers one example of the close and mutual relationship between the teacher’s actions, goals and orientations. The teacher has a clear content-related
goal for the lesson. At the same time, the planning and accomplishment of the lesson are done in a routinized and unreflective way, inspired by his previous experiences, values and beliefs.

One task is to find one fraction that is between \( \frac{1}{2} \) and \( \frac{1}{3} \). When the teacher asks the students for a solution he gets an unexpected answer from Student1, a male.

**Student1:** You can multiply the numerator and denominator by two, and then you can take two fifths.

**Teacher:** Can you repeat that, please?!

**Student1:** You multiply two by two and one by two.

**Teacher:** So, you’ll get two quarters? Right?

**Student1:** Right. And then you do the same with one third and get two sixths.

**Teacher:** And two fifths is in between. Yes, that’s one way to do it. That was pretty smart. But what do you think most of the rest of you have done? You’ve done it like this: you change one half to three sixths and one third to two sixths. And then you’ve changed three sixths to two twelfths and two thirds to four twelfths. And between them you can take, for example, five twelfths. But your answer was good. It shows understanding.

**Student2_F:** Does the first method always work?

**Teacher:** Well, the question is rather if you understand it, right? It’s a method that shows understanding. But sure, if you take a denominator that is in between, that’ll work. Actually, I’ve never thought about that method myself.

**Analysis:** As a teaching action, classroom dialogues are an important arena for the distribution of roles and responsibilities between the teacher and the students. During the lesson, the teacher presents a couple of standardized tasks, but gets into an unexpected situation when one student suggests a solution he cannot immediately understand. In this situation the teacher provides a standardized solution to the students, which is mainly based on CCK. There could be several reasons for providing the standardized way to solve the exercise: to provide a simple or comprehensive way to solve this kind of task, or it could be due to his orientation that the teacher should deliver a solution to all exercises. However, the main goal of his actions and statements seems to be to maintain control over the situation and avoid any further distribution of initiative to the student. According to his orientation, he is the most mathematically knowledgeable person in the classroom. His response to the student’s suggestion indicates that he is not prepared to risk getting into a conversation that he does not fully understand.

A closer look at what kind of MKT a teacher should need in order to act in such an unexpected situation shows that there is a need for quick and careful use of different kinds of MKT. Responding to an unexpected mathematical statement from a student not only requires an analysis of the student’s thinking and reasoning, which refers to KCS; there is also an instant need to use SCK to be able to further elaborate on the question. In his answer he needs CCK to be able to elaborate on the solution from a mathematical point of view. To provide an answer that is suitable and understandable with regard to the students and the situation, he also has to draw on KCT. Thus, the situation is complex and it is not surprising that this can
be a hard task for a newly qualified teacher. It is also obvious that it is the composition of the different kinds of MKT that is crucial, not the mathematical knowledge per se. It can be suggested that making quick variations between different kinds of MKT is particularly challenging to a newly qualified mathematics teacher.

The rest of the lesson, approximately 30 minutes, is spent on individual work with textbook exercises. During this time, the teacher gives 11 explanations of tasks to individual students. A majority of the questions concern arithmetic calculations with fractions, for example the rules for the addition and subtraction of fractions as well as the multiplication and division of fractions. These dialogues are characterized by the teacher’s account of the calculation rules. The following dialogue serves as an example of the features of the teacher’s explanations.

Student3: To multiply these (points in his notebook), I can just multiply them by each other? Is it that simple? Because in the other examples I’ve made the denominators equal.

Teacher: Well, you only have to do that when it’s plus and minus. When it’s multiplication and division, there’s no need for that. In this case you can just write three times four divided by four times two, and that’s it. You just have to put them on a common fraction bar, and then you’re done, when it’s multiplication.

After the lesson, the teacher takes the opportunity to comment on students’ mathematical pre-knowledge in more general terms, but the examples seem clearly influenced by the present lesson.

Teacher: When pupils start here, they don’t know things they should know from lower secondary school. Even if they multiplied and divided fractions there, they don’t know how to do it when they arrive here. At least from a general point of view.

Interviewer: What are the reasons or explanations for that?

Teacher: I don’t think they know what they’re doing. They just copy strategies for the tests, but they lack insight into what’s going on or what they’re doing. I don’t think they know what a fraction is. (Interview after the lesson)

Analysis: One important scene of actions for the development of knowledge is individual dialogues between the teacher and the students. During the lesson, the students ask the teacher questions about textbook exercises they cannot manage to solve. Characteristic of the teacher’s response to these questions is to give a quick instruction for how to solve the specific task at hand and sometimes provide a more general explanation of the mathematical content, for example how to add or divide fractions, use the power rules or factorize an expression. The teacher dominates these situations, without specifically considering the student’s solution or exploring what he or she has done. Thus, SCK is absent in the teacher’s interactions with students. The leading indicator for what he talks about with them is guided by the task rather than the student’s solution at hand or the problems the student may have encountered. The teacher’s explanations are guided by CCK, in this case features of fractions themselves, rather than how the students understand (or do not understand) division of fractions (KCS). Also, there are no indications of the teacher consciously varying his way of
explaining things on the basis that the students are different, have different pre-conditions and pre-knowledge, or might have encountered different problems with the same task. However, by answering several similar questions, the teacher develops a local KCS with respect to the current tasks.

This way of explaining may have consequences for the actual development of students’ knowledge. In terms of short-term goals, this can be a way to meet students’ instant need for help with exercises. One could also regard it as an effective way to provide explanations to as many students as possible during a lesson. A clear and standardized explanation that is easy to follow can also be interpreted as an understandable explanation, which makes the students happy. The teacher is confirmed in his role as teacher, and in his orientation about what a good teacher should do. However, the dominant use of CCK and the way of elaborating with KCT as a way of maintaining control of the situation result in the students not being challenged in their ways of thinking and thus remaining at a more surface level of learning. The teacher’s self-monitoring process when teaching is focused on himself rather than on the students. Because the teacher does not attempt to provide an explanation based on the students’ understanding and reasoning, situations requiring SCK do not emerge.

SUMMARY AND CONCLUDING REMARKS

The aim of this paper was to examine the relationship between mathematical knowledge for teaching and teaching actions. Based on empirical data from interviews with a novice mathematics teacher and observation of a mathematics lesson, some connections have been identified between mathematical knowledge for teaching and teaching actions. Characteristic of the episode is the novice teacher’s tendency to draw on CCK when his preferences are in the mathematics itself, rather than the problems and difficulties the students encounter with the mathematical content. On the level of the overall organization of the teaching and learning situation, the teacher draws on CCK when planning the lesson, even though elements of KCS can be found. Also in his interactions with students, he mainly draws on CCK with the aim of maintaining control over the situation. Actions on the level of developing the students’ knowledge and the teacher’s self-monitoring process still focus on CCK, with the teacher providing standardized explanations. His preference for CCK may be regarded as a typical feature of a novice teacher undergoing the transition from teacher education to teaching practice, and the transformation from academic mathematics to school mathematics and mathematical knowledge for teaching. However, the interaction with the students provides opportunities to further develop his KCS that may result in initiatives for situations involving SCK.

In this paper, Ball’s categories of MKT have been used. However, there are other categorizations of mathematical knowledge for teaching, and more thorough consideration of the most suitable categorization could be further scrutinized. In this study, the results indicate a new angle of CCK as a valuable tool to allow the teacher to keep the teaching situation under control. Strategically used, this kind of knowledge can be used to silence students’ questions, make an example of oneself as superior concerning content knowledge, and avoid discussions and reasoning about unfamiliar mathematical territory. This is a suggestion for
supplementary evaluation of different categorizations of mathematical knowledge for
teaching concerning how to use the mathematical content to retain authority in the classroom.

The results also imply the need for further elaboration on the connections and relationships
between actions, goals and orientations on the one hand and the use of mathematical
knowledge for teaching on the other. In the analyses, it was possible to connect actions with
categories of mathematical knowledge for teaching. However, similar actions can be due to
different goals and orientations. There seems to be a need for an additional theoretical or
methodological framework for analysing the intentions and rationale for actions. Also, clear
definitions are needed for how to operationally define an action, and how the specific analysis
of goals and orientations should be accomplished. This relationship can be elaborated on from
two perspectives: what the teacher actually does (which is analysed within these frameworks)
and on a more theoretical level (the relationship between the frameworks themselves).

Studying the relationship between knowledge and actions also initiates a more general
discussion about different theoretical perspectives on the teaching and learning of
mathematics. Some readers may feel that knowledge and actions should be researched from a
socio-cultural perspective, whereby Activity Theory could be regarded as a more
comprehensive framework and analytic tool. However, there are crucial differences between
these research approaches. Whereas Activity Theory has a clearly socio-cultural perspective,
Schoenfeld adopts a more individual and cognitive approach. Therefore, some words that
occur in both frameworks have significantly different meanings. The coordination of the
different theoretical categorizations and frameworks in this paper has resulted in a more
individual-cognitive approach to the relationship between mathematical knowledge for
teaching and actions for teaching. I argue that an examination of the relationship between
actions and knowledge could benefit from a more local and focused analysis that does not
take into consideration the whole social perspective. Therefore, it is my intention to further
elaborate on mathematical knowledge for teaching and teaching actions with the approach
initiated in this paper.

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