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This is the published version of a paper published in *Journal of Early Childhood Education Research*.

Citation for the original published paper (version of record):

Palmér, H., Ebbelind, A., Patron, E., Wernholm, M., Danielsson, K. (2026)
How young students' creation of analog and digital representations, in combination,
may contribute to their learning of combinatorics
Journal of Early Childhood Education Research, 15(1): 95-116
<https://doi.org/10.58955/jecer.160806>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

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<http://urn.kb.se/resolve?urn=urn:nbn:se:lnu:diva-145283>



How young students' creation of analog and digital representations, in combination, may contribute to their learning of combinatorics

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ABSTRACT: In this study, we elaborate on whether – and if so, how – students' creation of analog and digital representations, in combination, may strengthen their meaning making in combinatorics while engaging in problem solving. An intervention was conducted in two Swedish preschool classes (six-year-olds) involving 25 students. The results indicate that creating a digital animation did not primarily enhance problem solving but rather served as an aid for students to redesign their analog knowledge representations, enabling them to further engage with the mathematical content. A conclusion is that the creation of analog and digital representations, in combination, contributed to students' explorations of combinatorics and that, when integrated into a learning-design sequence, digital animations enhanced students' understanding of combinatorics. This conclusion should be seen in relation to the small scale of the study, which means that more studies are needed in order to generalize the findings.

Keywords: representation, digital animation, early mathematics, combinatorics, early childhood education

Introduction

This study is part of a research project focusing on how the use of digital tools in education may strengthen young students' learning in different content areas. The study was conducted in a Swedish preschool class, which implies six-year-old students. Studying the role of digital tools in early education is of importance since these tools are already embedded in educational systems both nationally and internationally (OECD, 2023). However, more knowledge is needed on how digital tools can be pedagogically integrated to effectively support learning in classrooms and how education can support learners in using digital tools creatively and effectively (Agélii Genlott, 2020; Voogt & Knezek, 2018). This article focuses on how students' creation of analog and digital representations, in combination, may contribute to their learning of mathematical content when working on problem solving. A problem-solving task is a non-routine task that requires learners to select and apply new strategies in order to reach a solution, often involving reasoning, justification, and exploration of multiple pathways (Gavaz et al., 2021). Examples of analog representations include written symbols, pen-and-paper drawings, or physical manipulatives, while digital representations are technology-based formats, for example, animations and simulations (Goldin, 2020; Vágová, 2021).

Despite its central importance in mathematical education, there are few studies on problem solving in early mathematics (e.g., Suseelan et al., 2022; Vessonen et al., 2024). Further, in research, analog and digital representations are seldom considered simultaneously in learning processes (Arzarello & Robutti, 2010; Otterborn et al., 2024; Pedersen et al., 2021). Despite the limited number of studies, there are successful studies on problem solving in early mathematics (see, for example, Palmér & van Bommel, 2018, 2023a; Vessonen et al., 2024). There are also studies indicating that, when both analog and digital representations are used and created, they can add value to each other provided that the task design and instructional approach encourage students to compare, translate between, and reflect upon the representations (Maschietto & Soury-Lavergne, 2013; Pedersen et al., 2021). Thus, working on problem solving may enhance young students' learning of mathematics, where the use and/or creation of digital representations may add value to the use and/or creation of analog representations and vice versa. However, it is not self-evident that using and/or creating several representations while working on problem solving will reinforce young students' learning processes. To reinforce young students' learning processes, the combination of the representations must be carefully designed (Goldin, 2020).

In the study presented in this article, the students, while working on a mathematical problem-solving task, used and created several different representations such as pictures,

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physical objects, and digital animations. The problem-solving task was on combinatorics, that is, on combinations, permutations, and ways of organizing objects. The students were to consider how many different ways three different-colored toy bears could be arranged in a row on a sofa. To make the combinatorial task meaningful for the young students, it was presented as a conflict between the toy bears, where the bears could not agree on who should sit where on the sofa. One toy bear then suggested that they could change places every day. The task for the students was to find out on how many consecutive days the bears could sit in different ways on the sofa. Thus, it was an enumerative combinatorial task where students were supposed to count the permutations for $n = 3$. This task has previously been thoroughly studied by Palmér and van Bommel (see, for example, 2018, 2023b). However, as will be explained later, the study presented here differs from previous studies in terms of theoretical and methodological aspects as well as the inclusion of creation of digital animations. The question to be explored is whether – and if so, how – students’ creation of analog and digital representations, in combination, may contribute to their learning of combinatorics when working on problem solving.

Literature review

In this section, we will first present research on problem solving in early mathematics. After that follows a subsection on representations in mathematics, and finally, a subsection on combinatorics in early mathematics.

Problem solving and problem posing

As mentioned, a problem-solving task is a non-routine task that requires students to select and apply new strategies in order to reach a solution (Gavaz et al., 2021). Thus, the individual who solves the task must develop strategies and/or obtain new knowledge to be able to solve it. The purpose of problem solving in mathematics education is twofold: to strengthen students’ ability to learn mathematical content and to strengthen their ability to solve problems (Gavaz et al., 2021; English & Sriraman, 2010).

Problem solving promotes mathematical reasoning, flexibility in the use of representations and strategies, and conceptual understanding (Gavaz, et al., 2021; Vessonen, et al., 2024; Palmér & van Bommel, 2023b) as well as the development of a view on mathematics that reinforces learning (Ebbelind et al., 2024). It is unsurprising, then, that researchers have long emphasized the importance of young students becoming familiar and comfortable with problem solving (see, for example, English, 2004; English & Sriraman, 2010). By working on problem solving, students can explore different

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mathematical methods and recognize that multiple solution pathways can exist. Such experiences can not only develop methodological flexibility but also reinforce that mathematics is exploratory and creative. By encouraging students to justify their solutions and consider alternative strategies, problem-solving tasks connect procedural skills to broader conceptual knowledge (Vessonen et al., 2024). In 2004, English found that young students often engage spontaneously in problem-solving activities outside school, and that these informal experiences can be built upon in problem-solving activities in early education. New studies support this as well as the finding that the learning of mathematical concepts is strengthened by problem solving. Thus, early education offering problem-solving opportunities lays a foundation for later mathematical proficiency (Gavaz et al., 2021; Suseelan, et al., 2022).

Representations in mathematics

In mathematics education, a representation is typically a sign or a configuration of signs, characters, or objects. The important thing is that it can stand for (symbolize, depict, encode, or represent) something other than itself (Goldin & Shteingold, 2001, p. 3). In research on mathematical representations (hereafter, representations), there is a consensus that learning mathematics involves and is strengthened by the ability to interpret and make connections within and between representations (see Duval, 2006, 2017; Goldin, 2020; Lesh et al., 1987). Lesh (1981) and Lesh et al. (1987) emphasize five types of representations: real-world situations, pictures, verbal symbols, written symbols, and manipulatives. These representations may be perceptually similar but mathematically different (for example, two blocks versus five blocks) or perceptually different but mathematically similar (for example, five blocks versus the symbol 5). Further, Heddens (1986) emphasizes that representations have four levels of abstraction: concrete (real items), semi-concrete (pictures of real items – often called pictographic representations), semi-abstract (symbolic representations of the real items – often called iconic representations), and abstract (symbols). Becoming able to use different representations, including knowing when and how it is possible to make connections within and between them, is important for learning mathematics (Duval, 2006, 2017; Goldin, 2020; Lesh et al., 1987). However, children do not automatically recognize such connections; rather, they must see them demonstrated (Goldin, 2020; Björklund & Palmér, 2020, 2022). Thus, it is important that education makes it possible for students to experience and connect different representations. However, commonly, the representation most valued by teachers is documentation with paper and pen, while representations demonstrated with other semiotic modes (that is, a system of resources with specific affordances and constraints for meaning making), including animations, may be less valued (Arifin et al., 2025; Selander & Kress, 2010). However, Selander (2024)

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emphasizes that multimodality is essential in learning, as each semiotic mode offers unique potentials for meaning making. Therefore, several modalities should be considered and valued according to their affordances within the specific learning situation.

Research shows that the integration of digital representations can enhance mathematical understanding by enabling students to connect visual, symbolic, and verbal modalities (see Ainsworth et al., 2002; Pedersen et al., 2021). According to Arzarello and Robutti (2010), digital animations are of particular importance as they make it possible to explore temporal and transformational aspects of mathematical processes that static images cannot capture. More recent studies have further emphasized this dynamic potential of digital representations. For instance, Kynigos and Karavakou (2023) demonstrate how learners, through the creation and manipulation of digital figural animations, can engage with the evolving nature of mathematical objects, thereby developing deeper conceptual understanding. Similarly, Baccaglini-Frank et al. (2025) argue that digital artifacts in mathematics education enable students to perceive and reason about change and process in ways that transcend static representations. Furthermore, digital tools, including animated representations, contribute to the cultivation of mathematical digital competencies, that is, the ability to flexibly use digital resources for mathematical inquiry and communication (Geraniou & Jankvist, 2019).

Besides enhancing mathematical understanding, digital tools have been shown to positively influence motivation. Fadda et al. (2022) conducted a meta-analysis on the effects of digital games on K–12 students' motivation in mathematics and found a significant positive impact compared to traditional teaching. A newer example is Zourmpakis et al. (2024), who examined adaptive gamification in STEM education and found that digitally supported adaptive approaches led to higher motivation and engagement than traditional inquiry-based learning. Similarly, Saccardo et al. (2024) studied emotions and digital tools in primary school mathematics, showing that digital tools improved performance and engagement. Overall, these findings suggest that digital tools can positively influence motivation in early mathematics.

Young children and combinatorics

English (1991, 1996, 2005) has shown that a meaningful context makes it possible for young children to work effectively to find permutations in combinatorial situations. To be able to solve combinatorial tasks successfully, systematic variation, constancy, exhaustion, and completion are the four principles students need to understand (English, 1996). The principle of systematic variation implies that a new permutation will occur if

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at least one item is varied systematically. The principle of constancy implies that a different permutation will occur if at least one item is kept constant while at least one other is varied systematically. The principle of exhaustion implies that a constant item is exhausted when it no longer generates new permutations when the other items are varied. The principle of completion means that when all constant items have been exhausted, all possible permutations have been found.

In a study from 1991, English identified six strategies used by young children when working with combinatorial tasks: 1) trial-and-error selection of objects (with duplicates); 2) trial-and-error selection of objects (with rejection of duplicates); 3) emerging pattern for the choice of objects (with rejection of duplicates); 4) consistent and complete patterned cyclical item selection (with rejection of duplicates); 5) emerging odometer pattern in item selection with possible item rejection; and finally 6) complete odometer pattern. These strategies are hierarchical in that the latter are more effective when it comes to finding all possible permutations. In another study, from 1996, English further elaborated on these strategies, referring to them as stages: 1) the random stage, 2) the transitional stage, and 3) the odometer stage. In the random stage, children use trial and error, and checking becomes important to succeeding in a task. At the transition stage, children start to adopt patterns in their documentations, but the patterns are not kept throughout the tasks. Instead, the children revert to the random stage. The transition stage, however, indicates knowledge of the principle of systematic variation and the principle of constancy. At the odometer stage, the children use an organized structure for the selection of permutations, where one item is held constant while the others are varied systematically. This stage indicates knowledge also of the principles of exhaustion and completion.

The results from English have been confirmed in newer studies on combinatorics in early mathematics (see, for example, Palmér & van Bommel, 2018; Wathne & Carlsen, 2022). In connection to the problem-solving task on combinatorics used in this study, the systematization of the permutations as well as the use of representations has been shown to be important (Palmér & van Bommel, 2018, 2023b). In Palmér and van Bommel's studies (2018, 2023b), students who used pictographic representations made fewer duplications, even in the random stage, compared to students using iconic representations. However, among students in the transitional and odometer stages, more students used iconic representations. The students who used pictographic representations made fewer duplications in their solutions than students who used iconic representations, which may be due to the time it takes to draw bears, or because the context of the task became clearer when drawing bears compared to when drawing lines

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or dots. Sometimes, the students also started to draw bears but changed to lines or dots after one or two permutations.

Theoretical and methodological framework

This study is part of a larger project based on the multimodal theoretical perspective *Designs for learning* (Selander, 2021; Selander & Kress, 2010). Previously, using this multimodal theoretical perspective, the project has focused on six-year-olds' meaning making in chemistry (Patron et al., 2024) and literacy (Wernholm et al., 2023). According to this perspective, a teacher designs learning activities that provide students with access to various learning resources. Learning resources can include artifacts in the form of pens and tablets, as well as modes, such as writing, speech, physical actions, images, gestures, etcetera. Based on their prior knowledge and experiences, students transform the subject matter by using such learning resources. For instance, when working on the problem-solving task on combinatorics, modes such as spoken language, gestures, and images are combined. The choices of modes imply different potentials for meaning making depending on the specific context, including the participants involved (Caiman & Kjällander, 2024; Selander, 2021). For example, when teaching combinatorics, teachers are likely to choose different words and visualization methods depending on whether the instruction is aimed at six-year-olds in a preschool class or at sixteen-year-old high school students.

Within *Designs for learning* (Selander, 2021; Selander & Kress, 2010), the learning-design sequence model has been developed (Figure 1). This model is particularly valuable in studies that aim to design and analyze teaching encompassing multiple modalities as it allows researchers to consider how each semiotic mode contributes to meaning making (Selander, 2024). The model is used as both a methodological and analytical tool in this study. Below, the different parts of the model are explained in relation to the study, following a description of participant selection and ethical considerations.

Participants and ethics

The study was conducted in two Swedish preschool classes (six-year-olds) with 25 students. The limited number of classes is due to the exploratory nature of the study, where smaller case studies are recommended prior to scaling up (Yin, 2018). The teachers in these classes have collaborated with researchers for several years in a longitudinal study. Due to that collaboration, the teachers are well acquainted with the problem-solving task on combinatorics, having previously conducted it with several groups of

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students (Palmér & van Bommel, 2021). What was new for the teachers in this study was the addition of students creating digital animations. Following Swedish ethical research guidelines, the students' caregivers gave informed written consent for their children's participation in the study. Data was selected only from children with this consent (Swedish Research Council, 2024). The team of researchers and teachers has extensive experience with research involving young students. To further ensure that students' participation in the study was based on informed consent, their verbal and non-verbal expressions (for example, facial expressions such as smiling or furrowing their brows, as well as body language such as nodding or shaking their heads) were carefully observed during the classroom implementation. The teachers together with one researcher attended the classroom during lessons, but the teachers led the lessons. Before each lesson, it was emphasized for the students that participation was voluntary and that alternatives were available for those who preferred not to take part. Moreover, efforts were made to ensure that collaboration within groups proceeded without conflict. However, no student who had provided written consent indicated any reluctance to participate; on the contrary, they expressed positive attitudes both prior to and during the evaluation of the lessons.

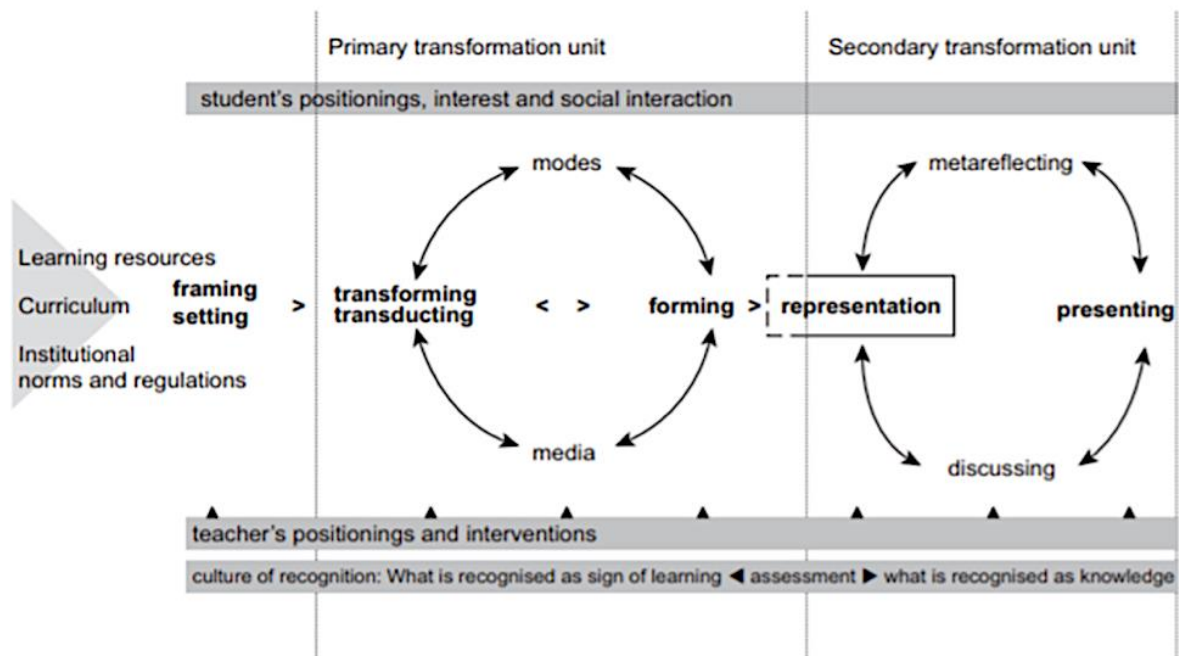


FIGURE 1 Learning Design Sequence Model (Björklund Boistrup & Selander, 2022, p. 4). (Picture used with authors' permission)

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The design of the lessons

The implementation of the problem-solving task on combinatorics was divided into two lessons. The first lesson started with the teacher showing the students three plastic toy bears in three different colors (setting). The teacher told the story about the dilemma of choosing seats on the sofa (see introduction section). Then, the students were divided into pairs to work together to solve the task during the primary transformation unit. Using the hands-on material (plastic toy bears), they were expected to create representations using paper and colored pens, thus transforming the content from one mode (plastic toy bears) to another (static visual representation on paper). This type of transformation between different modes is called transduction in the learning-design sequence (Caiman & Kjällander, 2024; Selander, 2021). During this primary transformation unit, students chose whether and how they would use the available resources. The first lesson ended with the teacher gathering all the students together, and the student pairs presented their documentations. During this secondary transformation unit, the focus was on the representations created during the first transformation unit. An essential aspect of the theoretical perspective Designs for learning is which expressions are recognized as signs of learning (Selander, 2021). Questions discussed with the students included the similarities and differences in the students' ways of documenting, how many different solutions (permutations) they had found, how the students had structured their solutions, and whether and how they knew if they had found all permutations (referred to as combinations in the instruction).

The second lesson was conducted the following day, with students working in the same pairs as in the first lesson. The students were given their drawn documentations from the day before, and the complete set of permutations was still visible on the whiteboard in the classroom (setting). In the primary transformation unit, the students were told to create a digital animation of the solution they had produced the previous day. In other words, they were asked to redesign their documentations into a new representation in another semiotic mode (i.e., transduction). All students were familiar with the application and knew how to handle the characters that were included in the animation. However, they had not previously used the application in connection with mathematics. In the secondary transformation unit, the students together with the teachers and researcher watched the animations, and the students were given opportunities to discuss their redesigned representations.

The empirical material for this study consists of video documentation from the two lessons. In line with the purpose of the study, a critical incident technique (Butterfield et al., 2005) was used to identify sequences in the video documentation where signs of

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learning were observed. The critical incident technique involved analyzing episodes where qualitative changes in students' reasoning on mathematical content (i.e. signs of learning), became visible. In the first lesson, such episodes included 1) the representations used (e.g. whether the students used pictographic and/or iconic representations), 2) the systematization of permutations (e.g. systematic variation, constancy, exhaustion, or completion visible in the documentations), and 3) students' reasoning on whether or not they had found all permutations (e.g. utterances where children negotiate whether they have finished or not). Then, these episodes were compared with the same students' reasoning when creating digital animations in the second lesson, identifying indications of meaning making. Episodes during the second lesson included students' reasoning on 1) their documentations from the first day, and 2) whether or not they had included all permutations in their animations. Also, this analysis focused on how the documentations from the first lesson became a resource the students could use in their work as well as in their collaboration. Finally, by comparing students' reasoning on the mathematical content on combinatorics (for example, knowing whether or not they had found all permutations) when working with analog and digital representations respectively, we could analyze whether – and if so, how – students' creation of analog and digital representations, in combination, contributed to their learning of mathematical content when working on problem solving.

Results

The results are divided according to the two lessons, where each lesson is viewed as a separate learning-design sequence. The question explored is whether – and if so, how – students' creation of analog and digital representations, in combination, may contribute to their learning of combinatorics when working on problem solving.

First learning-design sequence: Creating analog representations

The focus of the results of the first learning-design sequence is on the representations used, the systematization of permutations, and students' reasoning on whether or not they had found all permutations.

Primary transformation unit

There were 11 pairs and one group with three students (to enhance readability, these are referred to as 12 pairs in the results). Thus, a total of 25 students worked on the task. The collaborations were decided by the teacher. During the primary transformation unit, the

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students worked on the task for approximately 20–30 minutes. When working on the task, the students most often took turns drawing the permutations. At no point did the teacher guide the students by suggesting strategies or possible ways to represent solutions; instead, they inquired whether there were other ways the toy bears could sit on the sofa. The students could choose how to document the different permutations.

The students created paper-based representations of permutations. Some students drew bears (pictographic representation), while others represented the toy bears with dots or lines in the same colors as the toy bears (iconic representation). Of the 12 pairs, seven used pictographic representations, four used iconic representations, and one pair used both pictographic and iconic representations (see examples in Figure 2). Three of the documentations had duplicates (2 pictographic, 1 iconic, for example, see Figure 3). One of these pairs had made eight permutations and the other two pairs had made six permutations. Of the documentations with unique permutations, one documentation had two permutations (pictographic), four had three permutations (3 pictographic, 1 iconic), one had four permutations (pictographic), one had five permutations (iconic), and two had six permutations (1 iconic, 1 both).

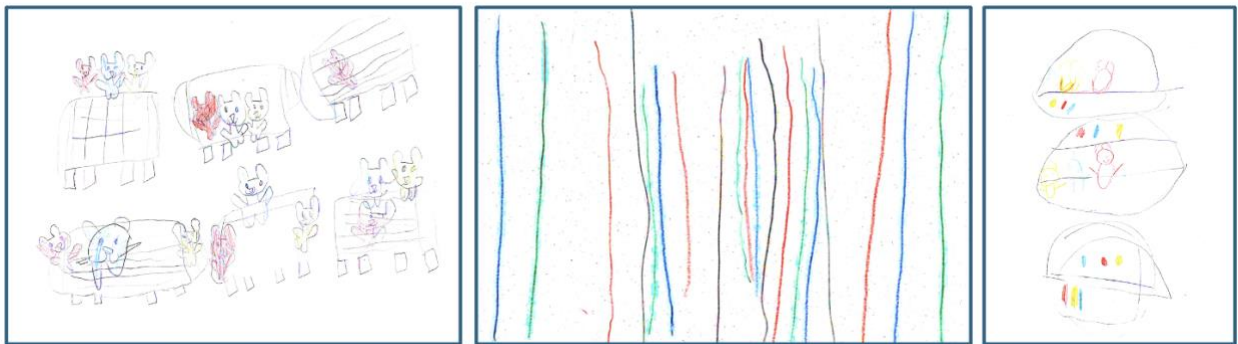


FIGURE 2 Examples of documentations with pictographic representation, iconic representation, and pictographic *and* iconic representation



FIGURE 3 Example of documentation with duplicate permutations

In students' work, systematization was most visible in connection to the pairs that found three unique permutations, that is, the bears were seated once in each seat on the sofa. This was often conducted quite quickly. For example, in one pair, a girl orally presented three different permutations after 38 seconds of work. Among the pairs who found four or more unique permutations, systematization was only visible twice and then once by chance. One example is provided below.

One pair who found six permutations did this by chance. After finding three permutations, one girl playfully moved around the colored pens. When one of the pens fell to the floor and was put back on the table, she discovered an additional solution, a fourth permutation. This led the pair to develop a systematic strategy using the pens to find the remaining permutations. Using the pens, they discovered a systematic way to find all six permutations, starting at a transition stage but ending in the odometer stage (English, 1996). Besides this pair, only one other pair worked systematically to find all six permutations, and they did so without using any other representations than paper and pen; this is the pair presented in the first example in the second learning-design sequence section below.

Secondary transformation unit

The teacher and students came together in a circle on the floor with all the students' documentations on paper. The teacher had one set of plastic toy bears as a learning resource. The teacher-led discussion mainly revolved around the various numbers of permutations in the students' documentations, how they had chosen to represent the bears, and how the collaboration had progressed. The students expressed that the work had been exciting and challenging and that finding the first three permutations was easy, but then it became trickier. For example: *"The task was interesting, but it took time. You get three quickly, but it takes a while because you need to think."* Another pair described a strategy: *"We think it becomes six different [ways], but we have drawn boxes instead of toy*

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bears.” Another example from the secondary transformation unit was a boy who responded, “*We have made sofas in six different ways: one, two, three, four, five, and six.*” He continued by saying that the task was challenging, pointing out that it took a while before they got the fourth permutation. When no one else during the joint discussion could find the last permutation, he said:

I have another one, yellow, red, and blue. Look, everything is in twos (he points at the six permutations and shows that each bear can be in the same spot twice), so this is done in different ways.

The teacher concluded that each toy bear could be placed twice in the same spot. The pair with eight permutations now realized they had duplicates, as the teacher explicitly asked them to consider why they had eight solutions. One boy then sat for almost a minute, looking at their documentation, and then said, “*Two are the same.*” The teacher ended the lesson by reviewing the six permutations, illustrating them on the board and with the plastic toy bears. The students who had not documented six permutations added to their documentations any permutations they had missed before. The two pairs with six permutations but with duplicates had not noticed their duplications, and thus, they thought they had found the correct solution. Further, the way of knowing when all permutations had been found was not elaborated on. When the teacher asked the whole group what they thought of the task, everyone but the boy quoted above said it was a good lesson. The boy quoted above answered, “*It was not good. How are we then supposed to know [if we have found all permutations]?*”

To summarize, in this first learning-design sequence, the teachers recognized all representations from the students as valid knowledge representations, regardless of whether the students had drawn bears, lines, circles, or something else. However, strategies and ways of systematization did not become the focus of the lesson, even though one boy in the end showed a strategy of “twos.” The six possible permutations were shown using different representations, but there was no elaboration on how to determine whether all permutations had been found.

Second learning-design sequence: Creating digital animations

The focus of the results of the second learning-design sequence is on students’ reasoning on their documentations from the first day and on whether or not they had included all permutations in their animations.

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Primary transformation unit

The purpose of the second learning-design sequence was for the students to redesign their representations of permutations from the previous day. Redesign should be understood as adopting a new mode; it may result in a different solution, but this is not a necessity. Creating a digital animation on a tablet application allowed the students to use movements to illustrate the permutations along with a recorded verbal explanation. The students had their drawn documentations from the day before, and the complete set of solutions was visible on the whiteboard in the classroom. All students created a digital animation, redesigning the solutions they had come up with the day before. Three main observations were made during this activity. The first concerns how all pairs used their documentations from the previous day in a systematic way. Thus, their documentations became a resource guiding their work. Second, the tablet became a shared space for collaboration, which was not observed in the first learning-design sequence when they took turns working with paper and pen. What was significant with most student pairs was that the division of work changed when the students were using the digital tool, and some students who had been inactive during the analog activity became more active. In contrast, none of the students who had previously been active became inactive. Third, creating a digital animation gave the students an opportunity to repeat the mathematical content, and in their recorded verbal explanations, all pairs included reasoning on the permutations. Even the two pairs that had six permutations but with duplicates from the first lesson now paid attention to their duplications. Together, these three observations indicate that creating a digital representation may contribute to students' learning of combinatorics. Two examples of the second learning-design sequence serve as illustrations below.

In the first example, a boy and girl collaborated. The first lesson they had during pair work documented six permutations. The researcher started by asking what task they had solved the day before. The students were instructed to tell a fairy tale about the bears, and they were to start with the animation. The boy leaned back in his chair, held up the paper, and said, *"Once upon a time,"* prompting the girl to continue with the blue bear sitting first. The boy asked the girl, *"Should I tell the whole story?"* and she nodded. She took charge of the tablet, indicating that she would move the bears on the tablet. The boy resumed, *"Once upon a time, three teddy bears lived in a house deep in the forest. One day, they wanted to watch something on TV but could not agree on how to sit on the sofa. So, they had to think."* He continued to verbally tell a story involving all six permutations, sometimes pausing to make sure the way the girl moved the bears on the tablet was correct. Thus, he kept track of the narrative, the mathematical content, and the work on the tablet.

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In the second example, two girls were collaborating. During the first lesson, they had first identified three permutations quite quickly and then the other three during whole-class discussion. The researcher prompted them to advise another group on how to solve the problem. Then, one of the girls explained how she had used the pens the day before: *“If you put the yellow one in the middle, you put the blue and red one there, and if you put the yellow one here, you put the others on each side.”* She fetched pens from another table. She then thoroughly explained how she had used them when solving the task the day before. The other girl paid attention to the explanation, and then the two girls were to create the digital animation. The girl observing remained silent while the girl who had explained the strategy tried to solve the task by herself. However, she could not simultaneously explain and document in the application, and she said, *“Will you move the figures, and I’ll talk?”* The reason for handing over the tablet seemed to be that she needed to move the pens while telling the story. As such (Figure 4), the girls used three modes of representation: the iconic representation on paper, the digital animation, and the pens.



FIGURE 4 Three modes of representation

Secondary transformation unit

In the secondary transformation unit, the students were given opportunities to view and discuss their animations. The pairs reacted in different ways after watching the animations. For example, one pair wanted to start over, making a new animation, as they found faults in their “story”. Another pair concluded that they had found all six permutations. Another pair commented on the potential of the application to use moveable pictures and sound. Thus, creating a digital representation offered students an opportunity to engage both in the content of combinatorics and in the digital application itself.

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To summarize, creating a digital animation gave the students an opportunity to repeat the mathematical content and to systematically explore the permutations in action. In their recorded verbal explanations, all pairs included reasoning on the permutations. That is, creating the digital animation enabled the students to explore combinatorics in a systematic way, which not all pairs had done during the first learning-design sequence. However, this would not have been possible without the first learning-design sequence, particularly the secondary transformation unit in that learning-design sequence, which laid the groundwork for the second learning-design sequence.

Discussion

In this study, the focus is on whether – and if so, how – students’ creation of analog and digital representations, in combination, may contribute to their learning of combinatorics while working on problem solving. The task was shown to be a problem-solving task for the students in the sense that it was a non-routine task that required the students to select and apply new strategies in order to reach a solution (Gavaz et al., 2021).

Based on previous studies with same-aged children working on this combinatorics task (see Palmér & van Bommel 2018, 2023b), the addition of creating a digital animation allowed more time to be spent on the task and provided opportunities to revisit and consolidate the mathematical content. Creating a digital animation gave students the opportunity to repeat and refine their understanding of the mathematical content, and in all animations, students included reasoning on the permutations. Further, the creation of the digital animation enabled students to use movements to illustrate permutations alongside a recorded verbal explanation, which, according to Ainsworth et al. (2002) and Pedersen et al. (2021), may enhance mathematical understanding. Furthermore, the recorded verbal explanations offered examples of students justifying their solutions and considering alternative strategies, supporting the connection between procedural skills and broader conceptual understanding (Vessonen et al., 2024). Thus, in line with previous studies by Kynigos and Karavakou (2023) and Baccaglioni-Frank et al. (2025), this study demonstrated how the creation of digital animations enabled students to perceive and reason about mathematical content in ways that transcend static representations. Also, it is possible that the digital animation, consistent with previous studies, contributed to students’ motivation and engagement (Fadda et al., 2022; Saccardo et al., 2024; Zourmpakis et al., 2024).

In the first lesson, some students drew bears (pictographic representations), while others represented the toy bears using dots or lines with the same colors as the toy bears (iconic

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representations). As such, all groups transduced the content from one mode (plastic toy bears) into another (pictographic or iconic representations) (Caiman & Kjällander, 2024; Selander & Kress, 2010). This is similar to previous studies on young students working on this task (Palmér & Van Bommel, 2018, 2023b). The teachers recognized all representations created by the students as valid knowledge representations, regardless of whether the students had drawn bears, lines, circles, or something else. Allowing young students to work with the mathematical representation they themselves choose and master has been shown to be important in connection with problem solving (Palmér & van Bommel, 2023b).

When working on the task in the first lesson, the level of systematizing among the students differed from the random to the odometer stage (English, 1996), which is also similar to previous studies on young students working on this task (Palmér & van Bommel, 2018, 2023b). Thus, there were students who showed understanding of all four principles necessary for working successfully with combinatorics (systematic variation, constancy, exhaustion, completion) while other students showed understanding of only one or two of these principles. To make all principles visible to the students, a desirable development of the first learning-design sequence would be for the teacher, in the concluding joint discussion on permutations, to also focus on the strategies used by the students.

In the second lesson, when the students created digital animations, they redesigned their final solutions from the day before. When creating the digital animation, all pairs included reasoning on the permutations. Thus, all students more or less touched upon systematic variation, constancy, exhaustion, and completion (English, 1996). This did not become evident in previous studies on this task where digital animations were not used (Palmér & van Bommel, 2018, 2023b). As such, the addition of digital representation in the form of animations provided opportunities for all students to work systematically on permutations. However, the creation of the digital animation would not have been possible without the analog representation. Hence, this is an example of how analog representations can add value to the use and creation of digital representations, and vice versa. According to Goldin (2020) and Duval (2006, 2017), the ability to interpret and make connections within and between representations strengthens the learning of mathematical content.

The sequence of lessons explored in this study exemplifies how digital animations can illuminate temporal and transformational aspects of mathematics that static representations cannot (Arzarello & Robutti, 2010; Pedersen et al., 2021). The digital animations in this study functioned as semiotic mediators where movement and verbal

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explanation were then connected to formal mathematical representations of permutations. The hybrid approach in this study allowed students to construct and manipulate dynamic representations, verbalize their reasoning, and receive feedback through peer and teacher interactions. In line with Otterborn et al. (2024), these results demonstrate that analog and digital representations are complementary; they can extend and deepen learning by integrating multiple dimensions of understanding, including conceptual, procedural, and social aspects.

The results presented in this article can be described as enhancement (Hughes et al., 2006), referring to how a digital tool, in this case the digital animation, provided increased learning opportunities without altering the content or form of the knowledge that students were expected to develop. However, the second learning-design sequence in the second lesson did not involve a whole-class discussion. Thus, we conclude that this part of the lesson design could potentially be expanded, with the whole class viewing the animations together and discussing systematic variation, constancy, exhaustion, and completion. Regardless, we view the creation of digital animation not primarily as a resource for problem solving but as a means for students to redesign their knowledge representations, thereby consolidating their understanding through further engagement with the content. Furthermore, digital tools, including animated representations, may contribute to the cultivation of mathematical digital competencies, that is, the ability to flexibly use digital resources for mathematical inquiry and communication (Geraniou & Jankvist, 2019) as well as motivation and engagement in mathematics (Fadda et al., 2022; Saccardo et al., 2024; Zourmpakis et al., 2024).

Limitations

A limitation of this study is that it included only a small number of classes and students, and therefore studies with more students are needed to make more general conclusions. It should also be noted that the teachers in the study were highly experienced, and results may not be similar where teachers have less experience with problem solving or combinatorics.

Conclusion

Based on our results, we propose that creating analog and digital representations, in combination, may contribute to young students' learning of combinatorics. When working with analog representations, some students showed understanding of systematic

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variation, constancy, exhaustion, and completion (the four principles students need to understand to work successfully with combinatorics), while others did not. The creation of a digital animation gave the students an opportunity to repeat the mathematical content, and in the recorded verbal explanations, all pairs included reasoning on the permutations, reasoning that more or less touched upon systematic variation, constancy, exhaustion, and completion. Based on the results, it can be concluded that the creation of analog and digital representations, in combination, contributed to students' explorations of combinatorics and that, when integrated into a learning-design sequence, digital animations enhanced students' understanding of combinatorics.

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