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Proposal for a Failure Surface for Orthotropic Composite Materials

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The paper presents a novel failure surface, designed for orthotropic composite materials such as wood and wood-based products but also fibre reinforced plastics. These materials show a distinctive difference of mechanical properties such as stiffness and strength in the principal material directions.

The failure surface description of Tsai and Wu [4] has been applied regularly for describing failure of wood, usually in the form $\Phi(\sigma) = a_{ij}\,\sigma_{ij} + b_{ijkl}\,\sigma_{ij}\,\sigma_{kl} - 1 = 0$ with the coefficients $a_{ij} = 0$ for $i = j$, the symmetries $b_{iijj} = b_{jjii}$ and $b_{ijij} = b_{jiji}$ and neglecting interaction between normal stresses and shear stresses so that twelve independent parameters remain (e.g. [2]). A further reduction to nine independent parameters is obtained by setting the interaction terms $b_{iijj} = 0$ for $i \neq j$ (e.g. [1]). Strength values can be chosen differently in tension and compression in the respective material direction. It is usually deployed as a closed, single surface in the stress-space which is relatively simple to implement in numerical models.

A multi-surface failure description (e.g. [3]) is often chosen on grounds of better approximation to experimental data and the separation of failure modes of brittle (in shear and tension) and plastic (compression) characteristics, which is especially helpful when allowing for hardening/softening models. The drawback is a more complicated implementation in to models compared to single-surface models.

The proposed model intends to combine the simplicity of a closed single-surface model and the better fit of a multi-surface model. It is based on a generalized superellipse in the six-dimensional stress-space:

$$\Phi(\sigma) = \left|\frac{\sigma_{LL} - \sigma_{LL}^0}{a_{LL}}\right|^{m_{LL}} + \left|\frac{\sigma_{RR} - \sigma_{RR}^0}{a_{RR}}\right|^{m_{RR}} + \left|\frac{\sigma_{TT} - \sigma_{TT}^0}{a_{TT}}\right|^{m_{TT}} + \left|\frac{\sigma_{RT}}{a_{RT}}\right|^{m_{RT}} + \left|\frac{\sigma_{TL}}{a_{TL}}\right|^{m_{TL}} + \left|\frac{\sigma_{LR}}{a_{LR}}\right|^{m_{LR}} - 1 = 0$$

The parameters a_{ij}, σ_{ij}^0 and m_{ij} can be interpreted as the offset from the point of origin and the length of the semi-axis in the respective material axis as well as the “roundness” of the corner regions, respectively. They may be obtained by curve-fitting of experimental data or using uni-axial strength data and solving equation consecutively. Due to the mathematical form, the failure surface is always closed.
A comparison of different failure models for interaction of stresses parallel and perpendicular

to the grain is provided in Figure 1. It is shown that the proposed failure surface is able to provide a much more "edgy" shape, reaching the corner regions of the stress-interactions better than a single-surface, therefore coming close to what is found in the multi-surface proposal.

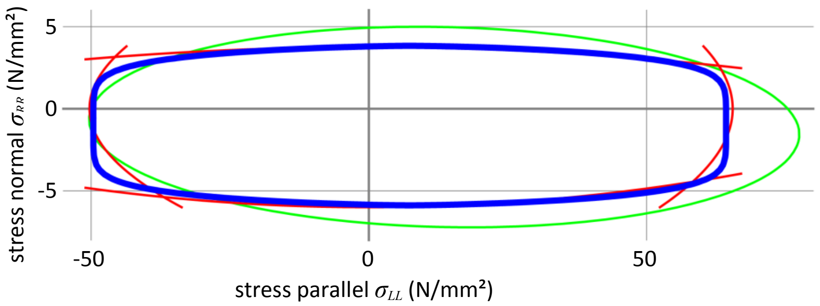

Figure 1: Comparison of the proposed failure surface (blue) to failure surfaces by Eberhardsteiner [2] (green), and Schmidt and Kaliske [3] (red).

The Tsai-Wu failure surface with terms $b_{iijj} = 0$ for $i \neq j$, as used in e.g. [1], is a particular case of the proposed failure model and regained by setting all powers $m_{ij} = 2$. Further cases of interest for other applications may be obtained by setting m_{ij} close but larger than 1 so that the surface resembles a rhomboid (in the case of $m_{ij} \leq 1$, points on the semi-axis become singularities) or by setting m_{ij} large so that a criterion similar to a maximum normal stress criterion is achieved.

The failure surface has been implemented into the Finite-Element software Abaqus by means of a user-defined, elasto-plastic material model for wood. Simulations for applications with primarily compressive (plastic) failure modes show good agreement with experimental data.

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