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ADAPTIVE CONCEPTUAL FRAMEWORKS FOR PROFESSIONAL DEVELOPMENT

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In this paper, I present the notion of adaptive conceptual frameworks that I have used to conduct design-based research with the aim of developing ICT supported mathematics instruction. In this approach, empirical data is connected with various theories in an adaptive and iterative process. I differentiate between Conceptual Framework for Development (CFD) and Conceptual Framework for Understanding (CFU) depending on how the frameworks are used in the design process. Using adaptive conceptual frameworks contribute to the transparency in the design process by making explicit the levels at which different theories operate.

INTRODUCTION

During the last decades, several similar methodologies have emerged that address the desire to conduct educational research with relevance for school practices. For example, design-based research aims explicitly at developing theories that could do “real work” by providing theoretically underpinned guidance on how to create educational improvement in authentic settings (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; McKenney & Reeves, 2012). A common feature of these approaches is the design of teaching activities in an iterative design process that shares many similarities with teachers’ daily work.

This paper contributes to research by describing how the design process may be co-determined by the interaction between different stakeholders such as researchers, teachers, disciplinary knowledge, theoretical frameworks, and other resources. At the core is the development of adaptive conceptual frameworks that were used to guide and justify an intervention in a lower secondary school with the overall aim of developing ICT supported mathematics instruction. These efforts have been inspired by co-design, as a design methodology that highlights the importance of involving different stakeholders such as teachers in the design research process in order to address the issue of ownership of innovation (Penuel, Roschelle, & Shechtman, 2007). Furthermore, working in close collaboration with teachers deepens our knowledge about pragmatic issues and promotes development of “innovations that fit into real classroom contexts” (ibid. p.52). Following the conceptualization of knowledge proposed by Chevallard (2007) in the Anthropological Theory of the Didactic (ATD), the two different perspectives of understanding and development could be viewed as two inseparable aspects of knowledge, integrating a practice that includes the things teachers do to solve different educational tasks (Praxis) with a
discursive environment that is used to describe, explain, and justify that practice (Logos). The adaptive conceptual frameworks explicitly address both perspectives.

The case study presented in this paper involves two lower secondary mathematics teachers. Empirical data is only used to motivate the development of the adaptive conceptual frameworks. Thus, a full analysis of the empirical data with respect to the intended learning objectives is not provided. The purpose of this paper is to describe how the use of adaptive conceptual frameworks has contributed to meet the emerging needs in a design process of ICT supported mathematics instruction during one design cycle.

**ADAPTIVE CONCEPTUAL FRAMEWORKS**

In this approach the researcher connects empirical data with various existing theories that are chosen in retrospect and that are used to generate additional empirical data in an iterative, incremental and adaptive process. Thus, theory is not applied onto practice, it is more about a “progressive interaction between theory and practice, by means of appropriating existing theoretical tools” (Bartolini Bussi, 1994, p. 127). Furthermore, the adaptive conceptual frameworks are considered in a state of flux and changeable according to the different challenges that might emerge when conducting design-based research. Thus, the adaptive conceptual frameworks should be regarded as tentative and a result of a research work that has similarities with research that sometimes is portrayed by the “bricolage” metaphor (Kincheloe, 2001), particularly regarding the efforts of embracing methodological flexibility and plurality of theories. From this perspective, this research approach aligns with the Singerian inquiry system (Churchman, 1971; Lester, 2005).

The workflow of the formal stages of a design cycle is illustrated in Figure 1. Each design cycle starts with a planning phase, followed by an implementation phase involving the teachers. The cycle is completed with an evaluation of outcomes. Three different frameworks are distinguished depending on their role in the different phases:

- methodological framework for professional development (MFPD),
- conceptual framework for development (CFD),
- conceptual framework for understanding (CFU).

![Figure 1: The adaptive frameworks for research and professional development](image)

The researcher uses the methodological framework for professional development (MFPD) to plan the interventions involving the teachers and to operationalize his
current understandings before engaging in a new design cycle. The conceptual framework for development (CFD) is used to describe and justify the different activities that the researcher engages in together with the teachers. Finally, the conceptual framework for understanding (CFU) is used to understand the outcomes of an intervention and to plan the next design cycle. While the CFD and CFU naturally share similarities, since they both put focus on the design process, the MFPD should be regarded as a separate framework for organizing and supporting the teachers’ professional development.

The different frameworks consist of multiple components, which need to be considered carefully how they interact. For this purpose, the categorization presented by Prediger, Bikner-Ahsbahs, and Arzarello (2008) of different levels of connected theoretical approaches was used. In their landscape of different levels of integration, the authors present a scale ranging from one extreme of ignoring other theories to the other extreme of unifying theories globally. Those strategies that are intermediate are called networking strategies. Networking strategies include strategies such as comparing, combining, coordinating and integrating locally. According to Prediger et al. (2008) the strategies of coordinating and combining are mostly used for a networked understanding of an empirical phenomenon or a piece of data and are typical for conceptual frameworks that, as in our case, not necessarily aim for a coherent theory. While comparing and contrasting always are possible the strategies of coordinating and combining can be a more difficult task especially if the theories are not compatible relative a specific purpose. The coordinating strategy is in turn used when a conceptual framework is built on well-fitting theoretical elements (ibid.). The networking strategies used in this study were comparing and coordinating.

THE BACKGROUND OF THE CASE STUDY

The participating teachers were involved in a developmental project in their school on how ICT could enhance their students’ learning of mathematics. The teachers participated in a one-day event with lectures and hands-on learning activities developed by researchers from media technology and mathematics education. One specific learning activity was designed to stimulate students to communicate, collaborate and generate general problem solving strategies (Sollervall & Milrad, 2012). Mobile phones were used in this activity to bridge between formal and informal learning spaces. During the discussions about the activity the teachers seemed to be more worried about the practical issues rather than the didactical issues. This made the researcher aware of a possible misunderstanding. My concern was that connecting between the students’ actions outdoors and a mathematical content is not necessarily a straightforward task. A successful orchestration would depend on the quality of the student-generated artifacts as well as the teachers’ ability to orchestrate this remaining part of the activity performed indoors.
Later on, two of the mathematics teachers from the school and I met to discuss the prospects of developing new activities supported by ICT. The teachers expressed their concerns about their students’ inability to use the distributive law and we all agreed on that it would be interesting to focus on algebra. The teachers had themselves completed the above-mentioned activity, which also could be used to address students’ conception of the distributive law by connecting multiple representations (ibid.). Using the activity with this particular focus towards the distributive law would not require any modifications of the activity itself but would require the teacher to orchestrate the activity towards this goal. None of the teachers seemed to perceive this opportunity and the continued discussions revealed that they did not know about possible geometrical representations of the distributive law.

These circumstances influenced the design process in a very straightforward manner. For the planning phase of the design, the researcher decided to address the teachers’ ability to adapt ICT to different situations and towards different goals. At the moment, perhaps this was more important than developing new activities with the teachers. With this pre-understanding the planning phase of the design was initiated.

**METHODODOLOGICAL FRAMEWORK FOR PROFESSIONAL DEVELOPMENT**

The methodology of collaborative design based research is at the same time a process of professional development for the teachers (Penuel et al., 2007) and any change in teachers’ knowledge base, attitudes and beliefs that this process may require should be regarded as a gradual and difficult (Guskey, 2002). The teachers’ insufficient understanding of mathematical representations was taken as a constraining factor for the teachers’ participation in the design process. To address this issue, two complementary theories were used to guide and plan for the teachers’ professional development. One of the frameworks specifically focuses on knowledge for teaching mathematics: Mathematical knowledge for Teaching (Loewenberg Ball, Thames, & Phelps, 2008) and the other framework focus on the affordances provided by ICT and on the integration of ICT in different subject areas: Technological Pedagogical Content Knowledge (Koehler & Mischra, 2008).

The strategy of *comparing* (Prediger et al., 2008) was used to identify common principles in these two theories related to the use of ICT to support students learning mathematics. Based on this comparison, the researcher decided to specifically recognize and support teachers’ understanding of the affordances for representation and communication provided by ICT.

**CONCEPTUAL FRAMEWORK FOR DEVELOPMENT**

The idea was to use the dynamic geometry software GeoGebra ([www.geogebra.org](http://www.geogebra.org)) to develop an application, with focus put on providing affordances for representation, that the teachers could use in a learning activity to address their students’ conception
of the distributive law. The researcher was interested in understanding how the teachers would perceive and make use of the specific affordances for representation and communication provided by the application in “live” settings. Thus, the software was an instrument for the researcher to provide competence development as well a didactical tool for the teachers to use with their students. The teachers were not familiar the software so the application was designed for them as end-users to operate only by using “click and drag” features.

Inspired by the work of Duval (2006), the dynamics of GeoGebra is used to illustrate how numerical expression can be interpreted and represented geometrically. Although figures and expressions are organized in a determined order in the application (see Fig. 2), the teacher still needs to consider how to use the application and create a hypothetical learning trajectory (HLT), i.e. “the consideration of the learning goal, the learning activities, and the thinking and learning in which students might engage“ (Simon, 1995, p. 133). In other words, the researcher made the didactical design but the pedagogical design was intended for the teachers to decide.

![Figure 2: Snapshot of the application, implemented in GeoGebra](image)

When the application was presented to the teachers they wanted immediate access to it. They seemed to recognize the limitations of the explanations that they normally used that were exclusively based on instructions on how to manipulate different variables. The teachers were provided with the application and they agreed on using it but they never did. Therefore, there was an additional meeting, where the researcher demonstrated a possible way to use the application in a learning activity. The demonstration was followed by a discussion about possible ways to orchestrate the interplay between different representations and the dynamical affordances (dragging mode, show/hide figures) supported by the application. By discussing related pedagogical issues and offering the teachers opportunities to adapt the application according to their needs, the researcher wanted to challenge the teachers to create their own hypothetical learning trajectory (HLT).
The CFD was developed by using the networking strategy of coordinating (Prediger et al., 2008) theoretical components (i.e. representation, GeoGebra and HLT) for practical reasons without aiming for a deeper integration. In contrast to the other components, the notion of hypothetical learning trajectory (HLT) was not presented explicitly to the teachers. In the next section we continue by presenting the crosscutting features of the enacted lessons.

**Teachers orchestrating the application in a learning activity**

The teachers used different interpretations of multiplication simultaneously and alternately without making explicit why and when an interpretation was preferable in some situations and not in others. This lack of explicitness resulted in vague connections between the numerical and geometrical representations. Justifications were based on computations or algebraic manipulations instead of referring to the available geometrical representations in the application. When the teachers became uncertain on how to proceed with the activity they tended to rely more on the numerical and algebraic representations to maintain the flow of the lesson. A significant part of the lessons was also dedicated to what seemed to be other more familiar activities such as formulating expressions for area and perimeter.

Furthermore, the teacher-initiated communication with the students did not seem to support a discussion on how and why things work the way they do. Occasional misinterpretations of students’ responses, not acknowledging their responses as correct, and not connecting their responses to the available representations, further contributed to the activity not proceeding as intended.

**CONCEPTUAL FRAMEWORK FOR UNDERSTANDING**

The enacted lessons were also different compared to the suggestions the teachers had themselves when discussing different ways to orchestrate a lesson supported by the application (when discussing the HLT). During the first two phases of this design cycle, the focus was on teacher knowledge but the crosscutting features of the lessons revealed another dimension. How does teacher knowledge come into play in the moment of teaching? In order to understand why the teachers did not make use of the ICT-supported affordances for connecting representations, the researcher decided to go beyond the theories of representation and teacher knowledge used previously. In other words, a different representation was chosen to address this emerging challenge and to evaluate the design process so far.

**Developing the CFU**

The Anthropological Theory of the Didactic (ATD) provides a different conceptualization of knowledge. In this theory a body of knowledge (a praxeology) consist of two inseparable blocks, the *praxis* and the *logos*. The *praxis* block refers to the kind of given *tasks* that you aim to study and the different *techniques* used to face
these problematic tasks. In this sense the praxis block represents the “know-how” of the praxeology and is the minimal unit of human activity. The logos block provides a discourse that is structured in two levels with the purpose to justify the praxis. The first level of the logos is technology, which provides a discourse about the technique. The second level of the logos is theory, which provides a more general discourse that serves as explanation and justification of the technology itself (Chevallard, 2007) by providing a framework of notions, properties and relations to organize and generate technologies, techniques and problems (Barbé, Bosch, Espinoza, & Gascón, 2005).

The ATD includes the study of didactic transpositions processes, which concerns the transformation of knowledge through different institutions. The transposition is a process of de-constructing knowledge and rebuilding different elements of knowledge into a more or less integrated whole with the aim of establishing it as “teachable knowledge” while trying to keep its character and function (Bosch & Gascón, 2006). It consists of the four following steps; scholarly knowledge, knowledge to be taught, taught knowledge and learned knowledge. The different steps provided a new way to describe the intervention. In this case, the focus of the intervention was on the connection between intended and enacted knowledge, that is, between the second and third step of the transposition of knowledge (see Fig. 3).

Figure 3: The transposition of knowledge

Furthermore, teaching is a didactic type of task that teachers can solve in a complex process of didactical transposition by using a set of available resources (didactical techniques), both external resources (curriculum, textbooks, tests, ICT-tools, colleagues, manipulatives, etc.) and teachers’ internal resources that in our case of ICT-supported instruction could be related to technological-pedagogical content knowledge (Koehler & Mischra, 2008). The logos block of a didactical praxeology then serves as means to describe and justify teaching and learning practices in the considered institution (Rodríguez, Bosch, & Gascón, 2008).

The notion of HLT was replaced by the notion of routines (Berliner, 2001) with focus on the IRE sequence (Initiate, Response, Evaluate). The IRE sequence is a three-part pattern where the teachers ask a question, students reply, and teachers evaluate the response or gives feedback (Mehan, 1979; Schoenfeld, 2010). In its most basic form the teacher initiates the sequence by posing a question to a student to which the teacher already knows the answer. The student then replies and the teacher evaluates by using phrases such as “yes” or “that’s fine” and continues with the next question or next problem. This adaptation was made in order to better describe the teachers’ overt orchestration of the lessons and especially the communication patterns between teachers and students. Furthermore, communicational exchange patterns, such as the
IRE sequence, can be regarded a didactical technique that teachers use in the creation of a mathematical praxeology. This theoretical component was further developed in a second design cycle into a didactical resource (Perez, 2014).

Moreover, representations were placed within the notion of praxeologies instead of being treated as a separate theoretical component as in the CFD. The role of representation is multifaceted. From one perspective it is a generic property of many ICT tools (Koehler & Mischra, 2008). From a second perspective, mathematical representations have important didactical affordances (Ainsworth, 1999), and finally representations are essential to mathematics as a discipline (Duval, 2006). Thus, mathematical representation is closely related both to praxis and logos of a mathematical praxeology. Furthermore, instructional strategies that systematically focus on knowledge about representations could be conceptualized as an element of a didactical technique and consequently a part of a didactical praxeology. Thus, depending on the purpose in which representations are used, the role of representation for a discipline as mathematics could be attributed to both a mathematical and a didactical praxeology. These adaptations allowed the researcher to provide a more comprehensive description of the crosscutting features in the enacted lessons and to evaluate the efforts of providing competence development.

In summary, the conceptual framework for understanding (CFU) consists of several theoretical components where the ATD is used as the dominant theory. The purpose of the conceptual framework for understanding (CFU) was to better understand an emerging empirical phenomenon (the crosscutting features). The CFU was developed by the researcher by using the strategy of coordinating different theoretical components (Prediger et al., 2008). To achieve this, the theoretical components of representations and routines (the IRE sequence) were interpreted as knowledge resources in accordance with the ATD and its focus on the epistemic dimension of teaching and learning processes in different institutions.

**Evaluating the design process**

The theoretical notions provided by the CFU allowed the researcher to capture the essence of this part of the design process. In summary, the intention to introduce the geometrical representation as a technological element in a mathematical praxeology was instead treated by the teachers as a didactical technique to allow the students to work with more open-ended tasks. Thus, the affordances of the embedded geometrical representations as a technological element were not used as intended. Furthermore, the communicational patterns (IRE sequence) used by the teachers did in many cases not support the creation of a mathematical praxeology including a well-developed logos discourse. In summary, the underlying principle-based learning objectives did not survive the transposition from how the researcher intended the application to be used and how it was actually used by the teachers. The transposition of knowledge between “knowledge to be taught” and “taught knowledge” proved to
be of greater difficulty requiring more scaffolding than the researcher had anticipated and planed for. With this understanding, a new design cycle could be initiated.

SUMMARY

In this case study the possibility of viewing a design process as incremental and adaptive has been considered. This should not be interpreted as a matter of searching for whatever works in the current situation. Instead, it is about the problematic task of assuring that the activity of inquiry is meaningful relative to the research objectives, i.e. the problem of developing systems guarantors (Churchman, 1971). This is a basic problem for any researcher but in this case, the problem of guarantors were not settled a priory and once and for all. By questioning the assumptions of the inquiry system, the design problem of knowing when and how to revise becomes difficult because there is no a priory authority to rely on. Instead, the question of why revise would depend on the measure of the performance of the system relative to the purpose (Churchman, 1971). Furthermore, in order to make tactical decisions that require an authority, the researcher must be prepared to consider a “whole breadth of inquiry in its attempt to authorize and control its procedures” (ibid. p. 196). In this case study, the choice to change theoretical perspective during the design process was considered necessary in order to adapt to an unforeseen and, from the researchers perspective, problematic situation. Thus, the question of why revise was motivated by an emerging phenomenon that questioned the performance of the design process. This resulted in a more comprehensive conceptual framework for understanding (CFU) where the Anthropological Theory of the Didactic served as an overarching theoretical perspective. The development of adaptive conceptual frameworks could be understood as a modeling process that aims at developing system guarantors. But as any model it only provides different affordances and constraints that may be used with varying levels of success to justify the choices we make and explain different phenomena that we seek to understand. The use of adaptive conceptual frameworks specifically affords transparency in the design by making explicit the levels at which different theories operate and the measures that are used to evaluate the system performance.

Finally, allowing the design process to be co-determined by the interaction between different stakeholders and resources is far from an straightforward task, but I believe that it allows the researcher to make use of available resources to address authentic educational needs as expressed by practicing teachers.

REFERENCES


